# Discussion of *Crashes and Collateralized Lending* by Jakub Jurek and Erik Stafford

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Do we need to be using such an involved description?

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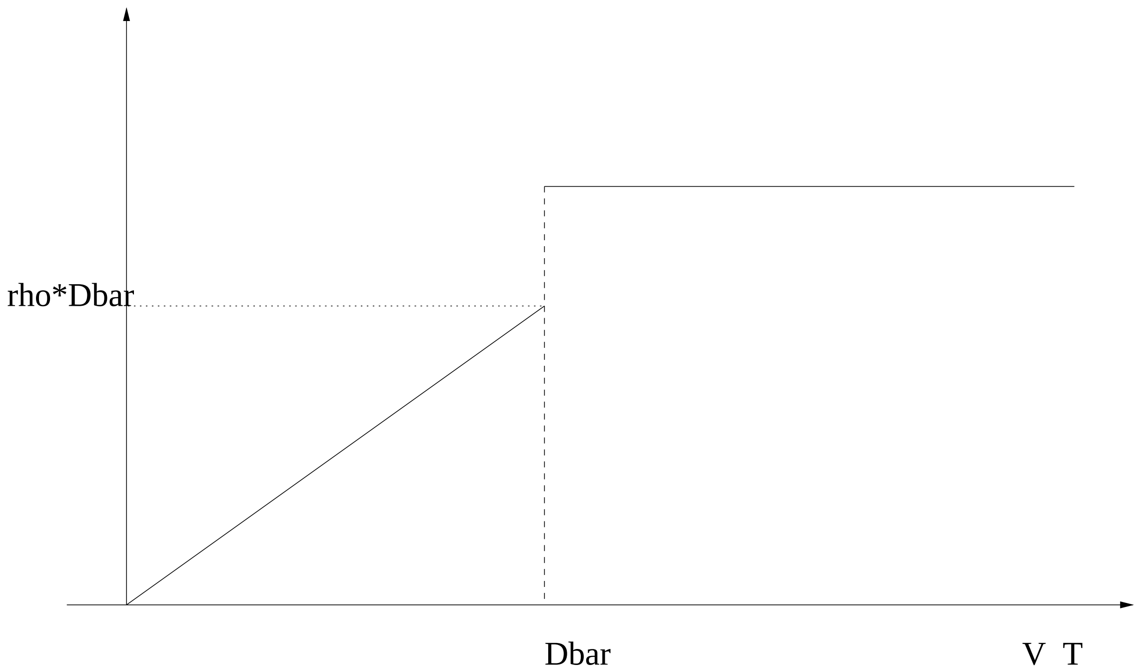

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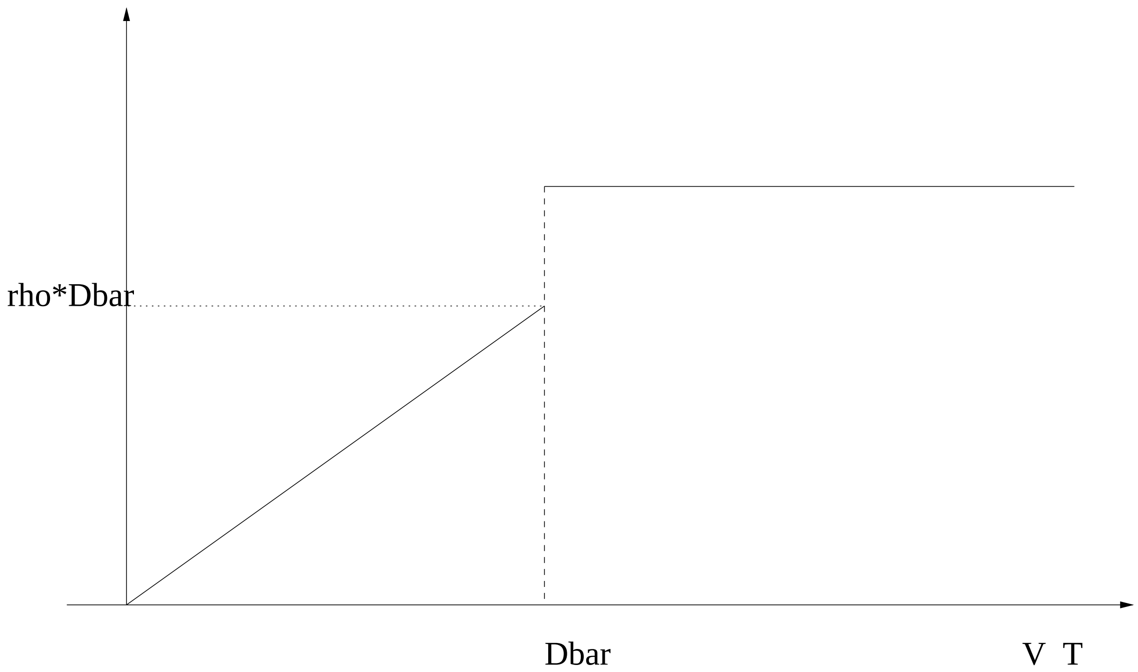


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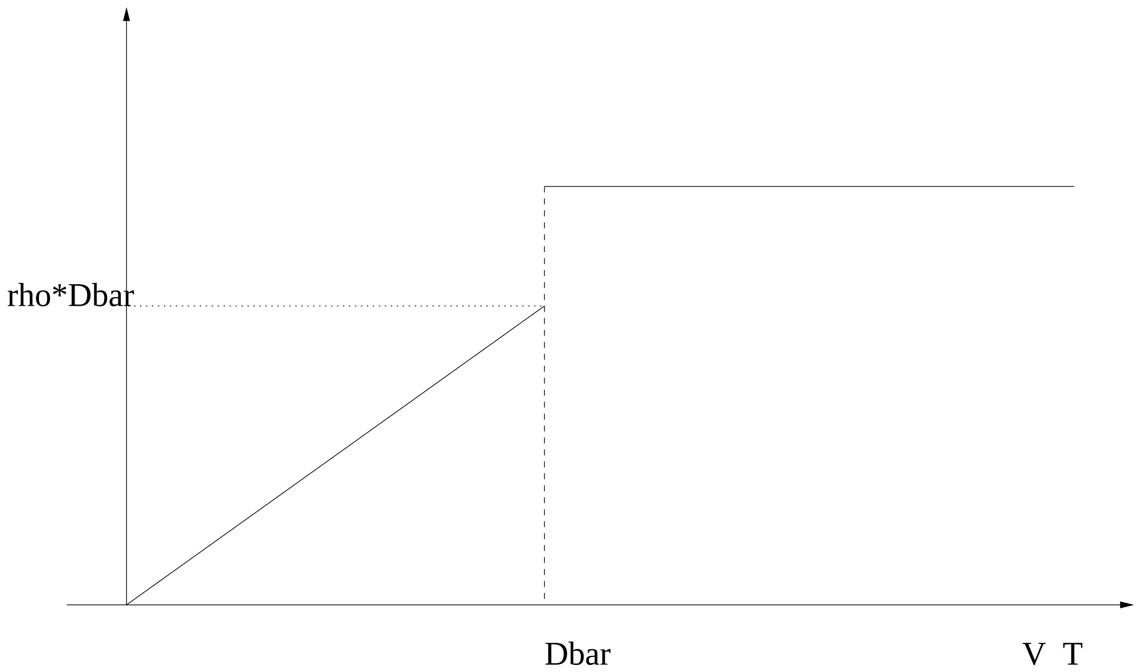


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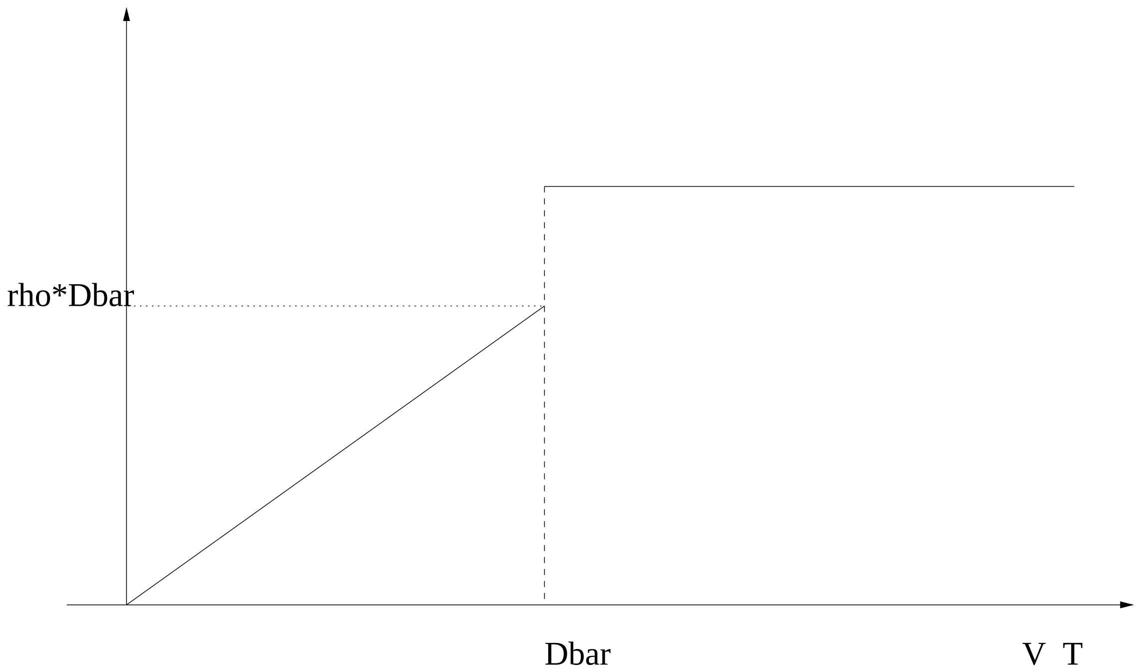


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because $C'(K) = -e^{-rT}\,\mathbb{P}(K < V_T)$

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Thus time-0 value of debt is

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which has to be equated to $D_0$ and solved for $s$.

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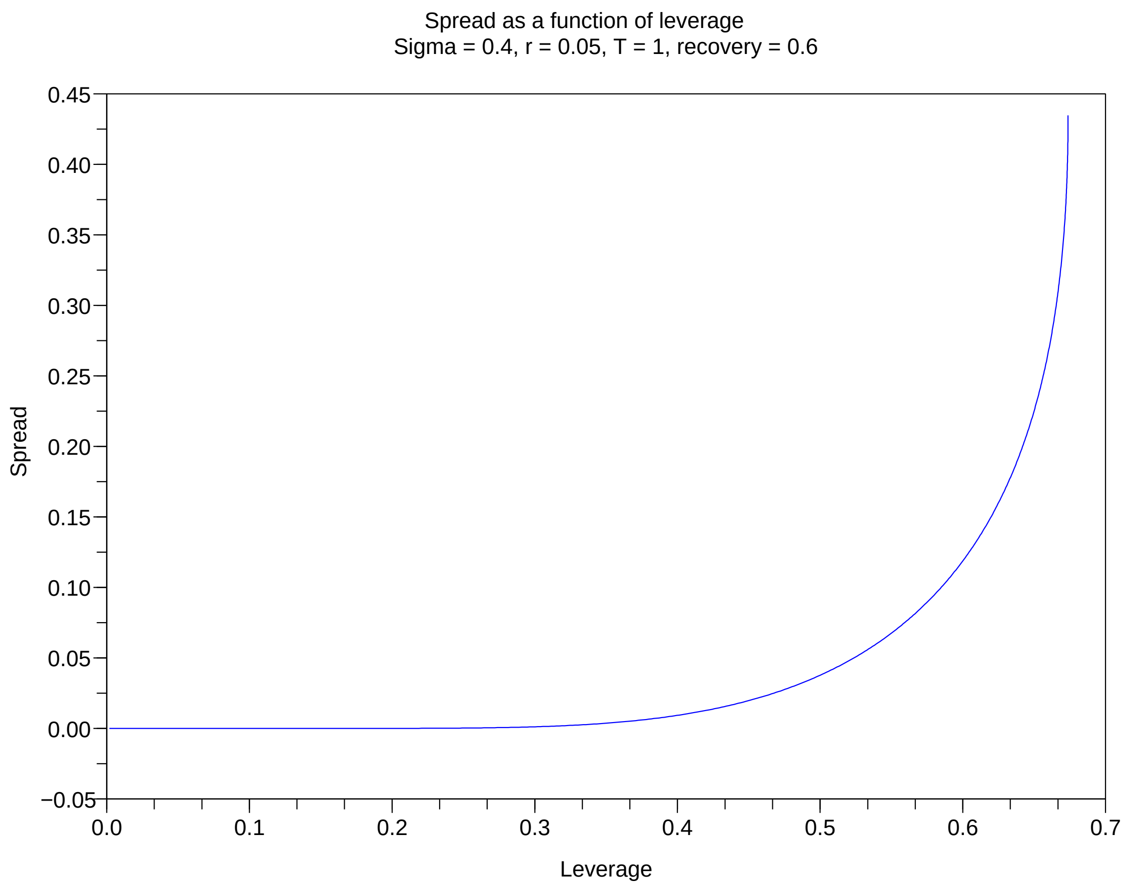

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None of this requires any distributional assumptions about $V$.

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$$\Psi(w) \equiv \mathbb{E}\big[\ \rho V_T - \rho(V_T - \bar{D})^+ + (1-\rho)\bar{D}\, I_{\{V_T \geq \bar{D}\}}\ \big|\ \bar{W}_T = w\ \big].$$

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A CDO tranche with attachment points $0 < a < b < 1$ will deliver

$$\frac{1}{b-a}\left\{(\frac{\Psi(w_T)}{\bar{D}} - (1-b))^+ \wedge (b-a)\right\}$$

at time $T$ .

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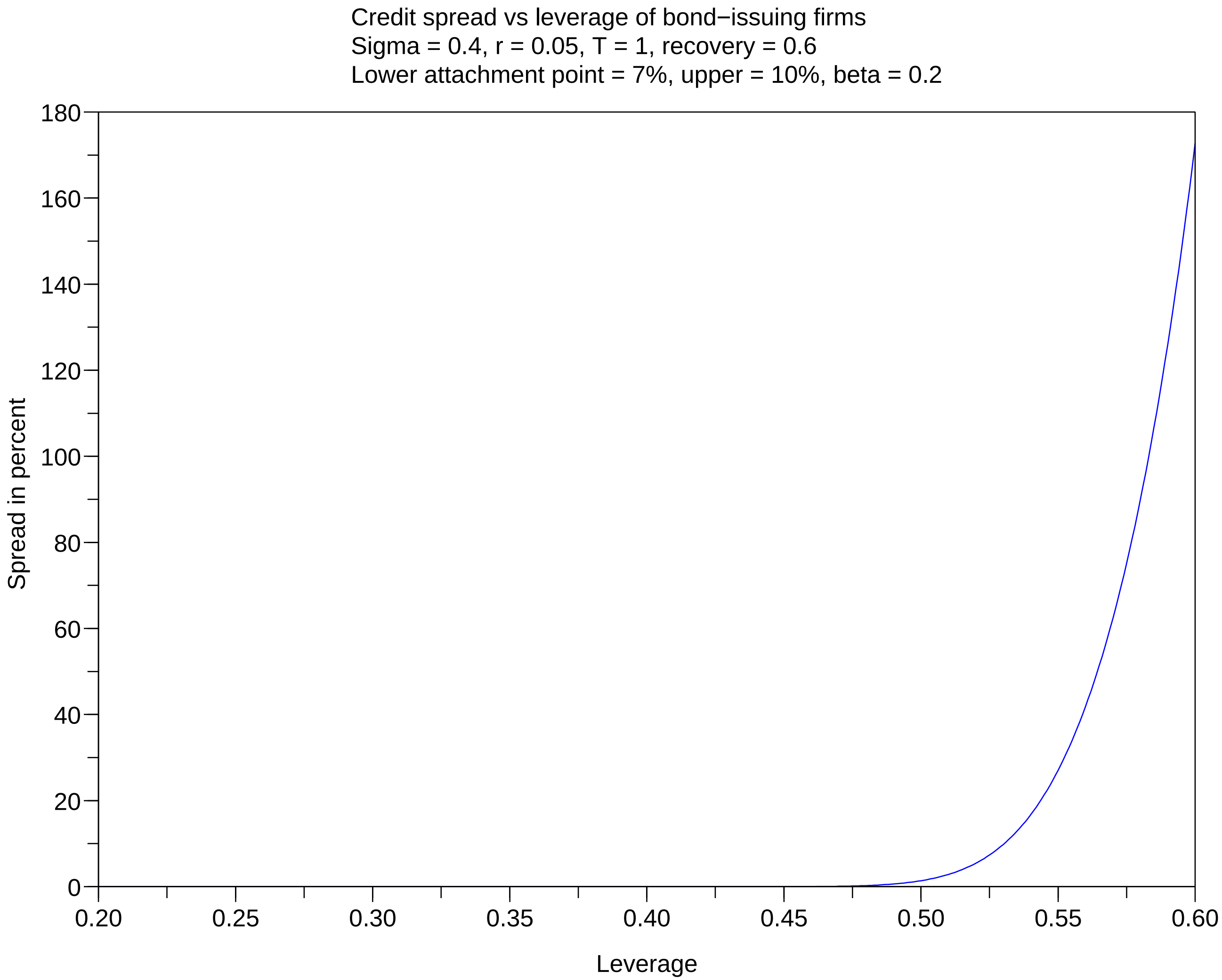

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