The Leverage Ratio, Risk-Taking and Bank Stability

Michael Grill, Jan Hannes Lang, Jonathan Smith*

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Abstract

Under the new Basel III banking regulations, a non-risk based leverage ratio will be introduced alongside the risk-based capital requirement. This move away from a solely risk-based capital framework has raised some concern of increased bank risk-taking; potentially offsetting any benefits from requiring highly leveraged banks to hold more capital. We address exactly this trade-off between additional loss-absorbing capacity and higher bank risk-taking associated with a leverage ratio requirement in both a theoretical and empirical setting. Using a theoretical micro model, we show that a leverage ratio requirement can indeed incentivise bound banks to slightly increase their risk-taking, but this increase in risk-taking should be more than outweighed by the increase in loss-absorbing capacity from higher capital, thus leading to more stable banks. These theoretical predictions are then tested and confirmed in an empirical analysis on a large sample of EU banks. Our baseline empirical model suggests that a leverage ratio requirement would lead to a significant decline in the failure probability of highly leveraged banks.

*Contact: Michael Grill, European Central Bank, michael.grill@ecb.europa.eu; Jan Hannes Lang, European Central Bank, jan-hannes.lang@ecb.europa.eu; Jonathan Smith, University of Cambridge, jejs3@cam.ac.uk. We would like to thank Sylvain Benoit, Petra Geraats, Mike Mariathasan, Barbara Meller, Alexander Popov, Gabriel Jimenez and Nicolas-Kirsi Scholtes for their comments and useful discussions. We are also grateful to participants at the 4th EBA Policy Research Workshop, the ECB 2nd Workshop of the Empirical Macro Workstream of the OMR Task Force, PTLR meetings at the Bank of Portugal and EBA, the 5th Paris Economix PhD Student Conference in International Macroeconomics and Financial Econometrics, the 2016 Scottish Economic Society Annual Conference, the Belgian Financial Research Forum and at seminars held at the European Central Bank, University of Cambridge, Bank of England, Riksbank and Bank of Mexico. Disclaimer: The views expressed in this paper are those of the authors and do not necessarily reflect those of the European Central Bank or the Eurosystem. All results are derived from publicly available information and do not imply any policy conclusions regarding individual banks.
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Contents

Non-Technical Summary i

1 Introduction 1

2 The Basel III Leverage Ratio Requirement 6

3 Theoretical model 7

3.1 The set-up of the model environment 7

3.2 The bank’s decision problem . . . . . . . . . . . . . . . . . . . . . . . 12

3.3 Main theoretical results . . . . . . . . . . . . . . . . . . . . . . . . . 13

3.3.1 Risk-taking under a risk-based capital requirement . . . . . . 13

3.3.2 Risk-taking with a leverage ratio requirement . . . . . . . . 14

3.3.3 Risk-taking vs. loss absorbing capacity . . . . . . . . . . . . . 18

4 Empirical analysis 20

4.1 Dataset . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 21

4.2 Effect of a leverage ratio constraint on bank risk-taking . . . . . . 21

4.3 Trade-off between loss-absorption and risk-taking . . . . . . . . . 31

4.4 Net effect of a leverage ratio constraint on bank stability . . . . . . 35

5 Conclusion 37

References 38

Appendix A: Alternative assumption 42

Appendix B: Proofs 51
Non-Technical Summary

As a response to the global financial crisis, the Basel Committee on Banking Supervision (BCBS) decided to undertake a major reform to the regulatory framework of the banking system. Under the new Basel III banking regulations, a non-risk based leverage ratio requirement (LRR) will be introduced alongside the risk-based capital framework with the aim to “restrict the build-up of excessive leverage in the banking sector to avoid destabilising deleveraging processes that can damage the broader financial system and the economy.” BCBS (2014a) The leverage ratio is a non-risk based capital measure and is defined as Tier 1 capital over a bank’s total exposure measure, which consists of on-balance sheet items as well as off-balance sheet items. It is widely expected that the LRR will become a Pillar I requirement for banks under Basel III, ever since the BCBS issued a consultative document that outlined a baseline proposal for the design of the LRR in December 2009.2

Nevertheless, the LRR has been subject to various criticism raised by market participants and other stakeholders. The main concern relates to the risk-insensitivity of the LRR: assets with the same nominal value but of different riskiness are treated equally and face the same capital requirement under the non-risk based LRR.3 Given that an LRR has a skewed impact, binding only for those banks with a large share of low risk-weighted assets on their balance sheets, this move away from a solely risk-based capital requirement may induce these banks to increase their risk-taking; potentially offsetting any benefits from requiring them to hold more capital. This paper addresses exactly this trade-off between additional loss-absorbing capacity and higher bank risk-taking associated with an LRR in both a theoretical and empirical setting.

First, we build a simple theoretical model that is able to capture the trade-off between risk-taking and higher loss-absorption associated with an LRR. The model yields two key results. First, if equity is sufficiently costly, imposing an LRR indeed always incentivises banks that are bound by it to modestly increase risk-taking. This occurs because the non-risk based nature of the LRR effectively reduces the marginal cost of risk-taking. Under an LRR bound banks are no longer forced to hold additional capital when they take greater risk, and greater risk is associated with a greater expected return. If capital is expensive, under a risk-based framework

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1 The Basel III regulations also include a strengthened risk-based capital framework and two new liquidity requirements, i.e. the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). See BCBS (2014a) for further details.
2 See BCBS (2009).
3 See for example ESRB (2015).
this incentivises banks to reduce their risk-taking as adding capital contributes to marginal costs. Under a binding LRR, this marginal cost disappears and hence banks increase their risk-taking since they can now increase risk and return without the penalty of having to hold greater capital.

Nevertheless, this increase in risk-taking is not unbounded. On the one hand, the risk-based capital framework underlies the LRR, such that if the bank takes too much additional risk it will simply move back into the risk-based capital framework. On the other hand, there exists an offsetting effect on risk-taking incentives from the fact that banks are required to hold greater capital, as this to some extent makes them more cautious (banks have more “skin in the game”). Consequently, the second key result from the model suggests that imposing an LRR should be beneficial for bank stability as the additional loss-absorbing capacity of banks dominates the increase in risk-taking. In particular, the model suggests that adding an LRR to the risk-based capital framework will both weakly decrease banks’ probability of failure, and if the distribution of banks is not such that the majority of banks are concentrated around the LRR minimum, which is arguably the case in reality, will strictly decrease expected losses for all parameter values, if the LRR is not set excessively high.

The theoretical banking model that we develop therefore yields two testable hypotheses. First, if equity is costly, the introduction of an LRR should incentivise banks for which it is a binding constraint to modestly increase risk-taking. Second, the negative impact of increased risk-taking induced by an LRR should be outweighed by the beneficial impact of increased loss-absorbing capacity, resulting in more stable banks. We take these two hypotheses and test them empirically on a large dataset of EU banks that encompasses a unique collection of bank distress events.

The empirical analysis follows in three steps. We first test whether banks with low leverage ratios started to increase their risk-taking and capital positions after the announcement of the Basel III leverage ratio regime at the end of 2009 using a difference-in-difference type approach. We then estimate the joint effects of the leverage ratio and risk-taking on bank distress probabilities in a logit model framework, in order to quantify the risk-stability trade-off associated with an LRR. Finally, we combine the first and second stage empirical results into a counterfactual simulation to test whether the negative impact of the estimated increase in risk-taking is outweighed by the benefits of increasing loss-absorbing capacity, i.e. whether an LRR is beneficial for bank stability.

The empirical evidence provided in the paper lends support to both hypotheses.
Our estimates suggest that banks bound by the LRR increase their risk-weighted assets to total assets ratio by around 1.5 - 2.5 percentage points more than they otherwise would without an LRR. Importantly, this small increase in risk-taking is more than compensated for by the substantial increase in capital positions for highly leveraged banks, which results in significantly lower estimated distress probabilities for banks bound by the LRR.

The theoretical and empirical results of our paper therefore support the introduction of an LRR alongside the risk-based capital framework for banks. The analysis further suggests that the LRR and the risk-based capital framework reinforce each other by covering risks which the other is less able to capture, making sure banks do not operate with excessive leverage and at the same time, have sufficient incentives for keeping risk-taking in check.
1 Introduction

Excessive leverage has been identified as a key driver of the recent financial crisis and of many past crises.\(^4\) Moreover, in the recent crisis a significant number of banks were found to have built up excessive leverage while apparently maintaining strong risk-based capital ratios (BCBS, 2014a). As a response, the Basel Committee on Banking Supervision (BCBS) decided to introduce into the Basel III regulatory framework, a non-risk based leverage ratio requirement (LRR) alongside the risk-based capital requirement. This was done to help contain the build-up of excessive leverage and to increase the stability of the banking system.\(^5\) The leverage ratio is a non-risk based capital measure and is defined as Tier 1 capital over a bank’s total exposure measure, which consists of on-balance sheet items as well as off-balance sheet items. The main idea is that imposing a cap on leverage will improve the loss absorbing capacity of highly leveraged banks and therefore reduce their failure probability, ultimately reducing the likelihood of a repeat crisis.

Nevertheless, the LRR has been subject to various criticism raised by market participants and other stakeholders.\(^6\) The main concern relates to its risk-insensitivity: as a non-risk based measure, assets of the same nominal value but of different riskiness are treated equally and face the same capital requirement. This has raised some anxiety that a move away from a solely risk-based capital framework will simply lead banks constrained by the LRR to increase their risk-taking; potentially offsetting any benefits from holding higher capital.\(^7\)

This paper addresses exactly this trade-off between additional loss-absorbing capacity and higher bank risk-taking associated with an LRR. This is done in both a theoretical and empirical model. We first build a simple micro model that suggests

\(^{4}\)Using a historical dataset for 14 developed countries over almost 140 years, Schularick and Taylor (2012) provide ample evidence that excessive leverage contributed to recurrent episodes of financial instability.

\(^{5}\)See BCBS (2009) and BCBS (2014a). The Basel III banking regulations also include a strengthened risk-based capital framework and two new liquidity requirements, i.e. the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR).

\(^{6}\)For example the ex-CEO of Barclays Antony Jenkins expressed concern about the LRR saying it needed “to be interpreted with care to avoid unintended consequences such as credit restriction and asset quality dilution” (see Treanor (2013)). Other examples include the Financial Supervisory Authority in Sweden (Finansinspektionen (2015)) which noted that “if non-risk-sensitive capital requirements - such as a leverage ratio requirement or standardised floor - are set at a level that makes them the binding capital restriction, Sweden may end up with a smaller, but riskier banking system. [...] A high leverage ratio requirement could consequently result in less financial stability”.

\(^{7}\)When considering the potential disadvantages of the LRR, the ESRB (2015) Handbook chapter on the LRR states on page 14 that “Most importantly, the leverage ratio is insensitive to assessments of the riskiness of different assets. Used on its own, it can incentivise banks to regulatory arbitrage by taking on riskier assets.”
if equity is sufficiently costly, indeed there always exists an increased incentive to take further risk once banks become constrained by the LRR. Nonetheless, our theoretical analysis suggests this increase in risk-taking should be limited and outweighed by the beneficial impact of the concurrent increase in loss-absorbing capacity arising from a higher capital requirement; so banks should become more stable with an LRR.

These theoretical results are then tested and confirmed within a three-stage empirical analysis on a large sample of EU banks for the period 2005 - 2014. First, we provide evidence of moderate increases in bank risk-taking using a difference-in-difference type approach taking the Basel III LRR announcement at the end of 2009 as a treatment that only affects a subset of banks that are highly leveraged. Second, we show in a logit model framework that the marginal beneficial impact of increasing a bank’s leverage ratio is much bigger than the marginal negative impact of increased bank risk-taking, especially if highly leveraged banks are forced to increase their leverage ratios to levels that are close to the currently discussed minimum standards in the range of 3 - 5%. Third, we combine the first and second stage empirical results into a counterfactual simulation to show that the negative impact of the estimated increase in risk-taking is outweighed by the benefits of increasing loss-absorbing capacity, i.e. that an LRR should be beneficial for financial stability by significantly reducing the failure probability of highly leveraged banks.

For our theoretical analysis, we develop a bank micro model along the lines of Dell’Ariccia et al. (2014) that is able to capture the trade-off between risk-taking and higher loss-absorption associated with an LRR. In line with the Basel III regulatory framework, we consider a setting in which the risk-based capital framework is complemented with a non-risk based LRR. Banks thus face the maximum of two capital charges. The LRR requires banks to hold capital against its assets independent of the riskiness of its portfolio, whereas the capital requirement of the risk-based framework depends on the risk choice of the bank. Banks can choose between two types of assets: a (relatively) safe asset and a risky asset. We then introduce the key friction of our model, a correlated system-wide shock that has a small probability of occurring, but hits both the safe and the risky asset. In our setting, the risk-weighted framework is not able to perfectly cover this correlated shock, therefore providing an opportunity for the LRR to improve upon a situation with only a risk-based framework. This friction relates directly to one of the Basel Committee’s key reasons for the imposition of an LRR: the build-up of leverage in

8Results are also robust to the shock hitting only the risky asset
low-risk assets and the imperfect coverage of rare shocks to these assets under the risk-based capital framework (BCBS, 2014b).

In a first step, we show that if equity is sufficiently costly, imposing an LRR always incentivises banks bound by it to take on more risk. This occurs because under an LRR, constrained banks are no longer forced to hold additional capital when they take greater risk, and greater risk is associated with a greater expected return. If capital is expensive, under a risk-based framework this incentivises banks to reduce their risk-taking as adding capital contributes to marginal costs. Under a binding LRR, this marginal cost disappears and hence banks increase their risk-taking since they can now increase risk without the penalty of having to hold greater capital.

Despite this however, we then show that imposing an LRR should be beneficial, both in terms of banks’ probability of failure and the expected loss of deposit funds (where this is defined as the expected amount of deposit funds the bank will be unable to repay on bankruptcy). In other words, the benefit of increased loss absorbing capacity brought about by the LRR should outweigh any negative impact from additional risk-taking. This is due to two reasons. First, there is a limit to how much additional risk a bank can take. If it takes too much additional risk, it will simply move back into the risk-based capital framework. Hence, as long as the risk-based capital requirement applies alongside the LRR, it acts to constrain this risk-taking incentive. Second, there exists a skin-in-the-game effect that somewhat offsets the incentive to increase risk-taking once a bank is bound by the LRR. Forcing banks to hold greater capital means they survive larger shocks. As a result, banks internalise losses which they otherwise would have ignored due to limited liability, and this slightly decreases their incentive to take further risk.

The model therefore illustrates both how incentives adjust under a combined LRR and risk-based capital framework, and how the trade-off between higher loss absorbing capacity and increased risk-taking looks like once banks become constrained by the LRR. The results add to the literature in a similar vain to Blum (2008) and Kiema and Jokivuolle (2014) who also look at the effects of imposing an LRR in addition to the risk-based capital framework, but with a different focus of the analysis.\footnote{Prior to Blum (2008), the literature had not considered a combined LRR, risk-based capital framework, thus we are one of the first to address the benefits and costs of imposing an LRR alongside the risk-based capital framework. The literature on the nexus between capital and risk-taking has been remarkably inconclusive. Theoretical predictions have ranged from suggesting higher requirements lead to riskier asset profiles (e.g. Kahane (1977), Michael Koehn (1980) and Kim and Santomero (1988)) to either suggesting the effect can be ambiguous (Gennotte and Pyle}
risk-independent capital ratio can improve bank stability through its disincentivising effect to conceal true risk-levels. Kiema and Jokivuolle (2014) consider a similar question but through a model risk perspective. They show that the introduction of an LRR can induce formerly low risk banks to increase risk-taking, however in the presence of model risk, which arises if some loans get incorrectly rated, an LRR can improve stability due to the presence of a greater capital buffer should these mispriced loans become toxic. We move away from these papers by abstracting from this gaming and model risk perspective and instead show that the LRR (combined with a risk-based capital requirement) is beneficial for bank stability not just because banks wish to game the system, but also due to its additional loss absorbing capacity.\(^{10}\)

Our theoretical model allows us to derive two main hypotheses, which we test empirically. To our knowledge, we are the first paper to combine a theoretical and empirical analysis of the imposition of an LRR. In particular, our two hypotheses suggest that: 1) Introducing an LRR incentivises those banks bound by it to modestly increase risk-taking; 2) Forcing banks to hold greater capital via an LRR is beneficial for bank stability. Using a panel data set of EU banks over the period 2005-2014, we find evidence in support of both our hypotheses.

First, we investigate risk-taking incentives via a difference-in-difference type approach. The announcement of the Basel III LRR at the end of 2009 is taken as a treatment that only affects banks below the LRR, which allows us to carve out treatment and control groups. Banks with leverage ratios below the minimum requirement of 3% (currently being assessed by the BCBS) are the treatment group, while banks with leverage ratios above the threshold are the control group.\(^{11}\) We use the risk-weighted asset (RWA) to total assets ratio as a proxy for risk-taking, which directly relates to our theoretical model. The results confirm our first hypothesis: an LRR leads banks to increase risk-taking, but this increase is relatively contained (in the region of a 1.5 to 2.5 p.p. increase in the RWA ratio), and small relative to the required capital increase from an LRR. This finding is also in line with the previous empirical literature that has suggested a positive relationship between higher capital and greater bank risk-taking (see e.g. Shrieves and Dahl (1992); Aggarwal (1991); Calem and Rob (1999); Blum (1999)) or lead to lower risk-taking incentives (Keeley and Furlong (1990); Flannery (1989); Hellmann et al. (2000); Repullo (2004); Repullo and Suarez (2004)).

\(^{10}\)We take as given that banks truthfully report both their risk and capital levels. The model could easily be extended to include both gaming and model risk, with results further in favour of the LRR.

\(^{11}\)We also test a 4% and 5% LRR to classify banks into treatment and control groups - the results remain robust.
and Jacques (2001); Rime (2001); Jokipii and Milne (2011)). Yet this literature has been plagued by endogeneity issues since capital and risk are inextricably linked. Since we focus on a regime change, moving from a fully risk-based capital framework to one in which there also exists an LRR, we are better able to identify any risk-taking effect without concern for reverse causality.

Second, we estimate the joint effects of the leverage ratio and risk-taking on bank distress probabilities in a logit model framework. We build on the early warning literature along the lines of Betz et al. (2014) and Estrella et al. (2000) who use logit models to analyse out-of-sample forecasting properties of specific variables. Both papers emphasise the benefits of higher capital levels for financial stability, while Berger and Bouwman (2013) have shown that banks with higher capital levels are more likely to survive a financial crisis. We refine this analysis within the context of the leverage ratio. We use our unique dataset of EU bank distress events between 2005 - 2014 and build a logit model to analyse the relationship between higher leverage ratios, risk-taking and bank distress probabilities, in order to quantify the risk-stability trade-off associated with an LRR. We show that the leverage ratio is a very important determinant for bank distress probabilities, both economically and statistically. Importantly, the marginal benefit of increasing a bank’s leverage ratio from low levels is an order of magnitude larger than the marginal negative impact from taking on greater risk.

Third, we use the results from the first two empirical exercises to analyse whether given our estimated increase in risk-taking, bank distress probabilities would decline following the imposition of an LRR. In particular, the results from the logit model are combined with the estimated increase in risk-taking from the difference-in-difference model in a counterfactual simulation. We ask whether bank distress probabilities significantly decline if an LRR forces banks to increase their leverage ratios to the minimum level, but at the same time this has the side effect of increased risk-taking (represented via higher RWA ratios). We perform the exercise with a 3%, 4% and 5% leverage ratio minimum and in all cases bank distress probabilities decline, even for our two most conservative exercises where banks are assumed to increase their risk-taking by triple the estimated amount, and by the maximum amount before moving back into the risk-based capital framework. The results therefore support the second hypothesis that banks should become more stable with the imposition of an LRR despite the slight increases in bank risk-taking.

The remainder of the paper is organised as follows. Section 2 presents a brief

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12 Other studies have also suggested a negative relationship (see e.g. Jacques and Nigro (1997)).
overview of the Basel III LRR framework. Section 3 develops the bank micro model and derives testable hypotheses regarding the effect of an LRR on risk-taking and bank stability. Section 4 tests the hypotheses empirically, and section 5 concludes.

2 The Basel III Leverage Ratio Requirement

As a response to the global financial crisis, the BCBS decided to undertake a major reform to the regulatory framework of the banking system. Under the new Basel III banking regulations, a non-risk based LRR will be introduced alongside the risk-based capital framework with the aim to “restrict the build-up of excessive leverage in the banking sector to avoid destabilising deleveraging processes that can damage the broader financial system and the economy”.\textsuperscript{13} The leverage ratio is a non-risk based capital measure and is defined as Tier 1 capital over a bank’s total exposure measure, which consists of on-balance sheet items as well as off-balance sheet items.

It is widely expected that the LRR will become a Pillar I requirement for banks under Basel III, ever since the BCBS issued a consultative document\textsuperscript{14} that outlined a baseline proposal for the design of the LRR in December 2009. Following further public consultations and revisions to the design, the BCBS issued the (almost) final LRR framework in January 2014 and is currently assessing a minimum Tier 1 leverage ratio of 3% until 1 January 2017 with a view to migrating to a Pillar I treatment on 1 January 2018. The BCBS will review the calibration of a minimum required leverage ratio framework and make any final adjustment to the definition by 2017.\textsuperscript{15} Figure 1 summarises the key regulatory milestones related to the LRR which will be used in the empirical analysis in section 4.2 to motivate the econometric set-up to identify the impact of an LRR on bank risk-taking.

\textsuperscript{13}See BCBS (2014a). The Basel III regulations also include a strengthened risk-based capital framework and two new liquidity requirements, i.e. the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR).

\textsuperscript{14}See BCBS (2009).

\textsuperscript{15}In Europe, the EBA is currently preparing a report on the impact and the potential calibration of the Leverage Ratio. Based on the results of the report, the European Commission shall submit by the end of 2016 a report on the impact and effectiveness of the leverage ratio to the European Parliament and the Council.
3 Theoretical model

The following section presents a simple microeconomic model that captures the trade-off between risk-taking incentives and higher loss-absorbing capacity associated with the introduction of an LRR.

3.1 The set-up of the model environment

Consider a one-period economy with three types of agent: banks, investors and depositors. There are \( n > 1 \) banks, run by risk-neutral penniless bankers. The size of the bank’s balance sheet is normalised to one. The bank finances itself with equity/capital \( k \) and deposits \((1 - k)\) subject to two capital requirements (a risk-based capital requirement and a leverage ratio requirement). There exists a continuum of identical, risk-neutral depositors. These depositors are negligible in size relative to banks. Depositors have two options: they either invest their endowment in bank deposits which yield a gross return of \( i \), or alternatively deposit their endowment in a storage asset, which yields a gross return of 1. Banks are covered by limited liability, they therefore repay depositors only in the case of survival. Nevertheless, there exists full deposit insurance.\(^{16}\) This implies deposits are insensitive to risk-taking and will receive a deposit rate equal to the expected return on the safe asset \( i = 1.\)\(^{17}\)

\(^{16}\)For simplicity, as in Hellmann et al. (2000), we assume the insurance premium is zero. Nonetheless, our results hold for any fixed insurance premium.

\(^{17}\)Deposits will be insensitive to risk-taking since whether banks fail or survive, depositors will be fully compensated. As a result, as long as \( i \geq 1 \), depositors will prefer bank deposits (where at
In addition to deposits, given bankers are wealth constrained (and must satisfy capital requirements), they can also raise funds by issuing equity. Investors are risk-neutral, they are not covered by deposit insurance, and they have an outside option yielding a gross return of \( \rho \) per unit of capital. As a result, banks must ensure the return they offer to shareholders is at least as large as \( \rho \) in expected terms in order to satisfy the investors’ participation constraint. Similar to Dell’Ariccia et al. (2014), throughout the analysis, \( \rho \) is assumed to be constant. This simplifies the analysis, but also allows us to assess the industry argument against the LRR head-on. Banks often argue that they must achieve certain high targets for return on equity in order to raise equity funding (see Johnson, 2011), this is in turn, since equity is more expensive than deposits, drives greater risk-taking (see Treanor (2013)). By assuming a constant \( \rho \), we are able to assess this scenario head-on. If bankers believe a certain required return on equity must be achieved, or their salary package is a function of achieved targets, then they will act as if \( \rho \) is fixed. Indeed, assuming a constant \( \rho \) puts the LRR at an inherent disadvantage since it is the higher cost of equity (relative to deposit costs) which drives greater risk-taking. Following the argument of those such as Admati and Hellwig (2013), one could suggest that as capital ratios rise, the required \( \rho \) should fall, since ceteris paribus, banks are becoming safer. Given that the higher is \( \rho \), the higher the increase in risk-taking, relaxing this assumption such that it declines as bank leverage ratios rise, will reduce this increase in risk-taking. Thus if the LRR is beneficial under a constant \( \rho \) assumption, it will also be beneficial when this assumption is relaxed; indeed the result should be strengthened. This is discussed in further detail in section 3.3.3. Assuming a constant \( \rho \) therefore allows us to tackle the industry case head-on, with the understanding that if the assumption is relaxed, the results would not only continue to hold, but would be strengthened in favour of the LRR, since the incentive to increase risk would decline.

Each bank may invest its funds into two assets: a risky asset and a (relatively) safe asset. Denote by \( \omega \) investment in the safe asset and by \( (1 - \omega) \) investment in the risky asset. As in Allen and Gale (2000), there exists a convex non-pecuniary investment cost to risky investment \( c(\omega) \), where \( c'(\omega) < 0 \) and \( c''(\omega) \leq 0 \), so investing in the risky asset becomes increasingly expensive. Banks face two types of capital regulation: a risk-based requirement and a non-risk-based leverage ratio requirement.

equality, depositors will be indifferent). Knowing this, banks will offer the lowest rate possible and thus set \( i = 1 \). Keeley and Furlong (1990) formally show that when there exists deposit insurance, deposit supply will not be a function of bank risk.
Since assets differ in their riskiness, the risk-based capital requirement is increasing in holdings of the risky asset. Specifically, as in the Basel risk-based capital framework, on each asset banks are required to hold sufficient capital such that they cover expected and unexpected losses with some probability $(1 - \alpha) \in (0, 1)$, where in the Basel requirements $\alpha = 0.001$. Therefore, there exists a capital requirement $k_{\text{safe}}$ on the safer asset, and $k_{\text{risky}}$ on the risky asset, where $k_{\text{safe}} < k_{\text{risky}}$. Given asset holdings of $\omega$, the risk-based capital requirement can be written as

\[ k_{\text{rw}}(\omega) = \omega k_{\text{safe}} + (1 - \omega) k_{\text{risky}}. \]

In addition, banks are subject to an LRR which states that banks must hold a minimum level of capital $k_{\text{lev}}$ independent of risk. The combined capital framework will be such that the bank must hold a capital level $k$ greater than or equal to the higher of the two requirements, namely $k \geq \max\{k_{\text{rw}}, k_{\text{lev}}\}$. Which constraint requires the higher capital level depends on the riskiness of the bank’s balance sheet. Figure 2 illustrates this. Since the risk-based requirement increases in holdings of the riskier asset, at low-risk holdings, the risk-based requirement (see the dashed diagonal line) lies below the LRR. As holdings of the riskier asset increase, the requirement also increases until beyond some level, denoted $(1 - \omega_{\text{crit}})$ in Figure 2, it starts to exceed the LRR. As a result, the combined capital framework exhibits a kinked structure.

There exist two possible states of nature, state 1, denoted $s_1$, which can be thought of as a good state, and state 2, denoted $s_2$, which can be thought of as a bad state. These states occur with probability $\mu$ and $(1 - \mu)$ respectively. Each asset’s return is a function of the state of the world. The safe asset offers a gross return of $R_1 \geq 1$ if state $s_1$ occurs, and $(1 - \lambda_1) \in (0, 1]$ if state $s_2$ occurs. On the other hand, in state $s_1$, the risky asset offers a gross return of $R_h^1 > R_1$ with probability $\pi$ and $(1 - \lambda_2) \in (0, 1)$ with probability $(1 - \pi)$, while in state $s_2$, it returns $(1 - \lambda_3) \in (0, 1)$ with probability $\pi$, and 0 otherwise. The expected return on the risky asset is assumed to be greater than the expected return on the safe asset. The setup can be seen in Figure 3, where $0 \leq \lambda_1 < \lambda_2 < \lambda_3$, and $\rho > [\mu R_1 + (1 - \mu)(1 - \lambda_1)] \geq 1$, so it is envisioned that losses on the risky asset are larger in the bad state, but losses on the safer asset are smaller than for the risky asset. The risk of the bank’s portfolio is thus determined by the investment proportion devoted to the risky asset relative to the safe asset.\(^{19}\)

\(^{18}\)In this simplified world, the only source of provisions is the bank’s own funds, hence it is analogous to bank capital.

\(^{19}\)The model assumes probabilities and payoffs are known with certainty, i.e. there exists no model risk or gaming concerns. We impose this assumption to illustrate the benefits of an LRR even in the absence of these concerns. Clearly if there exists model risk in addition, or if banks
Figure 2: Capital requirements under a combined leverage ratio requirement and risk-based framework.

Notes: The graph shows the interaction between a leverage ratio requirement ($k_{lev}$) and the risk-based requirement which is increasing in $(1 - \omega)$. $(1 - \omega_{crit})$ is the point at which the capital requirement under both the risk-based and the leverage ratio requirement are equalised.

Figure 3: Payoff of the risky and safe asset

Notes: Figure shows the payoff function dependent on the state of the world for the safe and risky asset.
As discussed above, under the risk-based framework, the exact capital requirement will be a function of how the probabilities $\mu$ and $\pi$ relate to $\alpha$. For generality, we consider all cases. For clarity, we discuss the cases separately. The baseline case $(1 - \mu) \leq \alpha$ will be discussed in this section. The alternative case, $(1 - \mu) > \alpha$, is discussed in the appendix. All results continue to hold.

Since $(1 - \mu) \leq \alpha$, the bank is only required to hold enough capital to survive state $s_1$. So immediately, it is clear that the capital charge on the safer asset, $k_{safe}$, is zero. For the risky asset, since $(1 - \mu) \leq \alpha$, the bank does not need to cover losses in state $s_2$ and thus the capital charge will be either $\lambda_2$ or zero. This can be seen in figure 4. If $(1 - \mu) + \mu(1 - \pi) > \alpha$, then the bank must cover the loss in state $s_1$, as otherwise the requirement is not satisfied, hence $k_{risky} = \lambda_2$. If $(1 - \mu) + \mu(1 - \pi) \leq \alpha$, then the probability of loss in state $s_1$ is so small that the bank does not need to hold capital against it, and so $k_{risky} = 0$. Since this case entails a zero capital requirement under both assets and hence there is no risk-based nature to it, indeed there is no capital requirement (both assets have a zero capital charge), we ignore this case for the more realistic previous case. So, if $(1 - \mu) \leq \alpha$, $k(\omega) = (1 - \omega)\lambda_2$.

The setup attempts to capture one of Basel’s key reasons for the imposition of an LRR: the inability of the risk-based framework to cover correlated shocks that can also impact lower risk assets. We envisage state $s_2$ as a low probability event, but it is an event that can hit both assets. Thus it may be that the risk-weighted framework is not able to perfectly cover this correlated shock, thereby providing an opportunity for the LRR to potentially improve upon a situation with only a risk-based framework.

game their risk weights, the benefit from an LRR will be further enhanced. See for example Blum (2008) and Kiema and Jokivuolle (2014).
3.2 The bank’s decision problem

The objective for the bank is to maximise expected profits after paying out shareholders, and conditional on survival, also taking into account the investment cost. In order to achieve this, each bank must determine the structure of its portfolio in terms of both its asset and liability side. Each bank must optimally choose the amount of capital and deposits to hold (subject to both a risk-adjusted capital requirement and a leverage ratio constraint), how much to pay depositors and equity holders, and their investment \((ω, 1−ω)\) in each asset. In order to raise funds, banks must satisfy both depositors and equity holders’ participation constraints. As noted above, for depositors this implies banks must satisfy \(i ≥ 1\) since their outside option is to store their assets with a gross return of 1. In optimum, since banks wish to minimise costs, the bank will set \(i = 1\). Investors on the other hand have an outside option \(ρ\). Unlike depositors, they are not covered by deposit insurance, so banks must ensure they earn an expected gross return of at least their opportunity cost. Suppose \((1−θ)\) is the share of profits given to equity holders as compensation, then it must be that the bank ensures the following participation constraint is satisfied:

\[
(1−θ)Π ≥ ρk
\]

where \(Π\) is expected profits, with

\[
Π = μπ[ωR_1 + (1−ω)R_2^h - id] + μ(1−π)\max{[ωR_1 + (1−ω)(1−λ_2) - id], 0}
\]

\[
+(1−μ)π\max{[ω(1−λ_1)+(1−ω)(1−λ_3)−id], 0}+(1−μ)(1−π)\max{[ω(1−λ_1)−id], 0}
\]

As with deposits, since banks treat this like a cost, in optimum this constraint must hold with equality.

Considering the entire setup together, we can write each bank’s problem formally as:

\[
\max_{ω,θ,i,k} \{θΠ − c(ω)\}
\]

subject to

\[
(1−θ)Π ≥ ρk
\]

\[
d + k = 1
\]

\[
i ≥ 1
\]

\[
k ≥ \max\{k_{lev}, k(ω)\}
\]
\[ \Pi = \mu \pi [\omega R_1 + (1 - \omega) R^h_2 - id] + \mu (1 - \pi) \max \{[\omega R_1 + (1 - \omega)(1 - \lambda_2) - id], 0\} + (1 - \mu) \pi \max \{[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) - id], 0\} + (1 - \mu)(1 - \pi) \max \{[\omega(1 - \lambda_1) - id], 0\} \]

where \( d \) is deposits, and following Dell’Ariccia et al. (2014), we parameterise the cost function as \( c(\omega) = (c/2)(1 - \omega)^2 \).

It is worth noting that the above problem illustrates how bankers and equity holders are covered by limited liability. Whenever returns are negative, payoffs become zero. Furthermore, the problem illustrates how banks can adjust their probability of survival in two ways. First, banks can choose to directly decrease risk-taking, i.e. increase \( \omega \). Second, banks can increase their probability of survival by choosing to hold more capital. Should losses then occur, the bank is able to withstand them.

### 3.3 Main theoretical results

#### 3.3.1 Risk-taking under a risk-based capital requirement

Let us first analyse the solution to the model when there exists only a risk-based capital requirement. The problem will be identical except since there does not exist an LRR, the capital constraint will reduce to \( k \geq k(\omega) \). As outlined in the previous paragraph, if desired, banks could choose to hold enough capital such they survive all potential losses. This has two effects. First, holding additional capital is costly since equity holders require an expected return of at least \( \rho > i = 1 \). Second, increasing capital sufficiently will enable banks to survive further shocks in state \( s_2 \), and this will both decrease the bank’s probability of default and generate additional return. Yet, state \( s_2 \) is a loss state; the assets yield a gross return of less than 1. As Lemma 1 shows, the bank does not find it optimal to increase capital to survive these states. The cost of holding greater capital outweighs the benefit of obtaining the residual value in these states. Capital must return on average \( \rho \) to satisfy shareholders, while depositors will accept \( i = 1 \). Since \( \rho \) is larger, ceteris paribus, banks will prefer to fund themselves with cheaper deposits. Banks will therefore never wish to hold more than the required capital amount. Indeed, banks would prefer to be 100% deposit financed, but due to the capital requirement, banks are forced to hold at least the minimum. As a result, the capital constraint binds. Lemma 1 formalises this.

**Lemma 1** Banks always wish to hold as little capital as possible; therefore the capital requirement will bind.
Proof. See the appendix.

Since the risk-weighted capital requirement binds, it will impact risk-taking decisions. Holding more of the risky asset entails holding greater capital and as we have noted, this is expensive. Hence, there exists a trade-off between holding more of the risky asset, which in expected terms yields more, and the cost of doing so. The bank will choose the point at which the marginal revenue from greater investment in the risky asset equals the marginal cost. The first order condition (FOC) depicts this:

$$\mu \left[ \pi R_2^h + (1 - \pi)(1 - \lambda_2) - R_1 \right] = -(\rho - \mu)k'(\omega) - c'(\omega)$$

The left hand side (LHS) of this expression shows the marginal benefit from increasing holdings of the risky asset $(1 - \omega)$, while the right hand side (RHS) illustrates the marginal cost. The marginal benefit comprises the increased potential payoff the risky asset offers. By shifting funds from the safer asset to the risky asset, the bank forgoes $R_1$, but gains $\pi R_2^h + (1 - \pi)(1 - \lambda_2)$ which is larger. On the other hand, the marginal cost takes into account both the cost of investing an additional unit in the risky asset, $c'(\omega) < 0$, and the fact that holding greater quantities of the risky asset require higher capital levels (shown in the $k'(\omega) < 0$ terms) which is more expensive than deposits. Replacing one unit of deposits with one unit of capital saves $i \cdot \mu = 1 \cdot \mu$ in expected terms, but costs $\rho$; hence the $(\rho - \mu)$ term; the net cost of replacing deposits with capital. In the risk-based framework there is therefore a trade-off the bank can exploit in terms of capital and risk; by choosing to hold less risk, the bank somewhat offsets the lower return by its ability to lower expensive capital. Banks trade off this potential loss of profits with the cost of risky investment, and hence choose a risk level such that the marginal benefit from increasing $(1 - \omega)$ is zero.

The condition illustrates the trade-off banks possess when risk-taking under a risk-based framework. Increasing the weight on the risky asset increases potential returns, but at the same time entails costs related to investment and capital raising. A risk-weighted capital requirement thus disincentivizes risk-taking, as it forces banks to hold more capital if they wish to take on more risk.

### 3.3.2 Risk-taking with a leverage ratio requirement

Suppose that now banks are subject to an additional constraint, namely, a constraint on leverage such that $k \geq k_{lev}$ regardless of $\omega$. Given the LRR exists alongside the risk-based capital framework, any LRR below the risk-weighted requirement will
have no effect (since it does not bind) and the results of the previous paragraph still hold. In order to make the LRR bite, the LRR must be set such that it is above the risk-weighted capital requirement of a bank. Suppose the LRR is set to $k_{lev} > k(\omega^*_{rw})$ such that it is the binding constraint, where $\omega^*_{rw}$ denotes the optimal safer asset holdings under the risk-based framework. Although banks can now potentially survive larger losses (since they hold greater capital), it may be that as a result of the LRR, bound banks shift so much of their portfolio into the risky asset that even with this higher level of capital, they cannot withstand these now more probable, larger losses. Whether this increase in capital is beneficial depends on how much (if at all) the bank is incentivised to shift its portfolio into the risky asset (which is more likely to fail and its residual value is lower).

The change in risk incentives can be clearly seen by comparing the FOC with respect to $\omega$ under a risk-based framework to the FOC if the LRR is binding. Suppose the LRR is set just above the risk-based capital requirement, then the FOC is characterised by:

$$\mu[\pi R^h_2 + (1 - \pi)(1 - \lambda_2) - R_1] = -c'(\omega)$$

As can be seen, all terms related to the risk-weighted capital requirement have disappeared due to the binding LRR. Removing this dependence on risk means banks can now increase risk without having to hold additional capital. In other words, the marginal cost of risk-taking declines as there is no longer a requirement to increase expensive capital if the bank increases $(1 - \omega)$. By removing the link between capital and risk-taking, the bank will be incentivised to take more risk. This can be seen in the FOC above. The LHS of the equality (i.e. the marginal benefit) is identical to before, whereas the RHS, the marginal cost, is lower. Thus the $\omega$ that solves this equation must be lower than the $\omega$ that solves the risk-based FOC, hence implying greater risk-taking. As the LRR rises however, and banks begin to hold more capital, it is possible that at the same time, depending on the LRR level, the marginal benefit can also change. The marginal benefit of increasing risk can decline, since with higher capital, banks survive larger shocks, and as a result, banks are forced to internalise these returns they otherwise would have ignored - so called “skin-in-the-game”. Due to the discrete nature of the asset setup, this effect first appears when the LRR is set high enough that banks are forced to also survive state
s₂ when the risky asset pays off \((1 - \lambda_3)\).²⁰ The FOC becomes:

\[
\mu [\pi R^h₂ + (1 - \pi)(1 - \lambda_2) - R_1] - (1 - \mu)\pi (\lambda_3 - \lambda_1) = -c'(\omega)
\]

Compared to the previous FOC, one can clearly see the presence of a “skin-in-the-game” effect, \((1 - \mu)\pi (\lambda_3 - \lambda_1)\), which brings down the chosen level of risk slightly. As capital holdings rise, banks survive larger and larger shocks. Since banks then attach value to these returns, this to some extent decreases the benefit of higher risk-taking, since the residual value of the risky asset is lower, and hence this reduces the optimal risk level chosen. There can therefore exist two opposing effects from the imposition of an LRR. The first effect (i.e. removing the link between risk and capital) - the loss of the \(k'(\omega)\) terms in the FOC - incentivises greater risk-taking, whereas the second effect - the skin-in-the-game effect, as banks are forced to increase capital by more and more - incentivises less risk-taking since banks begin to internalise returns they otherwise would have ignored. Proposition 2 formalises this discussion and shows that when equity is sufficiently costly, the first effect always dominates and banks increase risk-taking with an LRR.

**Proposition 2**

If \(k_{lev} < \lambda_1 + (\lambda_3 - \lambda_1)\frac{\mu R^h₂ + (1 - \lambda_2)\pi - R_1}{c}\), imposing a leverage ratio requirement will always incentivise banks to take more risk.

If \(k_{lev} \geq \lambda_1 + (\lambda_3 - \lambda_1)\frac{\mu R^h₂ + (1 - \lambda_2)\pi - R_1}{c}\), imposing a leverage ratio requirement will still always incentivise banks to take more risk if equity is sufficiently costly, i.e.:

\[
\rho > \begin{cases} 
\mu + (1 - \mu)\frac{\pi(\lambda_3 - \lambda_1)}{\lambda_2} & \text{if } k_{lev} \in [k_1, k_2) \\
\mu + (1 - \mu)\frac{(1 - \lambda_1) - \pi(1 - \lambda_3)}{\lambda_2} & \text{if } k_{lev} \geq k_2
\end{cases}
\]

where \(k_1 \equiv \lambda_1 + (\lambda_3 - \lambda_1)\frac{\mu R^h₂ + (1 - \lambda_2)\pi - R_1}{c}\) and \(k_2 \equiv \lambda_1 + (1 - \lambda_1)\frac{\mu [\pi R^h₂ + (1 - \lambda_2)\pi - R_1] - (1 - \mu)\pi (\lambda_3 - \lambda_1)}{c}\)

**Proof.** See the appendix.

Proposition 2 summarises the two effects that determine whether a leverage ratio will incentivise greater risk-taking. The first condition illustrates that for levels of the LRR just above the risk-based requirement, the “skin-in-the-game” effect is so

²⁰Whether the LRR can be set at such a level that banks begin to survive state s₂ shocks depends on the extent to which banks risk-up under an LRR, since if they increase risk to the maximum, this case is not possible. This will depend on the exact parameter values of the model. Nevertheless, for some parameter values, it is possible.
small, indeed in this region it is zero due to the discrete nature of the set-up, that the
only incentive driving risk-taking is the move away from linking risk to capital, which
simply incentivises the bank to risk-up. As the LRR rises however, as discussed
before, banks will begin to survive shocks in state $s_2$, and thus this “skin-in-the-
game” effect will begin to appear. The second condition illustrates that as long as
equity is sufficiently expensive,$^{21}$ the move away from a risk-based requirement will
always dominate the bank’s decision making, and thus banks will shift more of their
portfolio into the risky asset. This is because, compared to the cost of equity that
incentivised lower risk-taking under a risk-based capital requirement, and for which
banks are now released from considering, this “skin-in-the-game” effect is small;
state $s_2$ is a low probability state and any additional payoff is multiplied by
$(1-\mu)$ which is very small. To give an idea of the magnitude required, consider the
higher threshold for $\rho$ when $k_{lev} \geq k_2$, and the reasonable parametrisation $\pi = 0.8,$
$\lambda_1 = 0.02$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.8$, then $\rho$ must be larger than 1.0031.

Taking proposition 2 as a whole therefore, we can conclude that if equity is
sufficiently expensive, once the LRR binds, risk-taking will increase because the
LRR in effect allows banks to engage in greater risk-shifting. Removing the binding
risk-weighted capital requirement allows banks to increase risk while imposing most
of that risk onto the funds raised from depositors (ultimately the responsibility of
taxpayers) - since banks are not forced to raise any further capital. Since there exists
full deposit insurance, depositors are not sensitive to this risk-taking; hence banks
increase risk without incurring higher funding costs. With a risk-weighted capital
requirement, this ability to risk-shift is somewhat offset since taking on further
risk implies increasing capital, which is expensive. Once the risk-weighted capital
requirement ceases to bind, banks can increase risk-taking without needing further
additions of capital. This was a major inhibitor to risk-taking, hence under an LRR,
banks have a greater incentive to risk-shift.

Lastly, under the cases in which $\rho$ is less than the sufficient level, we cannot
immediately conclude that risk-taking will therefore be lower. It may be the case,
and if so, then clearly bank stability will improve as banks are more highly capitalised
and take lower risk, however we cannot generalise. This is because for larger values
of $k$, it may be that the optimal level of risk chosen by the FOC is not sufficient to
satisfy the shareholders’ participation constraint, since $\rho > \mu R_1 + (1-\mu)(1-\lambda_1)$.
If so, then banks are obliged to choose a higher risk level than desired, as otherwise
they are unable to raise equity, and this level could be higher than the risk-based

$^{21}$Where if $(1-\lambda_1) - \pi(1-\lambda_3) < \lambda_2$, this is always the case since $\rho \geq 1$
### 3.3.3 Risk-taking vs. loss absorbing capacity

Proposition 2 showed that imposing an LRR will always incentivise banks to increase risk-taking if equity is sufficiently costly. Nonetheless, this does not imply that an LRR is detrimental. Quite the contrary, whether the LRR improves outcomes depends on the extent of this risk-taking compared to increased loss absorbing capacity. We assess this in two important ways: first, via the impact on the bank’s probability of default, and second, via the impact on the expected loss of deposit funds.\(^{22}\) With an LRR, banks may potentially survive a state \(s_2\) shock, but in order to generate a benefit, it must be that any additional risk is outweighed by this loss-absorbing capacity. At the same time, even if the probability of default remains the same, the LRR may induce a benefit via its effect on the expected loss of deposit funds - since any losses that do occur are absorbed by capital rather than deposit funds. Proposition 3 formalises this discussion.

**Proposition 3** Relative to a solely risk-based capital framework:

1. Imposing a leverage ratio requirement, leads to weakly lower bank failure probabilities.

2. If \(\rho \leq \hat{\rho}\), imposing an LRR leads to a strictly lower expected loss of deposit funds if \(k > k_0\).

3. If \(\rho > \hat{\rho}\), imposing an LRR also leads to a strictly lower expected loss of deposit funds if \(k \in (k_0, k_0^\max)\), but for \(k \in (k_0, k^\max)\), where \(k^\max < 1\) and \(k > k^\max\) is infeasible, it is not possible to rule out that the expected loss of deposit funds can be larger under an LRR.

where \(\hat{\rho}\), \(k_0\) and \(k_0^\max\) are defined in the appendix, and \(k_0 < k_0^\max\).

**Proof.** See the appendix.

Proposition 3 illustrates that an LRR can improve bank default probabilities and reduce the expected loss of deposit funds.\(^{23}\) In other words, the increase in risk-taking.

\(^{22}\)In particular, the expected loss of deposit funds is defined as the expected amount of deposit funds the bank will be unable to repay on bankruptcy.

\(^{23}\)As before, to give an indication of the magnitude required of \(\rho\), consider the reasonable parameterisation: \(\mu = 0.999\), \(\pi = 0.9\), \(R_1 = 1.02\), \(R_2^{h} = 1.2\), \(c = 9\) (set following Dell’Ariccia et al. (2014)), \(\lambda_1 = 0.02\), \(\lambda_2 = 0.1\), \(\lambda_3 = 0.8\). This gives \(\hat{\rho} = 1.21\), so in reality, the third statement is unlikely to be relevant.
taking identified previously is not sufficiently large to outweigh the loss-absorbing benefit. Indeed, an LRR improves outcomes on both criterions for all \((k_0, \bar{k}_0)\). This can be understood by considering two important points. First, the risk-based capital requirement still underlies the LRR. As such, there is a limit to how much additional risk a bank can take, since if it takes too much risk, it will simply move back into the risk-based framework. In terms of failure probabilities, this puts a floor on bank failure probabilities, since if the bank takes too much risk such that it no longer covers the shocks that were required under the risk-based capital requirement, e.g. to survive state \(s_1\), it must be that the risk-based requirement is the higher binding requirement again. Since this acts as a backstop to risk-taking, banks are limited in the extent to which they can increase risk. Second, as we noted before, the skin-in-the-game effect somewhat offsets the incentive to increase risk-taking, and thus banks will not risk-up by vast amounts, since this to an extent subdued the risk-taking incentive. These two effects combine to prevent excessive risk-taking, thus the LRR has a beneficial effect both on bank failure probabilities and on the expected loss of deposit funds, as greater losses are born by the bank’s capital.

The lower bound on the expected loss of deposit funds condition is related to the amount of loss absorbing capacity available. For example, if the LRR is set to an epsilon above the risk-weighted capital requirement for a bank, the LRR adds barely any additional loss absorbing capacity, yet, the bank will take on more risk; this therefore leads to an increase in the expected loss of deposit funds relative to the solely risk-based framework. At higher levels of capital however, the additional loss absorption is sufficient to outweigh any additional risk-taking. Since in reality it is arguably the case that banks are not all concentrated around the LRR minimum, but there exists a distribution of banks with different risk-based capital requirements, we can suggest that as long as this distribution is not concentrated around the LRR minimum, this lower bound should be less of a concern.

Lastly, proposition 3 shows there can exist a potential risk when \(\rho\) is large and the LRR is set very high. This however only occurs when \(\rho > \hat{\rho}\), where \(\hat{\rho}\) is greater than the expected return on the risky asset, so banks must be targeting very large ROEs.\(^{24}\) This occurs because at these levels of \(\rho\), once the LRR rises beyond some point, the optimal choice of risk that the bank would like to take no longer meets the shareholders’ participation constraint. As a result, banks can be forced to increase risk-taking further just to meet their required return on equity. When \(\rho < \hat{\rho}\), this can also potentially occur, but the increase in risk-taking is not sufficiently fast as

\(^{24}\)This may cover a situation in which the expected return on risky assets has declined, but banks have not adjusted their ROE targets.
to outweigh the benefit from loss absorbing capacity. Above \( \hat{\rho} \) however, risk-taking increases so fast with increases in the LRR (just to meet the shareholders’ participation constraint) that at higher levels of the LRR, it can lead to worse outcomes than under a solely risk-based framework. The point at which this arises will depend on the size of \( \hat{\rho} \), and as stated only occurs for large \( \hat{\rho} \). Nevertheless, at these higher levels of capital, the increase in risk-taking is not sufficiently constrained and thus an LRR can lead to a higher expected loss of deposit funds. It should be noted however, that this case is somewhat a consequence of the constant \( \rho \) assumption. If one considers that \( \rho \) will decline as \( k \) rises, this forced increase in risk will either not occur, or it will be subdued. This is because if \( \rho \) declines as the LRR rises, risk-taking would also decline as the target ROE falls, and hence risk-taking would not be forced to consistently rise. Indeed, if \( \rho \) falls back below the expected return on the risky asset, as would be reasonable, the upper bound would cease to exist.

Overall therefore, from proposition 3, we can suggest that the LRR should improve outcomes via the dominating effect of higher loss absorbing capacity.

4 Empirical analysis

The model presented in the previous section suggests two testable hypotheses. First, the introduction of an LRR should incentivise banks for which it is a binding constraint to modestly increase risk-taking. Second, the negative impact of increased risk-taking induced by a leverage ratio constraint should be outweighed by the beneficial impact of increased loss-absorbing capacity, resulting in more stable banks. We take these two hypotheses and test them empirically on a large dataset of EU banks that encompasses a unique collection of bank distress events. The empirical analysis follows in three steps. We first test whether banks with low leverage ratios started to increase their risk-taking and capital positions after the announcement of the Basel III LRR using a difference-in-difference type approach. We then estimate the joint effects of the leverage ratio and risk-taking on bank distress probabilities in a logit model framework, in order to quantify the risk-stability trade-off associated with an LRR. Finally, we combine the first and second stage empirical results into a counterfactual simulation to test whether the negative impact of the estimated increase in risk-taking is outweighed by the benefits of increasing loss-absorbing capacity, i.e. whether an LRR is beneficial for bank stability.

The empirical evidence provided in the following sections lend support to both hypotheses. Our estimates suggest that banks bound by the LRR increase their
risk-weighted assets to total assets ratio by around 1.5 - 2.5 percentage points more than they otherwise would without an LRR. Importantly, this small increase in risk-taking is more than compensated for by the substantial increase in capital positions for highly leveraged banks, which results in significantly lower estimated distress probabilities for banks constrained by the LRR. The remainder of this section describes the underlying dataset and detailed results of the three stages of the empirical analysis.

4.1 Dataset

The dataset consists of a large unbalanced panel of around 600 EU banks (see table 4 for the exact number of banks used in each regression) covering the years 2005 - 2014, and is based on publicly available data only. There are three main building blocks of the dataset: i) a large set of bank-specific variables based on publicly available annual financial statements from SNL Financial; ii) a unique collection of bank distress events that covers bankruptcies, defaults, liquidations, state-aid cases and distressed mergers from Bankscope, Moody's, Fitch, the European Commission, Reuters and Bloomberg. and; iii) various country-level macro-financial variables from the ECB’s Statistical Data Warehouse. The dataset builds upon and expands the dataset described in Betz et al. (2014) and Lang et al. (2015). Tables 1 and 2 display various descriptive statistics of the dataset by country.

4.2 Effect of a leverage ratio constraint on bank risk-taking

To identify how the risk-taking behaviour of a bank changes after the imposition of an LRR, we exploit the panel structure of our dataset in combination with the timing of the Basel III LRR announcement, as described in section 2. We attempt to achieve identification by borrowing from the programme evaluation literature and treating the announcement of the Basel III LRR as a treatment that only affects a subset of banks, i.e. only banks below the LRR. Since our dataset includes time periods where an LRR was not part of the regulatory regime (only the risk-based capital requirements applied), we can justifiably classify banks into treatment and control groups. This classification can be justified via the kinked structure of capital requirements under a combined leverage ratio and risk-based capital framework, which was illustrated in figure 2. The LRR will only bind for those banks with leverage ratios below the minimum requirement, or in other words for banks with low risk-weighted asset ratios. For all other banks, the risk-based capital framework will remain the binding constraint, so their behaviour should not be different in the pre-treatment and post-treatment periods, i.e. they can be seen as the control group.

This classification of banks into treatment and control groups can be justified via the kinked structure of capital requirements under a combined leverage ratio and risk-based capital framework, which was illustrated in figure 2. The LRR will only bind for those banks with leverage ratios below the minimum requirement, or in other words for banks with low risk-weighted asset ratios. For all other banks, the risk-based capital framework will remain the binding constraint, so their behaviour should not be different in the pre-treatment and post-treatment periods, i.e. they can be seen as the control group.
framework was in existence), we use a difference-in-difference type analysis in which the effect of an LRR on risk-taking is estimated through a treatment dummy, while controlling for a large set of bank-specific and country-level variables that capture systematic differences in bank behaviour pre- and post-treatment. Our econometric strategy therefore is to compare the periods before the existence of an LRR with the periods after and then to analyse whether those who were affected by the imposition of an LRR (i.e. those treated) increased their risk-taking behaviour. Table 3 tests the comparability between the two groups, with columns (2) and (3) directly testing the parallel trend assumption. Column (2) shows that on average there does not seem to be any significant differences between treated banks and control group banks until we consider the post-treatment era. As the first row of the table shows, the change in risk-weighted assets to total assets ratio is significantly larger for treated banks than for control group banks after 2010. Column (3) confirms this by looking at the pre-2010 and post-2010 era, there are no significant differences between the groups pre-2010, but post-2010, for a given increase in a bank’s RWA to total asset ratio, this increase is significantly larger for banks in the treatment group.

Our identification strategy is somewhat complicated by the fact that the LRR
<table>
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<th>Country</th>
<th>GDP growth</th>
<th>Inflation</th>
<th>Δ unemployment, y-on-y</th>
<th>Credit to GDP</th>
<th>10-yr yield</th>
<th>Government debt</th>
<th>Δ House price growth, y-on-y</th>
<th>Δ Stock market growth, y-on-y</th>
<th>Δ Bundspread</th>
<th>Private credit</th>
<th>Δ Banking sector securities to liabilities, y-on-y</th>
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Source: SNL Financial, Bloomberg, ECB
Table 3: Change in a bank’s leverage ratio and RWA to total assets ratio

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<td>Leverage Ratio ≤ 3% after 2010</td>
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<td>1.639***</td>
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<td>R2</td>
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Notes: The dependent variable in column (1) is the first difference in a bank’s leverage ratio (Tier 1 to total assets ratio). The dependent variable in column (2) is the first difference in a bank’s risk-weighted asset to total asset ratio. The regressions all include time fixed effects. Significance is based on clustered robust standard errors.

will not become a binding Pillar I regulatory requirement until 2018. Nevertheless, we rely on the assumption that banks already started to adjust their behaviour in response to the Basel III LRR announcement. Table 3 gives evidence for this. Column (1) illustrates with a simple regression that while on average there seems to be no significant difference in the change in a bank’s leverage ratio between bank’s with leverage ratios below and above 3%, post-announcement, significant differences begin to arise. For a given increase in a bank’s leverage ratio, LRR bound banks (with leverage ratios below 3%) increase their leverage ratios by significantly more than non-bound banks.26 Figure 5 seems to further illustrates this. While the percentage of bound versus non-bound banks remained around 17% in all years prior to 2010, there seems to have been shift in 2010 as banks started to bolster their leverage ratios. From 2009 to 2010, there is around a 5 p.p. decline in the percentage of bound banks, and this percentage continues to decline to around 7% as of 2014 as banks with leverage ratios below 3% are forced to increase them. The assumption that banks started to react already to the Basel III LRR announcement can also be argued by the fact that banks were already required to start reporting their leverage ratios (and its components) to supervisors from 1 January 2013 onwards. Moreover, adjusting balance sheet structures takes time, so that it is reasonable to assume that banks already started to react well in advance of the LRR becoming a binding regulatory requirement. Indeed, economic reasoning suggests that in order to properly identify the effect of the Basel III LRR, it is necessary to take into

26Next to this simple evidence, a more thorough analysis is performed later in this section and this result is shown to be robust. Banks with low leverage ratios indeed started to bolster them after the Basel III LRR announcement.
account anticipatory effects, since by 2018 all banks must already satisfy the LRR and thus any effects on risk-taking will probably already have occurred before that date. Formally, our empirical strategy consists of estimating various versions of the following general panel model, where the left-hand-side variable is a risk-taking proxy for bank $i$, located in country $j$, in year $t$:

\[
y_{i,j,t} = \alpha + \beta T_{i,j,t} + \theta' X_{i,j,t-1} + \varphi' Y_{j,t-1} + \mu_i + \lambda_t + \epsilon_{i,j,t}
\]  

The terms $\mu_i$ and $\lambda_t$ are bank and time fixed-effects respectively, $X_{i,j,t-1}$ and $Y_{j,t-1}$ are vectors of bank-specific and country-specific control variables (discussed below), and $\epsilon_{i,j,t}$ is an i.i.d error term. In the risk-taking model above, $T_{i,j,t}$ is the treatment dummy of interest. It is set equal to 1 for a given bank and year.
if its leverage ratio in the previous year was below the 3% minimum, but only for years following the first announcement of the Basel III LRR. The treatment dummy is set to 0 otherwise. Thus, the coefficient of interest for the first stage of the empirical analysis is $\beta$, which measures how the announcement of the Basel III LRR has affected the risk-taking behaviour of banks. 2010 is set as the treatment start date in reference to the December 2009 BCBS consultative document that outlined the baseline proposal for the LRR (see timeline presented in Figure 1). Moreover, 3% is taken as the relevant leverage ratio threshold since the BCBS is currently assessing a minimum leverage ratio of 3% until 1 January 2017. Indeed, on 10 January 2016, BCBS’s oversight body, the Group of Central Bank Governors and Heads of Supervision (GHOS) agreed that the minimum level of the Tier 1 LRR should be 3%.27 As our measure of bank risk-taking, we use the ratio of risk-weighted assets to total assets. While the ratio of risk-weighted assets to total assets is an imperfect measure of true bank risk-taking, it is the most direct measure of risk-taking, and it is the measure that should be affected by the introduction of an LRR.28 Since data for the Basel III definition of the leverage ratio is unavailable, as our leverage ratio proxy, the ratio of Tier 1 equity to total assets is used. This variable correlates very highly with the Basel III regulatory definition of the leverage ratio. On the available data of the Basel III leverage ratio (from 2013-2014), the average correlation coefficient is 0.92.

Various bank-specific and country-specific control variables are used in order to capture firm-specific variation and environmental factors that banks may have faced. The following bank-specific variables are used as control variables: balance sheet size (measured via the logarithm of total assets), since it may be that larger institutions take more risk; the ratio of total loans to total assets, to control for the business model of a bank; the loan-to-deposit ratio, to control for liquidity; pre-tax return on assets (ROA), to control for bank profitability, since it may be that more profitable banks take less risk in a skin-in-the-game type mechanism;

27 For robustness purposes we also test a 4% and 5% threshold level to classify treatment and control groups.
28 While the risk-weighted assets to total assets ratio is potentially imprecise for comparing the level of risk-taking across banks, changes in this measure for a given bank should in principle be highly correlated with actual changes in risk-taking. This should be true as long as risk-weight levels are positively correlated with true risk. In addition, control variables for the calculation method of risk weights are included in the panel regressions, which should partly account for the fact that risk-weight levels appear to differ systematically between the standardised approach and the internal ratings based approach for determining risk-weights. Furthermore, the risk-weighted assets to total assets ratio is the most direct measure of risk-taking, since any changes in risk should show up in the end of year results. This contrasts with other measures such as non-performing loans or the Z-score in which it is not obvious at what lag changes will show up, since they are ex-post lagged indicators.
and the tier 1 to total asset ratio, to control for the bank’s leverage ratio. We also include the following bank-specific dummy variables: first, a dummy variable called “Tier 1 capital ratio treatment” (see 4), which is defined in a similar way to the leverage ratio treatment dummy, but in reference to the bank’s risk-weighted capital requirements. This is included so as to control for the concurrent strengthening of the risk-based capital framework (see BCBS (2009)), so that results captured by the leverage ratio treatment dummy are not wrongly capturing responses to changes in the risk-based capital framework. Second, a dummy variable called “Dummy ($LR \leq 3\%$)” is included in order to control for the general effect of having a leverage ratio below 3%. In particular, for all years in the sample, the dummy is set equal to 1 for a given bank and year if that bank’s leverage ratio in the previous year was below 3%. It is set to 0 otherwise. Third, dummy variables are included for whether in a particular year a bank uses internal risk-based (IRB) models in its risk-based framework. In particular, there is a dummy for whether a bank is advanced IRB, foundation IRB or a mixture. This is complemented with further dummy variables controlling for the Basel regime prevailing at the time, which can be seen in the dummies Basel II, Basel II.5 and Basel III in table 4. Lastly, the following macro variables are controlled for: GDP growth, inflation, government debt to GDP, and the change in unemployment rate, to control for the economic environment; the 10-year government bond yield, to control for the monetary environment, including capturing potential effects from the risk-taking channel of monetary policy; and the ratio of total credit to GDP, stock price growth, and house price growth since these factors may impact risk-taking incentives for banks.

Table 4 presents the baseline estimation results for the effect of the Basel III LRR announcement on the risk-taking behaviour of EU banks. In line with the first hypothesis, the results suggest that since the Basel III LRR framework was announced at the end of 2009, EU banks with low leverage ratios have slightly increased their risk-taking, as measured by their risk-weighted assets to total assets ratio. This conclusion is robust to various specifications and estimation methods. The estimated coefficients for the treatment effect are always positive and highly significant for virtually all model specifications. Column (1) illustrates that without controlling for the fact low risk banks have lower capital requirements, and thus often

29 In particular, the dummy is set equal to 1 for a given bank and year if its Tier 1 capital ratio in the previous year was below the new Basel III risk-based capital requirements (including buffers), but only for years after 2009. The dummy is set to 0 otherwise.

30 Dynamic panel GMM as in Arellano and Bond (1991) is used since a lagged dependent variable is introduced in the model. In the GMM estimation, GDP growth, the change in unemployment and the Basel regime variables are considered exogenous; all other variables are instrumented using lags of the variable in question.
Table 4: Estimated effect of the Basel III leverage ratio on bank risk-taking

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<td>-0.214*</td>
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<td>0.478**</td>
<td>0.588***</td>
<td>0.505***</td>
<td>0.516*</td>
<td>0.569*</td>
<td>0.535*</td>
<td>0.535*</td>
</tr>
<tr>
<td>Unempl. rate change, y-on-y</td>
<td>-0.489***</td>
<td>0.130</td>
<td>0.331</td>
<td>0.465</td>
<td>0.461*</td>
<td>0.741*</td>
<td>0.754*</td>
<td>0.699*</td>
<td>0.800**</td>
</tr>
<tr>
<td>10-year yield</td>
<td>0.266***</td>
<td>-0.311</td>
<td>-0.249</td>
<td>-0.125</td>
<td>-0.0899</td>
<td>-0.341</td>
<td>-0.956</td>
<td>-0.137</td>
<td>-0.147</td>
</tr>
<tr>
<td>Total credit / GDP</td>
<td>-1.296-06</td>
<td>-0.0114</td>
<td>-0.0130</td>
<td>-0.0463*</td>
<td>-0.0330</td>
<td>-0.0413*</td>
<td>-0.0140*</td>
<td>-0.0019*</td>
<td>-0.0072**</td>
</tr>
<tr>
<td>Stock price growth, y-on-y</td>
<td>0.0212**</td>
<td>-0.0311*</td>
<td>-0.00343</td>
<td>-0.0130</td>
<td>-0.00773</td>
<td>-0.0191</td>
<td>-0.0174</td>
<td>0.00680</td>
<td>0.00841</td>
</tr>
<tr>
<td>Home price growth, y-on-y</td>
<td>-0.0049</td>
<td>-0.286-05</td>
<td>-0.00221</td>
<td>-0.0016</td>
<td>0.00418</td>
<td>7.654-05</td>
<td>0.00265</td>
<td>-0.0312</td>
<td>-0.0118</td>
</tr>
<tr>
<td>Government Debt / GDP</td>
<td>-0.0092**</td>
<td>0.132***</td>
<td>0.0098***</td>
<td>0.0736***</td>
<td>0.0699***</td>
<td>0.0927***</td>
<td>0.0038*</td>
<td>0.0968***</td>
<td>0.0837**</td>
</tr>
<tr>
<td>Intercept</td>
<td>53.49***</td>
<td>6.595***</td>
<td>27.357***</td>
<td>24.937***</td>
<td>40.267***</td>
<td>27.557***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy (LR ≤ 3)</td>
<td>-24.737***</td>
<td>-1.944***</td>
<td>-3.412***</td>
<td>-1.316**</td>
<td>-1.994**</td>
<td>-1.990**</td>
<td>-0.800</td>
<td>-0.599</td>
<td>-0.740</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the risk-weighted assets to total assets ratio (expressed as a percentage). In all models, explanatory variables are lagged by one period to avoid endogeneity issues. The models in columns 1-2 are estimated with simple pooled OLS. Columns 3-5 are estimated with bank and time fixed-effects (FE), while column 6 performs the same estimation by instruments the lagged dependent variable with its second lag. Columns 7-10 are estimated using GMM, where the dependent variable, bank-specific variables and macro variables (excluding GDP growth, and the change in unemployment) are instrumented with their own lags. Columns 7 and 9 use all valid lags as GMM-style instruments, while columns 8 and 10 use all valid lags up to lag 7 as GMM-style instruments. GDP growth, the change in unemployment and Basel regime variables are treated as exogenous and are therefore used as IV-style instruments in columns 7-10. AR1-p, AR2-p and Hansen-p refer to the p-values of the Arellano-Bond tests for first- and second-order autocorrelation of the differenced residuals and exogeneity of the instruments using the Hansen J statistic respectively. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

What is more, while the Basel III LRR announcement seems to have incentivised slightly higher risk-taking, the concurrent strengthening of the risk-based capital framework under Basel III seems to have had the opposite effect. This can be explained by lower capital ratios, there is an inherent bias in the results that gives a negative coefficient. This is somewhat offset when controls are added in column (2), but once one properly controls for this via fixed and time effects, and various control variables, the coefficient turns positive and significant (columns (3)-(10)). In terms of the quantitative impact, the point estimates for the treatment effect of a 3% LRR suggest that banks bound by it increase their risk-weighted assets ratio by around 1.5 to 2.5 percentage points more than they otherwise would, which appears rather muted.
seen via the variable "Tier 1 capital ratio treatment". Specifically, the range of point estimates presented in Table 4, once bank and time fixed effects are controlled for, suggests that banks with Tier 1 capital ratios below their regulatory minimum reduced their risk-weighted asset ratios by around 0.3 to 2.3 percentage points more than they otherwise would have, after the strengthening of the risk-based capital framework under Basel III was announced. Hence, the small estimated effects on bank risk-taking of the Basel III leverage ratio announcement are not a result of the concurrent strengthening of the risk-based capital framework since this effect is controlled for.\(^31\)

The small estimated increase in risk-taking for banks bound by the Basel III LRR remains robust to various other tests, both quantitatively and in terms of statistical significance. First, columns (1) - (3) in Table 5 show that the result is robust to using different bank and country samples. Second, the results in columns (4) - (6) tackle potential concerns that banks with vastly different leverage ratios are fundamentally different through a Regression Discontinuity Design (RDD).\(^32\) By restricting the estimation sample to banks that are close to either side of the LRR threshold it is more likely that these banks exhibit similar ex-ante behaviour. This allows us to estimate a local average treatment effect (LATE). The optimal bandwidth around the LRR threshold is determined via the procedure proposed by Imbens and Kalyanaraman (2012), and then half and double this bandwidth is tested. As can be seen from columns (4) - (6), our core result is left unchanged. The treatment dummy remains significant at all different bandwidth levels (we experiment with different bandwidth levels for robustness) and the coefficient remains within a similar region of magnitude, namely between 1.24 and 1.57 percentage points.

Columns (7) - (10) of Table 5 tackle concerns related to potential misclassifications of the treatment and control groups, given that uncertainty remains over the final level of the leverage ratio threshold. Column (7) shows that the significant small increase in risk-taking remains when the model is re-estimated excluding all

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\(^31\)A similar dummy for the new Basel III liquidity requirements is also desirable, but due to data availability, it is not possible to construct such a dummy variable. A control variable for liquidity is however included, namely the loan-to-deposit ratio, but it is insignificant. Furthermore, we suggest that if there is a bias, the bias would lead us to overestimate the risk-taking effect. This is because under the new liquidity regulations, banks must hold a sufficient amount of high liquid low risk assets. Hence the current estimates may not be taking into account the downward pressure on risk-levels that the liquidity regulations would impose. Nevertheless, for robustness, the simulations in section 4.4 also consider higher levels or risk-taking up to the maximum possible before moving back into the risk-based framework, in order to capture any potential underestimate.

\(^32\)Without controls, the validity of difference-in-difference crucially relies on the identical ex-ante behaviour of banks in the control and treatment groups, so that it is only the treatment that generates differing behaviour, not differences among group participants.
Table 5: Robustness of the estimated effect on bank risk-taking

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage ratio treatment, 3%</td>
<td>1.678***</td>
<td>1.925*</td>
<td>2.217***</td>
<td>1.238*</td>
<td>1.305*</td>
<td>1.566**</td>
<td>2.284**</td>
<td>2.072**</td>
<td>1.571***</td>
<td>1.834***</td>
</tr>
<tr>
<td>Leverage ratio treatment 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage ratio treatment, 4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,325</td>
<td>1,476</td>
<td>646</td>
<td>1,010</td>
<td>545</td>
<td>1,767</td>
<td>1,754</td>
<td>2,550</td>
<td>2,550</td>
<td>2,550</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.086</td>
<td>0.074</td>
<td>0.111</td>
<td>0.161</td>
<td>0.254</td>
<td>0.126</td>
<td>0.105</td>
<td>0.092</td>
<td>0.093</td>
<td>0.096</td>
</tr>
<tr>
<td>Number of banks</td>
<td>529</td>
<td>324</td>
<td>107</td>
<td>274</td>
<td>185</td>
<td>433</td>
<td>506</td>
<td>583</td>
<td>583</td>
<td>583</td>
</tr>
</tbody>
</table>

Bank sample
- Western Europe
- W. Europe excl. GIPS
- SSM SIs
- All EU
- All EU
- All EU, LR \( \notin (3,5) \)
- All EU
- All EU
- All EU

Estimation method
- FE
- FE
- FE
- FE RDD, optimal
- FE RDD, half
- FE RDD, double
- FE
- FE
- FE
- FE

Notes: The dependent variable is the risk-weighted assets to total assets ratio (expressed in percentage points). The same set of control variables as in the second column of Table 4 are included in all of the regressions, including bank and time fixed-effects. All explanatory variables are lagged by one period to avoid endogeneity issues. All EU sample means estimation is based on all of the EU banks contained in the dataset. Western Europe represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Spain, Finland, France, the UK, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Sweden. The Western Europe excl. GIPS sample represents the Western Europe sample excluding banks from Greece, Italy, Ireland, Portugal and Spain. The SSM SIs sample includes only significant institutions (SIs) which are directly supervised by the ECB’s Single Supervisory Mechanism (SSM). All EU, LR \( \notin (3,5) \) excludes all observations where a given bank had a leverage ratio greater or equal than 3% and smaller or equal than 5%. RDD refers to a Regression Discontinuity Design that restricts the estimation sample to banks that are close to the leverage ratio threshold on either side. The optimal bandwidth is plus / minus 1.895 around the baseline 3% leverage ratio threshold. The leverage ratio treatment variables are dummy variables that indicate whether a given bank had a leverage ratio below the threshold level in the previous year, 1 year after 2009. The leverage ratio treatment 2 variable measures the one-sided distance from the required minimum level. Formally: treatment variable 2 = treatment dummy \( \cdot (LR_{min} - LR) \). *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

banks with leverage ratios between 3% and 5%.

In addition, column (8) in Table 5 suggests that the induced increase in risk-taking due to the LRR is smaller, the closer a bank is to the 3% leverage ratio threshold.

The coefficient estimate suggests that banks with leverage ratios of 1.5%, 2% and 2.5% adjusted their risk-weighted asset ratios upward by 3.1, 2.1 and 1.0 percentage points respectively. This fits well with intuition, since the incentive for additional risk-taking should be smaller if a bank is required to increase capital only slightly, say from a 2.9% to a 3% leverage ratio, compared to if a bank is required to increase capital by a lot, for instance from a 1% to a 3% leverage ratio. Finally, columns (9) - (10) in Table 5 show that significant coefficient estimates with similar magnitudes as before are obtained if the leverage ratio treatment dummy is based on a 4% and 5% minimum LRR. In summary, the results from the first stage empirical exercise suggest that an LRR appears to incentivize additional risk-taking for banks bound by it, but this additional risk-taking appears limited, as suggested by our theoretical model of section 3.

To shed more light on banks’ reactions to the Basel III LRR announcement, the risk-taking regressions are also re-estimated with the change in a bank’s leverage ratio as the dependent variable, to see if treated banks were increasing their leverage

\[33\] This is done as it may be that banks with leverage ratios between 3% - 5% are fuzzy in whether they should be classified as treatment or control group banks. Banks with leverage ratios just above 3% may also act to some extent. Therefore, excluding all bank observations with leverage ratios in this range should alleviate potential misclassification problems of the treatment and control groups.

\[34\] The 3% leverage ratio treatment 2 variable measures the one-sided distance from the required minimum level. Hence, if the leverage ratio of a bank was 1%, the treatment variable would be 2%. Formally: treatment variable 2 = treatment dummy \( \cdot (LR_{min} - LR) \).
### Table 6: Estimated effect on banks’ leverage ratios

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage ratio treatment, 3%</td>
<td>0.831***</td>
<td>0.790***</td>
<td>1.146***</td>
<td>0.429***</td>
<td>0.518***</td>
<td>0.718***</td>
<td>1.081***</td>
<td>0.999***</td>
<td></td>
</tr>
<tr>
<td>Leverage ratio treatment 2, 3%</td>
<td>0.652***</td>
<td>0.432***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage ratio treatment, 4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tier 1 capital ratio treatment</td>
<td>0.400***</td>
<td>0.354***</td>
<td>0.662***</td>
<td>0.142</td>
<td>-0.132</td>
<td>0.019</td>
<td>0.473***</td>
<td>0.400***</td>
<td>0.419***</td>
</tr>
<tr>
<td>Observations</td>
<td>2,631</td>
<td>2,393</td>
<td>648</td>
<td>1,021</td>
<td>544</td>
<td>1,807</td>
<td>1,826</td>
<td>2,631</td>
<td>2,631</td>
</tr>
<tr>
<td>Number of banks</td>
<td>602</td>
<td>538</td>
<td>107</td>
<td>280</td>
<td>186</td>
<td>451</td>
<td>524</td>
<td>602</td>
<td>602</td>
</tr>
<tr>
<td>Estimation method</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
<td>FE RDD, optimal</td>
<td>FE RDD, half</td>
<td>FE RDD, double</td>
<td>FE</td>
<td>FE</td>
<td>FE</td>
</tr>
<tr>
<td>Bank sample</td>
<td>Western Europe excl. GIPS</td>
<td>W. Europe excl. GIPS</td>
<td>SSM SIs</td>
<td>All EU</td>
<td>All EU</td>
<td>All EU</td>
<td>All EU</td>
<td>All EU</td>
<td>All EU</td>
</tr>
</tbody>
</table>

**Notes:**
- The dependent variable is the first difference of the leverage ratio (expressed in percentage points).
- The same set of control variables as in the second column of Table 4 are included in all of the regressions, including bank and time fixed-effects. All explanatory variables are lagged by one period to avoid endogeneity issues. All EU sample means estimation is based on all of the EU banks contained in the dataset. Western Europe excl. GIPS represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Finland, France, the UK, Luxembourg, the Netherlands, and Sweden. The SSM SIs sample includes only significant institutions (SIs) which are directly supervised by the ECB’s Single Supervisory Mechanism (SSM). All EU LR $\notin (3,5)$ excludes all observations where a given bank had a leverage ratio greater or equal than 3% and smaller or equal than 5%. RDD refers to a Regression Discontinuity Design that restricts the estimation sample to banks that are close to the leverage ratio threshold on either side. The optimal bandwidth is plus/minus 1.86% around the baseline 3% leverage ratio threshold. The leverage ratio treatment variables are dummy variables that indicate whether a given bank had a leverage ratio below the threshold level in the previous year, for years after 2009. The leverage ratio treatment 2 variable measures the one-sided distance from the required minimum level. Formally: treatment variable 2 = treatment dummy · ($LR - LR_{min}$). *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

... ratios at the same time as taking on further risk. This indeed seems to have been the case, as can be seen from Table 6, with estimates of around a 0.44 - 1.1 percentage point greater increases in a bank’s leverage ratio than would have otherwise happened. This result is again robust to different country and bank samples, running various RDD specifications, and assuming different treatment thresholds. This finding also provides further support for the assumption that banks already started to react to the Basel III LRR upon announcement in 2009, well before it is planned to migrate to a binding Pillar I regulatory requirement in 2018. To summarise, while treated banks may have increased their risk-weighted assets to total assets ratios by around 1.5 to 2.5 p.p. more, they also increased their leverage ratios by up to 1 p.p. more over the period of consideration. This is a considerable increase in a bank’s capital position relative to the estimated increase in risk-weighted assets.

### 4.3 Trade-off between loss-absorption and risk-taking

For the second part of the empirical analysis, we use our unique dataset of EU bank distress events in a discrete choice modelling framework, to analyse the joint effects of the leverage ratio and risk-taking on bank distress probabilities. Table 7 provides details of the distress events. As can be seen, in addition to direct failures, distress events also include state interventions and distressed mergers, since these are effectively failures. In total, there are 278 distress events. This analysis is crucial in order to quantify the net impact of the risk-stability trade-off associated with an LRR. As discussed in van den Berg et al. (2008), a logit model is preferred over
Table 7: Distress events by category

<table>
<thead>
<tr>
<th>Distress category</th>
<th>Composition</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct failure</td>
<td></td>
<td>33</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Liquidation</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Distressed merger</td>
<td></td>
<td>49</td>
</tr>
<tr>
<td>Merger with state intervention</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>Merger with coverage ratio &lt; 0</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>State intervention</td>
<td></td>
<td>221</td>
</tr>
<tr>
<td>Capital injection</td>
<td></td>
<td>165</td>
</tr>
<tr>
<td>Asset protection</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td>Asset guarantee</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>303</td>
</tr>
</tbody>
</table>

a probit model, because the fatter tailed error distribution matches better to the empirical frequency of bank distress events. While the early-warning literature has commonly used a pooled logit approach (see e.g. Lo Duca and Peltonen (2013)) we control for both time and country fixed-effects, since in-sample fit and unbiased coefficient estimates are more important for our analysis than optimising out-of-sample predictive performance.\(^{35}\) Specifically, various versions of the following logit model are estimated, where the left-hand-side variable is the binary distress indicator for bank \(i\), located in country \(j\), in year \(t + 1\), \(\gamma_j\) and \(\lambda_{t+1}\) are country and time fixed-effects respectively, and \(X_{i,j,t}\) and \(Y_{j,t}\) are vectors of bank-specific and country-specific control variables respectively: \(^{36}\)

\(^{35}\)Controlling for time and country fixed-effects should lead to better in-sample fit, as shown by Fuertes and Kalotychou (2006).

\(^{36}\)In particular, in addition to the leverage ratio and the ratio of risk-weighted assets to total assets ratio, the following control variables are included. Bank-specific variables include: non-performing loans (NPLs) to total assets, in order to control for the bank’s portfolio; pre-tax ROA, to control for profitability, since more profitable banks may have lower probabilities of distress; the coverage ratio, to control for the bank’s ability to meet its financial obligations; interest expenses to total liabilities, which allows us to control for the amount of interest a bank pays on its liabilities; the loan-to-deposit ratio, to control for liquidity; and log total assets, to control for size, since it is probable that larger banks have lower distress probabilities. As in the risk-taking regressions, we also control via dummy variables for each bank-year observation whether banks were IRB, and what Basel regime prevailed at the time. The macro-financial variables include: GDP growth, inflation, government debt to GDP, and the unemployment rate, to control for the economic environment; the change in the Bund-spread as a measure of country risk; the change in the banking sector’s issued debt to liabilities as a measure of indebtedness in the banking system; and total credit to GDP, private sector credit flow and stock price growth as further controls for the macroeconomic environment.
\[ P(I_{i,j,t+1} = 1) = \frac{\exp(\alpha + \theta'X_{i,j,t} + \varphi'Y_{j,t} + \gamma_j + \lambda_{t+1})}{1 + \exp(\alpha + \theta'X_{i,j,t} + \varphi'Y_{j,t} + \gamma_j + \lambda_{t+1})} \] (2)

Table 8 presents the main results from our bank distress analysis, where the leverage ratio is proxied by the ratio of Tier 1 equity to total assets and risk-taking is proxied by the ratio of risk-weighted assets to total assets, as in the first stage empirical exercise above. Columns (1) - (2) present the baseline estimation results excluding and including country and time fixed-effects. In line with economic intuition, the leverage ratio has a negative coefficient and risk-taking a positive coefficient. Most importantly, in comparison to risk-taking, the leverage ratio seems to be much more important for determining a bank’s distress probability, both statistically and economically. For example, models (1) and (2) suggest that a 1 p.p. increase in a bank’s leverage ratio is associated with around a 35-39% decline in the relative probability of distress to non-distress (the odds ratio).\(^{37}\) This is much larger than the marginal impact from taking on greater risk. The coefficient estimates suggest that increasing a bank’s risk-weighted assets ratio by 1 p.p. is associated with an increase in its relative distress probability of only around 1-3.5%. This demonstrates the relative importance of the leverage ratio in determining bank distress probabilities.

The other models in Table 8 show that the results are robust to introducing non-linear effects in the leverage ratio and risk-weighted assets ratio (columns (3) - (4)) and to considering different country and bank samples (columns (5) - (7)). Adding squared terms for both variables of interest and a cubic term for the leverage ratio indeed improves the fit of the model, as measured by the Pseudo R-squared and the Area Under the Receiver Operating Characteristics Curve (AUROC), as well as the statistical significance of the estimated effect of risk-taking on bank distress probabilities. Figure 6 illustrates graphically the estimated non-linear effects of the leverage ratio and risk-taking on bank distress probabilities obtained from model (4), which is the most complete specification. There seems to be considerable benefit for bank stability from increasing the leverage ratio from low levels, but as a bank’s leverage ratio gets to around 5% the benefits from increasing it further start to diminish slightly. Moreover, the marginal beneficial impact for bank stability of increasing the leverage ratio from low levels is much stronger than the marginal negative impact of increasing a bank’s risk-weighted assets. Columns (5) - (7) confirm that this result remains robust if we restrict the estimation sample to banks.

\(^{37}\)For a detailed discussion on the interpretation of logit coefficients, see Cameron and Trivedi (2005).
Table 8: Estimated effect of the leverage ratio and risk-taking on bank distress probabilities

<table>
<thead>
<tr>
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<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage ratio proxy</td>
<td>-0.510***</td>
<td>-0.427***</td>
<td>-1.046***</td>
<td>-3.206***</td>
<td>-3.957***</td>
<td>-5.188***</td>
<td></td>
</tr>
<tr>
<td>Leverage ratio proxy, squared</td>
<td>0.045***</td>
<td>0.403***</td>
<td>0.420***</td>
<td>0.580***</td>
<td>0.465</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage ratio proxy, cubed</td>
<td></td>
<td>-0.023***</td>
<td>-0.021**</td>
<td>-0.029***</td>
<td>-0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RWA / Total assets</td>
<td>0.035***</td>
<td>0.011</td>
<td>0.166***</td>
<td>0.202***</td>
<td>0.188***</td>
<td>0.251***</td>
<td>0.406***</td>
</tr>
<tr>
<td>RWA / Total assets, squared</td>
<td></td>
<td>-0.001***</td>
<td>-0.002***</td>
<td>-0.002***</td>
<td>-0.002***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPLs / Total assets</td>
<td>0.072***</td>
<td>0.055</td>
<td>0.090***</td>
<td>0.101***</td>
<td>0.098***</td>
<td>0.097**</td>
<td>0.117</td>
</tr>
<tr>
<td>Coverage ratio</td>
<td>-0.014***</td>
<td>-0.011**</td>
<td>-0.012***</td>
<td>-0.012***</td>
<td>-0.012***</td>
<td>-0.013***</td>
<td>-0.026***</td>
</tr>
<tr>
<td>Pre-tax ROA</td>
<td>-0.013</td>
<td>-0.082</td>
<td>-0.018</td>
<td>-0.001</td>
<td>-0.031</td>
<td>-0.001</td>
<td>-0.402</td>
</tr>
<tr>
<td>Interest expenses / Total liabilities</td>
<td>0.203***</td>
<td>0.152**</td>
<td>0.127***</td>
<td>0.140***</td>
<td>0.132***</td>
<td>0.147***</td>
<td>0.149</td>
</tr>
<tr>
<td>Loan-to-Deposit ratio</td>
<td>0.092**</td>
<td>0.092***</td>
<td>0.003***</td>
<td>0.003***</td>
<td>0.003***</td>
<td>0.003***</td>
<td>0.005***</td>
</tr>
<tr>
<td>Total assets, log</td>
<td>0.314***</td>
<td>0.345***</td>
<td>0.323***</td>
<td>0.334***</td>
<td>0.330***</td>
<td>0.341***</td>
<td>0.438***</td>
</tr>
<tr>
<td>Basel II dummy</td>
<td>0.698*</td>
<td>0.175</td>
<td>-0.001</td>
<td>0.018</td>
<td>0.171</td>
<td>-0.104</td>
<td>-1.206</td>
</tr>
<tr>
<td>Basel II.5 dummy</td>
<td>-1.180</td>
<td>-1.256</td>
<td>-1.492</td>
<td>-1.632</td>
<td>-1.214</td>
<td>-1.660</td>
<td>-2.539</td>
</tr>
<tr>
<td>Advanced IRB dummy</td>
<td>-1.967***</td>
<td>-1.751***</td>
<td>-1.591***</td>
<td>-1.733***</td>
<td>-1.844***</td>
<td>-1.702***</td>
<td>-0.496</td>
</tr>
<tr>
<td>Foundations IRB dummy</td>
<td>0.627</td>
<td>0.612</td>
<td>0.527</td>
<td>0.537</td>
<td>0.625</td>
<td>0.564</td>
<td>1.125*</td>
</tr>
<tr>
<td>Mix IRB / SA dummy</td>
<td>0.222</td>
<td>0.116</td>
<td>0.088</td>
<td>0.127</td>
<td>0.126</td>
<td>0.115</td>
<td>1.711***</td>
</tr>
<tr>
<td>Bund-spread, y-on-y change</td>
<td>0.284***</td>
<td>0.495*</td>
<td>0.515**</td>
<td>0.485*</td>
<td>0.553*</td>
<td>0.354*</td>
<td>1.882</td>
</tr>
<tr>
<td>Government Debt / GDP</td>
<td>0.009*</td>
<td>-0.067***</td>
<td>-0.070***</td>
<td>-0.073***</td>
<td>-0.096***</td>
<td>-0.090***</td>
<td>0.050</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.105***</td>
<td>0.218**</td>
<td>0.185**</td>
<td>0.182**</td>
<td>0.262***</td>
<td>-0.002*</td>
<td>-0.725*</td>
</tr>
<tr>
<td>GDP growth, y-on-y</td>
<td>-0.009</td>
<td>-0.211</td>
<td>-0.156</td>
<td>-0.163</td>
<td>-0.209</td>
<td>-0.222</td>
<td>-0.470</td>
</tr>
<tr>
<td>Inflation, y-on-y</td>
<td>-0.099</td>
<td>-0.867***</td>
<td>-0.856***</td>
<td>-0.886***</td>
<td>-0.910***</td>
<td>-0.735***</td>
<td>-1.074*</td>
</tr>
<tr>
<td>Private sector credit flow</td>
<td>0.060***</td>
<td>0.096***</td>
<td>0.103***</td>
<td>0.108***</td>
<td>0.137***</td>
<td>-0.014</td>
<td>-0.027</td>
</tr>
<tr>
<td>Total credit / GDP</td>
<td>0.001</td>
<td>0.047**</td>
<td>0.057***</td>
<td>0.056**</td>
<td>0.089***</td>
<td>0.046*</td>
<td>-0.04</td>
</tr>
<tr>
<td>Bank issued debt / Liabilities, y-on-y change</td>
<td>-0.095</td>
<td>-0.200*</td>
<td>-0.205</td>
<td>-0.227*</td>
<td>-0.387***</td>
<td>-0.159</td>
<td>0.163</td>
</tr>
<tr>
<td>Stock price growth, y-on-y</td>
<td>-0.011</td>
<td>0.038*</td>
<td>0.039*</td>
<td>0.041**</td>
<td>0.044*</td>
<td>0.043</td>
<td>0.115</td>
</tr>
<tr>
<td>Intercept term</td>
<td>-6.157***</td>
<td>-26.26***</td>
<td>-29.96***</td>
<td>-26.80***</td>
<td>-34.80***</td>
<td>-22.84***</td>
<td>-10.76</td>
</tr>
</tbody>
</table>

Observations: 1,661
Pseudo R2: 0.284
AUROC: 0.870

Country and time fixed-effects: No, Yes
Non-linear effects: No, Yes
Bank sample: All EU, All EU, All EU, All EU, Euro Area, Western Europe, W. Europe excl. GIIPS

Notes: Logit model estimates are obtained on a binary bank distress variable (See Betz et al. (2014) and Lang et al. (2015) for details on the bank distress event definitions). The numbers in the table are logit model coefficients. All right hand side variables are lagged by one year. All EU sample means estimation is based on all the EU banks contained in the dataset. The Euro Area sample only includes banks from the 19 Euro Area countries. Western Europe represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Spain, Finland, France, the UK, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Sweden. The Western Europe excl. GIIPS sample represents the Western Europe sample excluding banks from Greece, Italy, Ireland, Portugal and Spain. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors. AUROC refers to the Area Under the Receiver Operating Characteristics Curve, which is a global measure of how well the model can classify observations into distressed and non-distressed periods. An uninformative model has an AUROC of 0.5, while a perfect model has an AUROC of 1.
4.4 Net effect of a leverage ratio constraint on bank stability

The two previous empirical exercises suggest that while constrained banks slightly increase risk-taking with an LRR, the concurrent increase in their Tier 1 to asset ratio appears more important for bank stability considerations. To analyse this more formally, the results from the bank distress model are combined with the estimated increase in risk-taking in a counterfactual simulation. The simulation proceeds as follows. We first take all bank-year observations in our sample where the bank had a leverage ratio below the relevant minimum, and compute the associated distress probabilities using the true data. We then compute counterfactual distress probabilities for the same set of bank-year observations, assuming that banks increase their leverage ratios up to the required minimum, but at the same time also increase their risk-weighted assets by the estimated amount. Finally, we look at the average change in distress probabilities across all the relevant bank-year observations in the sample to see whether bank distress probabilities decline and whether any decline is statistically significant. In this way, we attempt to assess quantitatively the net effect of the potential trade-off between greater loss-absorbing capacity and higher bank risk-taking associated with an LRR. To allow for a conservative assessment, the mid-point in the range of the estimated increase in risk-taking is assumed, i.e.
Table 9: Simulated change in average bank distress probabilities

<table>
<thead>
<tr>
<th>LR threshold:</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>4%</th>
<th>5%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks with an LR of:</td>
<td>Less than 3%</td>
<td>Between 3-4%</td>
<td>Between 4-5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta(RWA/TA) = 2)</td>
<td>-0.840***</td>
<td>-0.993***</td>
<td>-1.000***</td>
<td>-0.645***</td>
<td>-0.985***</td>
<td>-0.662***</td>
</tr>
<tr>
<td>(\Delta(RWA/TA) = 4)</td>
<td>-0.768***</td>
<td>-0.990***</td>
<td>-1.000***</td>
<td>-0.471**</td>
<td>-0.978***</td>
<td>-0.501**</td>
</tr>
<tr>
<td>(\Delta(RWA/TA) = 6)</td>
<td>-0.664***</td>
<td>-0.985***</td>
<td>-0.999***</td>
<td>-0.212</td>
<td>-0.967***</td>
<td>-0.271</td>
</tr>
<tr>
<td>(\Delta(RWA/TA) = max)</td>
<td>-0.407</td>
<td>-0.809</td>
<td>-0.915</td>
<td>-0.0415</td>
<td>-0.586</td>
<td>-0.120</td>
</tr>
<tr>
<td>Number of Banks</td>
<td>91</td>
<td>91</td>
<td>91</td>
<td>88</td>
<td>88</td>
<td>128</td>
</tr>
</tbody>
</table>

Notes: The numbers represent the average simulated percentage change in the distress probability for the relevant bank sample between 2005 - 2014, expressed as decimal numbers (i.e. 0.1 represents 10%). Changes in distress probabilities are derived as follows. First, distress probabilities are estimated using the underlying data. Second, each bound bank has its leverage ratio increased to the stated percentage (e.g. 3%), while at the same time increasing its risk-weighted assets ratio by the stated amount (e.g. 2 p.p.). Using this adjusted data, new distress probabilities are estimated and the percentage change is taken. The table reports median values, where the median changes are reported separately for the sample of banks with a leverage ratio less than 3%, between 3-4% and between 4-5%. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on bootstrapped standard errors on 10,000 replications.

Table 9 reports median estimated figures from the various simulations. The numbers can be interpreted as the average percentage change in distress probability for the relevant banks in our sample between 2005 and 2014. So, for example, if a bank had a probability of distress of 0.02, a change of -0.840 (reported in the first row), would imply a fall by 84% to 0.0032. Since increasing the leverage ratio minimum increases the sample of banks below this minimum, to ensure comparability across simulations, results are reported separately for the sample of banks with a leverage ratio less than 3%, between 3-4% and between 4-5%. The results demonstrate that bank distress probabilities should significantly decline with an LRR, even when taking into account much higher increases in risk-taking than were estimated. For example, Table 9 shows that assuming a 3% LRR and an increase in the risk-weighted assets ratio of 2 p.p., the average distress probability declines by 84% for the given sample of bank-years. If the increase in the risk-weighted assets ratio is assumed to be 6 p.p., the average decline in distress probabilities would still be 66.4%. Even if we assume that banks increase their risk-weighted assets ratio by the maximum amount possible before moving back into the risk-based framework (denoted \(\Delta(RWA/TA) = max\) in the table), the results still indicate that bank distress probabilities will decline, although the result becomes insignificant. The simulation results therefore lend support to the second hypothesis, namely that the beneficial impact of higher capital holdings from an LRR should more than outweigh
the negative impact of increased risk-taking, thus leading to more stable banks.

5 Conclusion

Theoretical considerations and empirical evidence for EU banks provided in this paper suggest that the introduction of an LRR into the Basel III regulatory framework should lead to more stable banks. This paper has shown that although there can indeed exist an increased incentive to take risk once banks become bound by the LRR, this increase should be more than outweighed by the synchronous increase in loss-absorbing capacity due to higher capital. The analysis therefore supports the introduction of an LRR alongside the risk-based capital framework. The analysis further suggests that the LRR and the risk-based capital framework reinforce each other by covering risks which the other is less able to capture; making sure banks do not operate with excessive leverage and at the same time, have sufficient incentives for keeping risk-taking in check.
References


Appendix A: \((1 - \mu) > \alpha\)

This section solves the model under the alternative assumption that \((1 - \mu) > \alpha\). This influences the model in respect to the risk-based capital requirement. To see that it entails a different capital requirement, consider how this assumption maps into the requirement that on each asset banks must cover all shocks with some probability \((1 - \alpha)\). If \((1 - \mu) > \alpha\), then ensuring survival in only state \(s_1\) is no longer sufficient, therefore the capital charge on each asset must also ensure that some shocks in state \(s_2\) are covered. Consider the safe asset, since \((1 - \mu) > \alpha\), the capital charge on the safe asset must ensure that banks survive an additional shock in state \(s_2\), but there is only one additional shock in state \(s_2\), and thus \(k_{safe} = \lambda_1\); anything less would violate the requirement. Consider the risky asset, in state \(s_2\) the risky asset returns \((1 - \lambda_3)\) with probability \(\pi\) and 0 otherwise. If the bank therefore holds capital of only \(\lambda_{risky}\), it will fail to cover shocks with probability \((1 - \mu)(1 - \pi)\). Hence, if \((1 - \mu)(1 - \pi) \leq \alpha\), this is sufficient, and the capital charge on the risky asset will be \(k_{risky} = \lambda_3\). On the other hand, if \((1 - \mu)(1 - \pi) > \alpha\), \(\lambda_3\) is not sufficient to satisfy the requirement, and the capital charge on the risky asset will be \(k_{risky} = 1\). This second case is less realistic since it implies a zero probability of default; the risk-based capital requirement is so high that it covers all shocks. Nevertheless, we take both cases and show that the main results found in section 3 continue to apply.\(^{38}\)

Since \((1 - \mu) > \alpha\), the new capital requirement will be:

\[
k(\omega) = \begin{cases} 
\omega\lambda_1 + (1 - \omega)\lambda_3 & \text{if } (1 - \mu)(1 - \pi) \leq \alpha \\
\omega\lambda_1 + (1 - \omega)1 & \text{if } (1 - \mu)(1 - \pi) > \alpha 
\end{cases}
\]

Aside from this strengthened capital requirement, the problem will be identical to section 3.1. We begin by showing that as before for any \(\omega \in [0, 1]\), banks always wish to hold as little capital as possible, and therefore the capital requirement will bind.

First consider \(\omega = 1\). If this is the case, profits will be given by \(\mu R_1 + (1 - \mu)(1 - \lambda_1) - (1 - k) - \rho k - c(1)\). Clearly since \(\rho > 1\), this is maximised at \(k = 0\), and hence banks will choose the minimum capital level (this is true for any value of

\(^{38}\)While this section presents results assuming \((1 - \mu) > \alpha\), the results equally apply to the case in which \((1 - \mu) \leq \alpha\) but the risk-based capital framework is strengthened. This has the same effect as altering the assumption on the probabilities, namely increasing the capital charge on each asset, and thus it is equivalent
Consider now \(\omega \in [0, 1)\). Take the case in which \((1 - \mu)(1 - \pi)\leq \alpha\). Banks will prefer to hold the minimum capital and thus make the requirement bind if and only if profits under a binding capital requirement are higher than holding excess capital. To see that this is the case, first see that when the capital requirement binds, banks will only survive state \(s_2\) if the risky asset returns its residual value in this state, i.e. \((1 - \lambda_3)\). This is the case if and only if:

\[
\omega(1 - \lambda_1) \leq (1 - k(\omega))
\]

\[
0 \leq (1 - \omega)(1 - \lambda_3)
\]

which is true since \(\omega \in [0, 1)\) and \(\lambda_3 \in (0, 1)\). So if the capital requirement binds, and \((1 - \mu)(1 - \pi)\leq \alpha\), the bank can only survive if the risky asset pays off its residual value \((1 - \lambda_3)\) in state \(s_2\).

Compare profits under a binding capital requirement and when the bank holds excess capital. The bank will prefer the capital requirement to bind if and only if profits under a binding capital requirement are higher than holding excess capital, namely:

\[
\mu[\omega R_1 + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi
- (1 - k(\omega)) [\mu + (1 - \mu)\pi] - \rho k(\omega) - c(\omega)
\]

\[
\mu[\omega R_1 + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi
- (1 - k_{ex}) - \rho k_{ex} - c(\omega)
\]

where \(k_{ex} > k(\omega)\).

Rearranging, this is true if and only if:

\[
\rho > \mu + (1 - \mu) \frac{k_{ex} - (1 - \omega) [(1 - \pi) + \lambda_3 \pi] - \omega \lambda_1 (1 - \pi)}{k_{ex} - (1 - \omega) \lambda_3}
\]

which holds since \(\rho > 1\) and \(\mu + (1 - \mu) \frac{k_{ex} - (1 - \omega) [(1 - \pi) + \lambda_3 \pi] - \omega \lambda_1 (1 - \pi)}{k_{ex} - (1 - \omega) \lambda_3} < 1\) as \(\mu \in [0, 1]\) and \(\frac{k_{ex} - (1 - \omega) [(1 - \pi) + \lambda_3 \pi] - \omega \lambda_1 (1 - \pi)}{k_{ex} - (1 - \omega) \lambda_3} < 1\) since \([k_{ex} - (1 - \omega) [(1 - \pi) + \lambda_3 \pi] - \omega \lambda_1 (1 - \pi)] < [k_{ex} - (1 - \omega) \lambda_3 - \omega \lambda_1 (1 - \pi)] < [k_{ex} - (1 - \omega) \lambda_3]\).

Now consider \((1 - \mu)(1 - \pi) > \alpha\). Since this case implies a zero probability of
default, profits will be given by

$$
\mu [\omega R_1 + (1 - \omega)\pi R_2^b + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu) [\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)\pi] - (1 - k) - \rho k - c(\omega)
$$

where $k \geq k(\omega)$. Since $\rho > 1$, for any $\omega$, this is maximised by minimising $k$, i.e. the bank will hold as little capital as possible and thus the capital requirement will bind.

Therefore, as in section 3.3, banks always wish to hold the minimum capital requirement. Since this is the case, the risk-based capital requirement will impact risk-taking decisions. Suppose the LRR does not exist, then the first order condition that determines optimal bank risk-taking is given by:

$$
\begin{cases}
\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1] - (1 - \mu) [\lambda_3 - \lambda_1] = -k'(\omega) [\rho - (\mu + (1 - \mu)\pi)] - c'(\omega) & \text{if } (1 - \mu)(1 - \pi) \leq \alpha \\
\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1] - (1 - \mu) [(1 - \lambda_1) - (1 - \lambda_3)\pi] = -k'(\omega)[\rho - 1] - c'(\omega) & \text{if } (1 - \mu)(1 - \pi) > \alpha
\end{cases}
$$

As can be seen, as previously, the risk-based capital requirement disincentivises risk-taking, and this can be seen in the $k'(\omega)$ terms on the RHS of both FOCs. Let us compare these to the risk level chosen under an LRR. As before, we know there are two cases that can arise. Firstly, the risk level can be set by the FOC. Secondly, if this level is not sufficient to satisfy the shareholders’ participation constraint at the given LRR, the risk level can be set by the shareholders’ participation constraint itself. By definition, the risk level set by the shareholders’ participation constraint must be greater than the risk level chosen under the FOC, otherwise the original level would have satisfied the participation constraint. Thus, it is sufficient to show that the risk level chosen under the FOC is larger than the risk level under a solely risk-based framework. Suppose $\omega$ is set by the FOC therefore, if $(1 - \mu)(1 - \pi) \leq \alpha$, the FOC with respect to $\omega$ is given by:

$$
\begin{cases}
\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1] - (1 - \mu) [\lambda_3 - \lambda_1] = -c'(\omega) & \text{if } \omega(1 - \lambda_1) < (1 - k_{lev}) \\
\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1] - (1 - \mu) [(1 - \lambda_1) - (1 - \lambda_3)\pi] = -c'(\omega) & \text{if } \omega(1 - \lambda_1) \geq (1 - k_{lev})
\end{cases}
$$

whereas if $(1 - \mu)(1 - \pi) > \alpha$, the FOC is given by:

$$
\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1] - (1 - \mu) [(1 - \lambda_1) - (1 - \lambda_3)\pi] = -c'(\omega)
$$
When the LRR binds, all $k'(\omega)$ terms disappear, thereby reducing the marginal cost of risk-taking (the RHS). Consider first the case where $(1-\mu)(1-\pi) > \alpha$, clearly the $\omega$ that solves the FOC under a binding LRR is smaller than the $\omega$ that solves the FOC under a risk-based capital requirement - the LHS is identical, whereas the RHS is smaller. Consider now $(1-\mu)(1-\pi) \leq \alpha$. There are two cases to the FOC since it may be possible that at a higher LRR, even if the risky asset returns 0 in state $s_2$, the bank’s capital covers all losses. What can be seen by comparing these two cases is that risk is lower in the bottom case - this is the skin-in-the-game effect we discussed earlier. Comparing the upper case to the FOC under a solely risk-based framework, and it is clear that risk increases with an LRR; the LHS is identical, yet the RHS is smaller since all terms relating to $k'(\omega)$ disappear. There is no skin-in-the-game effect here due to the discrete nature of the set-up. The skin-in-the-game effect appears when banks survive an additional shock, and that is when the bottom case applies. As before, we show that risk can still be larger than under a solely risk-based framework. Plugging in the functional forms, risk under a binding LRR will be larger than under the risk-based requirement iff:

$$(\lambda_3 - \lambda_1) \left[ \rho - [\mu + (1-\mu)\pi] \right] > (1-\mu)(1-\pi)(1-\lambda_1)$$

Rearranging, this becomes:

$$\rho > \left[ \mu + (1-\mu)\pi \right] + (1-\mu)(1-\pi) \frac{(1-\lambda_1)}{(\lambda_3 - \lambda_1)}$$

So as before, if $\rho$ is sufficiently expensive, banks will always increase risk-taking under a binding LRR, where since we are in the case in which $(1-\mu)(1-\pi) \leq \alpha$, with $\alpha = 0.001$, $\rho$ does not need to be very large to exceed this.

Imposing an LRR therefore always incentivises banks to increase risk if equity is sufficiently expensive. Yet, as before, this does not imply an LRR is detrimental. In order to consider the consequences of imposing an LRR, we must consider this increase in risk-taking in comparison to loss absorbing capacity, and we do so as before in respect to the effect on the probability of default, and the expected loss of deposit funds. Let us first consider the less realistic case in which $(1-\mu)(1-\pi) > \alpha$. This implied the capital charge on the safe asset was $\lambda_1$ and the capital charge on the risky asset was 1. This implies a zero probability of default and hence a zero expected loss of deposit funds. Imposing an LRR will thus also yield a zero probability of [39]For this case to exist however, it must be that the bank invested very little into the risky asset, and as will be discussed further below, due to the shareholders’ participation constraint, this may not be possible.
default and a zero expected loss of deposit funds, hence it is weakly better. This case is unrealistic since it implies a zero probability of default under the risk-based framework, but illustrates even in this case that the LRR does not worsen outcomes.

Consider the more realistic case now, \((1 - \mu)(1 - \pi) \leq \alpha\). Under the risk-based framework, the probability of default is \((1 - \mu)(1 - \pi)\) since it defaults only if the risky asset pays off 0 in state \(s_2\). We show that under an LRR, the probability of default cannot be higher, and can be strictly lower. Suppose the bank takes the maximum possible risk: \(k(\omega) = \omega_{max} \lambda_1 + (1 - \omega_{max}) \lambda_3 = k_{lev}\), so \(\omega_{max} = \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)}\).

The probability of default will be at least as low as under a solely risk-based framework iff it can survive when the risky asset pays off \((1 - \lambda_3)\) in state \(s_2\), i.e.:

\[
\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{lev})
\]

Rearranging,

\[
\frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)}(1 - \lambda_1) + (1 - \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)})(1 - \lambda_3) \geq (1 - k_{lev})
\]

\[
(1 - k_{lev}) \geq (1 - k_{lev})
\]

Both sides are equalised, so the bank will survive a \(\lambda_3\) state \(s_2\) shock even if it takes the maximum risk.  

Let’s now consider when an LRR leads to a strict decline in the probability of default. This will be true iff:

\[
\omega(1 - \lambda_1) \geq (1 - k_{lev})
\]

\[
k_{lev} \geq 1 - \omega_{lev}(1 - \lambda_1)
\]

So there can exist a region where the probability of default is strictly lower. Overall therefore, imposing an LRR weakly decreases the probability of default.

Consider the expected loss of deposit funds now. Under the risk-based framework, the expected loss of deposit funds, which we denote as \(EL_{rw}\) will be:

\[
EL_{rw} = (1 - \mu)(1 - \pi) \left[(1 - k(\omega)) - \omega_{rw}(1 - \lambda_1)\right]
\]

\(\omega_{max}\) as defined above only applies for \(k_{lev} \leq \lambda_3\), nevertheless for levels above \(\lambda_3\), banks will still survive a \(\lambda_3\) shock by definition that they hold more capital than \(\lambda_3\).
Under an LRR, where \( k_{lev} > k(\omega) \), the expected loss of deposit funds will be:

\[
\max\{(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)], 0\}
\]

We start by showing that unlike distress probabilities, if the bank takes the maximal risk, denoted \( \omega_{max} \), the expected loss of deposit funds will be larger under an LRR. To see this, first see that if banks take the maximum risk, it is not possible to survive all shocks - banks will only survive state \( s_2 \) if the risky asset pays off its residual value \( (1 - \lambda_3) \). Thus the expected loss of deposit funds will be positive.

To survive both states of the world, it must be that \( \omega_{max} (1 - \lambda_1) > (1 - k_{lev}) \). Taking the maximal risk implies \( \omega_{lev} = \omega_{max} = \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)} \). Plugging this in and rearranging, the above expression simplifies to \( k_{lev} < \lambda_1 \), but this is a contradiction, it is not possible for \( k_{lev} < \lambda_1 \) since \( k_{lev} \geq k(\omega_{rw}) = \omega_{rw} \lambda_1 + (1 - \omega_{rw}) \lambda_3 \geq \lambda_1 \). Thus if the bank takes the maximum risk, the bank can only survive state \( s_2 \) if the risky asset also pays off its residual value \( (1 - \lambda_3) \) in this state. Hence, the expected loss of deposit funds will given by \( (1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)] > 0 \).

Given this, suppose the bank indeed takes the maximum possible risk level, plugging this into \( (1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)] \) and rearranging, we find that the expected loss of deposit funds will be lower under an LRR iff:

\[
k_{lev} > k(\omega) + \left[ \omega_{rw} - \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)} \right] (1 - \lambda_1)
\]

This simplifies to

\[
k_{lev} [\lambda_3 - 1] > k(\omega) [\lambda_3 - 1]
\]

but since \( \lambda_3 < 1 \), this is a contradiction as \( k_{lev} > k(\omega) \). So if the bank takes maximal risk, the expected loss of deposit funds will be larger under an LRR.

This suggests that if the bank takes too much risk under an LRR (i.e. approaches the maximal risk), the expected loss of deposit funds will be larger under an LRR. As noted before, there are two cases which determine the bank’s risk-taking. First, the bank’s optimal risk choice can be determined by its FOC. Second, it is possible that this optimal risk-level is not sufficient to satisfy the shareholders’ participation constraint and risk will be pinned down by the participation constraint. Let us take each case in turn.

Consider the first case in which the level of risk is pinned down by the FOC. Suppose the LRR is set just above the risk-based capital requirement such that the
expected loss of deposit funds is positive. Under this case, the expected loss of deposit funds will be lower under an LRR if:

\[
(1 - \mu)(1 - \pi) \left[(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)\right] < (1 - \mu) (1 - \pi) \left[(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1)\right]
\]

Plugging in the optimal solution and rearranging, we find:

\[
k_{lev} - k(\omega) > \omega_{rw} - \omega_{lev} (1 - \lambda_1)
\]

\[
k_{lev} > k(\omega) + (\lambda_3 - \lambda_1) [\rho - [\mu + (1 - \mu)\pi]] (1 - \lambda_1)
\]

\(k_{lev}\) can be set at any level greater than \(k(\omega)\), so there exists a region just above \(k(\omega)\) in which the expected loss of deposit funds is greater under an LRR - there is risk-shifting but little loss absorption.

Consider all \(k_{lev}\) above this level then, from above, we know that as long as the bank can choose its optimal level of risk (i.e. set by the FOC), \(\omega_{lev}\) will either stay constant or increase in \(k_{lev}\). Hence, if this is the case, as is clear from the expected loss of deposit funds function under a LRR, as \(k\) increases, the expected loss of deposit funds under an LRR will decrease and hence the expected loss of deposit funds will be strictly lower under an LRR for all \(k_{lev}\) greater than this level.

However, this optimal risk level must also be feasible, namely the solution must be less than the maximum possible risk level; so we must add an extra condition. As can be readily seen from the maximal possible risk level, \(\omega_{max} = \frac{\lambda_3 - k_{lev}}{(\lambda_3 - \lambda_1)}\), this function is decreasing in \(k_{lev}\). At low \(k_{lev}\) the bank’s interior solution may be larger than this maximal possible risk level, whereas at higher \(k_{lev}\), the interior solution is possible. To be beneficial in terms of the expected loss of deposit funds therefore, we must impose that the solution be an interior one, i.e. \(\omega_{lev} > \frac{\lambda_3 - k_{lev}}{(\lambda_3 - \lambda_1)}\) or \(k_{lev} > \omega_{lev}^* \lambda_1 + (1 - \omega_{lev}^*) \lambda_3\) where \(\omega_{lev}^*\) is the optimal risk choice.

Combining these two conditions, and denoting \(k_1\) the maximum of these conditions, we can conclude that when the optimal risk level is set by the FOC, to be beneficial in terms of the expected loss of deposit funds, \(k_{lev}\) must be set above \(k_1\), i.e.

\[
k_{lev} > k_1 \equiv \max\{\omega_{lev} \lambda_1 + (1 - \omega_{lev}) \lambda_3, k(\omega_{rw}) + (\lambda_3 - \lambda_1) [\rho - [\mu + (1 - \mu)\pi]] (1 - \lambda_1)\}
\]

Let us now consider the second case in which the shareholders’ participation constraint determines \(\omega\). As seen from above, for a \(k > k_1\) to be detrimental, it must be that we are in this second case, as otherwise the expected loss of deposit funds (under an LRR) will decrease in \(k\) and thus always be better above \(k_1\). When the shareholders’ participation constraint determines \(\omega\), risk is increasing in \(k_{lev}\), so
in this case, it may be that any benefit from an increase in \( k_{lev} \) is offset by increased holdings of the risky asset.

Suppose then that the shareholders’ participation constraint determines \( \omega_{lev} \). We show that for \( \rho \leq \hat{\rho} \) (defined below), \( \omega_{lev} \) will not decline fast enough to lead to a detriment, yet for \( \rho > \hat{\rho} \), the increase in risk-taking can outweigh increased loss absorption.

To be detrimental, it must be that

\[
(1 - \mu)(1 - \pi) \left[ (1 - k_{lev}) - \omega_{lev} (1 - \lambda_1) \right] > E_{rw}
\]

\[
\Leftrightarrow (1 - \lambda_1)(\omega_{rw} - \omega_{lev}) > (k_{lev} - k(\omega_{rw}))
\]

where \( \omega_{lev} \) is set by the shareholders’ participation constraint, i.e.

\[
\omega_{lev} = \frac{\mu R_1 + \mu (1 - \pi) (1 - \lambda_2) + (1 - \mu) \pi (1 - \lambda_3) - (\rho - [\mu + (1 - \mu) \pi]) k - [\mu + (1 - \mu) \pi]}{\mu R_1 + (1 - \mu)\pi (1 - \lambda_1)}
\]

Define \( E_c(\text{risky}) = [\mu R_1 + \mu (1 - \pi) (1 - \lambda_2) + (1 - \mu) \pi (1 - \lambda_3)] \) and \( E_c(\text{safe}) = [\mu R_1 + (1 - \mu)\pi (1 - \lambda_1)] \), we can rewrite this condition as:

\[
k_{lev} \left[ 1 - \frac{(1 - \lambda_1)(\rho - (\mu + (1 - \mu) \pi))}{E_c(\text{risky}) - E_c(\text{safe})} \right] < k(\omega_{rw}) \left[ 1 - \frac{(1 - \lambda_1)(\rho - (\mu + (1 - \mu) \pi))}{E_c(\text{risky}) - E_c(\text{safe})} \right]
\]

\[
+(1 - \lambda_1) \frac{[\omega_{rw}[E_c(\text{risky}) - E_c(\text{safe})] - [E_c(\text{risky}) - (\mu + (1 - \mu) \pi)] + (\rho - (\mu + (1 - \mu) \pi)(\omega_{rw} \lambda_1 + (1 - \omega_{rw}) \lambda_3)]}{E_c(\text{risky}) - E_c(\text{safe})}
\]

The second term on the RHS is negative, which can be seen as follows. By the shareholders’ participation constraint:

\[
\omega_{rw} E_c(\text{safe}) + (1 - \omega_{rw}) E_c(\text{risky}) - (1 - k(\omega_{rw})) (\mu + (1 - \mu) \pi) \geq \rho k(\omega_{rw})
\]

\[
\Leftrightarrow E_c(\text{risky}) - (\mu + (1 - \mu) \pi) - (\rho - (\mu + (1 - \mu) \pi))(\omega_{rw} \lambda_1 + (1 - \omega_{rw}) \lambda_3) \geq \omega_{rw}[E_c(\text{risky}) - E_c(\text{safe})]
\]

So the second term on the RHS must be negative. Since this is the case, we can immediately state that if \( \rho \leq (\mu + (1 - \mu) \pi) + E_c(\text{risky})/(1 - \lambda_1) - E_c(\text{safe})/(1 - \lambda_1) \), as \( k_{lev} \geq k(\omega) \), this will never hold. Whereas, if \( \rho > (\mu + (1 - \mu) \pi) + E_c(\text{risky})/(1 - \lambda_1) - E_c(\text{safe})/(1 - \lambda_1) \), this simplifies to:

\[
k_{lev} > \frac{k_1}{\mu} \equiv k(\omega_{rw})
\]
bound level was defined at the point where \( \rho > \hat{\rho} \), if this to be possible, it must be that the increase in loss absorption is outweighed by the increase in risk-taking. For this to hold it must be that (1 - \( k_1 \)) > \omega \), since the shareholders’ participation constraint does not determine \( \omega \). Nevertheless we can immediately conclude that for any \( \rho \leq \hat{\rho} \equiv \max \{(\mu + (1 - \mu)\pi) + \frac{[E_c(\text{risky}) - E_c(\text{safe})]}{(1 - \lambda_1)}), \hat{\rho} \}, \) the increase in risk-taking will not be sufficient to outweigh the increased loss-absorbing capacity. Combining this result with the result when the shareholders’ participation constraint does not determine \( \omega \), and we can conclude that for all \( k_{lev} > k_1 \), if \( \rho \leq \hat{\rho} \), the expected loss of deposit funds will be strictly lower under an LRR.

Let us now show that this upper bound \( \bar{k}_1 \) is strictly greater than the lower bound \( k_1 \). The lower bound level on the optimal risk level was defined at the point where \( k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^*(1 - \lambda_1) \) where * denotes optimal levels. The upper bound level was defined at the point where \( k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^{pc}(1 - \lambda_1) \) where \( pc \) denotes the level determined by the shareholders’ participation constraint. Since \( \omega_{lev}^{pc} < \omega_{lev}^* \), it must be that the upper bound is strictly greater than the lower bound. The upper bound is also larger than the level required for an interior solution. We can see this by comparing the two conditions. The upper bound will be larger if \( \omega_{lev}^* \lambda_1 + (1 - \omega_{lev}^*) \lambda_3 < \omega_{rw}^* \lambda_1 + (1 - \omega_{rw}^*) \lambda_3 + (1 - \lambda_1)(\omega_{rw}^* - \omega_{lev}^{pc}) \). Rearranging, we find: \((1 - \lambda_1)(\omega_{rw}^* - \omega_{lev}^{pc}) - (\omega_{rw}^* - \omega_{lev}^*)(\lambda_3 - \lambda_1) > 0 \), which is true since \( \omega_{lev}^{pc} < \omega_{lev}^* \) and \( \lambda_3 < 1 \).

We can conclude therefore that for all \( \rho \), if \( k_{lev} \in (k_1, \bar{k}_1) \) the expected loss of deposit funds will be lower under an LRR.

Lastly, suppose \( \rho > \hat{\rho} \), then \( \bar{k}_1 < 1 \), and there can potentially exist a region above \( \bar{k}_1 \) in which the expected loss of deposit funds are greater under an LRR, i.e. the increase in loss absorption is outweighed by the increase in risk-taking. For this to be possible, it must be that \( \rho > \hat{\rho} > E_c(\text{risky}) \). This can be seen as follows. For the expected loss of deposit funds to be positive, it must be that (1 - \( k_1 \)) > \omega_{lev}^{pc}(1 - \lambda_1). Plugging in \( \omega_{lev}^{pc} \) and rearranging, we find that if \( \rho > \hat{\rho} \), for this to hold it must be
that:

\[ k_{lev} > \frac{[E_c'(risky) - (\mu + (1 - \mu)\pi)](1 - \lambda_1) - [E_c'(risky) - E_c'(safe)]}{[\rho - (\mu + (1 - \mu)\pi)](1 - \lambda_1) - [E_c'(risky) - E_c'(safe)]} \]

which since \( k_{lev} \leq 1 \) is only possible if \( \rho > E_c'(risky) \). Since \( \rho > E_c'(risky) \), for these parameter values, the LRR can never be set high enough to ensure banks survive all states of the world since there exists an upper bound on the LRR below 1 at which banks can no longer satisfy their shareholders’ participation constraint. Since \( \overline{k}_1 < 1 \), it must be that risk-shifting increases at a faster rate than the benefit from loss absorption until the bank hits the corner solution of \( \omega = 0 \) at which point the LRR cannot be set above this since the bank would not be able to raise further equity without violating the shareholders’ participation constraint. Denote this point \( k_{max} \), which is defined at \( k_{max} \equiv \frac{E_c'(risky) - (\mu + (1 - \mu)\pi)\rho}{\rho - (\mu + (1 - \mu)\pi)} \). Thus, when \( \rho > \hat{\rho} \), it can be that for \( k > \overline{k}_1 \) until \( k_{max} \) at which point the LRR cannot be raised any further, the expected loss of deposit funds is larger than under a solely risk-based capital requirement. To show that this region is possible, suppose the parameters are such that \( \mu = 0.99, \pi = 0.9, R_1 = 1.02, R_2^\delta = 1.2, c = 9 \), \( \lambda_1 = 0.02, \lambda_2 = 0.1, \lambda_3 = 0.8 \). This gives \( \hat{\rho} = 1.1521 \). Suppose \( \rho = 1.155 \), this gives \( \overline{k}_1 = 0.8694 \) and \( k_{max} = 0.9692 \). Furthermore, the expected loss of deposit funds is positive throughout this region as can be seen by plotting \( \omega pc(1 - \lambda_1) - (1 - k_{lev}) \) for all \( k \in [\overline{k}_1, k_{max}] \). This can be seen in figure 7 which is strictly decreasing in \( k_{lev} \). At \( \overline{k}_1 \) this is equal to -0.0059, while at \( k_{max} \) this is equal to -0.0287. Thus, there can exist a region where the expected loss of deposit funds is larger under an LRR.

These are the same results as obtained in the main text, thus the results are robust to the alternative assumption that \( (1 - \mu) > \alpha \) or strengthening the risk-based framework.

Appendix B: Mathematical proofs

Proof of Lemma 1

We show that for any \( \omega \), a bank will prefer to hold the minimum capital requirement. First see that for any \( \omega \), if the capital requirement is binding, the bank will not be able to survive a state \( s_2 \) shock, thus it will always enter bankruptcy in state \( s_2 \).

\( c \) is set to 9 following Dell’Ariccia et al. (2014).
Figure 7

Note: The chart plots $\omega_{pc}(1 - \lambda_1) - (1 - k_{lev})$ for all $k \in [k_1, k_{max}]$. 
In state $s_2$, the safe asset returns $(1 - \lambda_1)$, while the risky asset returns a maximum of $(1 - \lambda_3)$. Suppose the bank holds the minimum capital requirement, i.e. $k(\omega) = (1 - \omega)\lambda_2$

To survive a shock in state $s_2$, it must be that:

$$\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{rw})$$

otherwise the return on the two assets are not sufficient to repay depositors even when the risky asset pays off its highest state $s_2$ return. Imposing the assumption that banks hold the minimum capital requirement and rearranging, this becomes:

$$\omega \lambda_1 + (1 - \omega)(\lambda_3 - \lambda_2) \leq 0$$

which is a contradiction, since $\lambda_3 > \lambda_2$. So for any $\omega \in [0, 1]$, this condition cannot hold. Hence if banks hold the minimum capital requirement, they can never survive state $s_2$.

Given this is the case, we show that for any $\omega$, banks will not find it optimal to hold excess capital.

The profit from holding the minimum capital requirement is:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega)$$

If the bank decides to hold excess capital, where $k_{ex}$ denotes a capital level above the minimum, then profit will be either:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi - (1 - k_{ex})[\mu + (1 - \mu)\pi] - \rho k_{ex} - c(\omega)$$

if the bank holds only enough excess capital to survive when the risky asset returns $(1 - \lambda_3)$, or:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi - (1 - k_{ex}) - \rho k_{ex} - c(\omega)$$

if the bank can hold enough excess capital to survive all shocks.

We show that holding the minimum capital requirement is preferred to both
these alternatives, namely:

\[
\mu [\omega R_1 + (1 - \omega) \pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega) >
\]

\[
\mu [\omega R_1 + (1 - \omega) \pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi
\]

\[-(1 - k_{ex}) [\mu + (1 - \mu)\pi] - \rho k_{ex} - c(\omega)
\]

and

\[
\mu [\omega R_1 + (1 - \omega) \pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega) >
\]

\[
\mu [\omega R_1 + (1 - \omega) \pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi - (1 - k_{ex}) - \rho k_{ex} - c(\omega)
\]

Let us proceed with the first condition. Plugging in the minimum capital requirement and simplifying, we find that this is true if and only if:

\[
\rho > \mu + (1 - \mu)\pi \frac{[k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]}
\]

which is true by definition, since \(\rho > 1\), and \(\mu + (1 - \mu)\pi \frac{[k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]} < 1\), since \([k_{ex} - (1 - \omega)\lambda_2] > [k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3]\) for any \(\omega\).

Performing the same exercise with the second condition, we find a similar condition stating that banks will prefer to hold the minimum capital requirement if and only if:

\[
\rho > \mu + (1 - \mu)\pi \frac{[k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]}
\]

which again is true by definition since \(\rho > 1\), and \(\mu + (1 - \mu)\pi \frac{[k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]} < 1\), since \([k_{ex} - \omega \lambda_1 - (1 - \omega)\lambda_3] < [k_{ex} - (1 - \omega)\lambda_2]\).

**Proof of Proposition 2**

The proof proceeds in two stages. First, we show the optimal solution under a solely risk-based framework. Second, we show that under a binding LRR, a bank’s optimal risk level will always be higher than this for a sufficiently large \(\rho\).

We know from lemma 1 that the bank will never survive state \(s_2\) under the risk-based framework. So, under a solely risk-based capital requirement, the bank will
choose an \( \omega \) that maximises:

\[
\mu [\omega R_1 + (1 - \omega)\pi R_2^b + (1 - \omega)(1 - \lambda_2)\pi] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega)
\]

The optimal choice can be written as:

\[
(1 - \omega) = \frac{\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1]}{c} - \lambda_2 (\rho - \mu)
\]

With an LRR, as discussed in the text, for large \( \rho \), there can be a point at which the shareholders’ participation constraint forces banks to take on further risk. By definition this risk level is larger than the optimal risk the bank would otherwise choose. As a result, it is sufficient to show that if the optimal level of risk is higher than the risk-based choice, then this level of risk will also be.

Let us suppose therefore that \( \rho \) is low enough that even at \( k_{lev} = 1 \), banks could choose their optimal risk level and they would still satisfy the shareholders’ participation constraint. This puts a lower bound on the bank’s chosen level of risk, which we can show is always larger than the risk-based choice for sufficiently large \( \rho \). Solving the bank’s maximisation, the optimal \((1 - \omega)\) can be characterised by:

\[
(1 - \omega) = \begin{cases} 
\frac{\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1]}{c} & \text{if } k_{lev} < k_1 \\
\frac{\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1]}{c} - (1 - \mu) \pi (\lambda_3 - \lambda_1)/c - (1 - \mu)(1 - \pi)(1 - \lambda_1)/c & \text{if } k_{lev} \in [k_1, k_2) \\
\frac{\mu [\pi R_2^b + (1 - \lambda_2)\pi - R_1]}{c} - (1 - \mu) \pi (\lambda_3 - \lambda_1)/c - (1 - \mu)(1 - \pi)(1 - \lambda_1)/c & \text{if } k_{lev} \geq k_2 
\end{cases}
\]

where \( k_1 \equiv \lambda_1 + (\lambda_3 - \lambda_1) \frac{\mu R_2^b + (1 - \lambda_2)\pi - R_1}{c} \) and \( k_2 \equiv \lambda_1 + (1 - \lambda_1) \frac{\mu R_2^b + (1 - \lambda_2)\pi - R_1}{c} - (1 - \mu) \pi (\lambda_3 - \lambda_1)/c - (1 - \mu)(1 - \pi)(1 - \lambda_1)/c \).

The first row is clearly larger than the solution under the risk-based requirement, and this proves the first statement. Comparing the other two rows with the risk-based choice, it is easy to derive a condition on \( \rho \)ho for which the risk choice under an LRR is always larger than the risk-based choice. Rearranging, we find that the risk choice will always be larger under an LRR, if and only if:

\[
\rho > \begin{cases} 
\mu + (1 - \mu) \frac{\pi (\lambda_1 - \lambda_2)}{\lambda_2} & \text{if } k_{lev} \in [k_1, k_2) \\
\mu + (1 - \mu) \frac{\pi (1 - \lambda_1)(1 - \lambda_2)}{\lambda_2} & \text{if } k_{lev} \geq k_2 
\end{cases}
\]

where \( k_1 \) and \( k_2 \) are defined as above. This proves the second statement.
Proof of Proposition 3

The proof proceeds in two steps. First, we look at failure probabilities, then we consider the expected loss of deposit funds.

Under the risk-based framework, by definition, the probability of default is \( (1 - \mu) \). When the LRR binds, the bank will have more capital, but at the same time, it will take more risk. This level of risk however is capped at the maximum possible level of risk before the bank moves back into the risk-based framework. We show that even if the bank takes this level of risk, default probabilities will not rise, and for some LRR levels, default probabilities will decline relative to the risk-based probability.

The maximum risk level occurs at the point where the risk-based capital requirement equals the LRR: i.e. \( k(\omega) = (1 - \omega_{\text{max}}) \lambda_2 = k_{\text{lev}} \). In other words, the maximum the bank can increase risk to is: \( (1 - \omega_{\text{max}}) = \frac{k_{\text{lev}}}{\lambda_2} \). Suppose this is the case, and the bank increases risk to the maximum, so \( (1 - \omega_{\text{lev}}) = (1 - \omega_{\text{max}}) = \frac{k_{\text{lev}}}{\lambda_2} \):

We show that even at this level, the bank will survive the shock in state \( s_1 \) and thus its probability of default will not be less than \( (1 - \mu) \). This is true if and only if \( \omega R_1 + (1 - \omega)(1 - \lambda_2) \geq (1 - k_{\text{lev}}) \)

Plugging the maximum risk level into the above:

\[
\left(1 - \frac{k_{\text{lev}}}{\lambda_2}\right) R_1 + \frac{k_{\text{lev}}}{\lambda_2} (1 - \lambda_2) \geq (1 - k_{\text{lev}})
\]

Rearranging:

\[
(R_1 - 1) \left(1 - \frac{k_{\text{lev}}}{\lambda_2}\right) \geq 0
\]

which is true for all \( k_{\text{lev}} \leq \lambda_2 \).

So, for all \( k_{\text{lev}} \leq \lambda_2 \), the bank can take the maximum risk and it will still survive the state \( s_1 \) shock. This is because, with the risk-based framework underlying the LRR, it cannot be that the LRR allows failure in this state, otherwise the risk-based capital requirement would have been higher. If \( k_{\text{lev}} > \lambda_2 \), the bank can still never enter bankruptcy in state \( s_1 \). To see this, denote \( k_{\text{lev}} = \lambda_2 + \varepsilon \) as any LRR above \( \lambda_2 \), where \( \varepsilon \in [0, 1 - \lambda_2] \). For any \( \omega \in [0, 1] \) and \( \varepsilon \in [0, 1 - \lambda_2] \), \( \omega R_1 + (1 - \omega)(1 - \lambda_2) > (1 - k_{\text{lev}}) = (1 - \lambda_2 - \varepsilon) \). So for any \( k_{\text{lev}} \), the probability of default will not fall below \( (1 - \mu) \).

We now show that the probability of default can be strictly lower under an LRR.

56
The probability of default will be strictly lower under an LRR if the bank can survive a shock in state $s_2$. Suppose the parameters are such that the optimal solution lies below the maximum risk level discussed above. A bank will survive a $\lambda_3$ shock in state $s_2$ iff:

$$\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{lev})$$

Plugging in the optimal $\omega$

$$k_{lev} \geq 1 - \omega_{lev}^*(1 - \lambda_1) - (1 - \omega_{lev}^*)(1 - \lambda_3)$$

So for $k_{lev}$ greater than this, the probability of default can be strictly lower.

Now, consider the expected loss of deposit funds. Under a risk-based framework, the expected loss of deposit funds will be:

$$EL_{rw} \equiv (1 - \mu) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1) - (1 - \omega_{rw})(1 - \lambda_3)\pi]$$

Under an LRR, the expected loss of deposit funds will be:

$$(1 - \mu) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1) - (1 - \omega_{lev})(1 - \lambda_3)\pi]$$

if at the level the LRR is set and the risk banks take, they can only survive state $s_1$ shocks.

$$(1 - \pi) (1 - \mu) (1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)]$$

if at the level the LRR is set and risk level taken, banks can survive all state $s_1$ shocks and also survive state $s_2$ with probability $\pi$, i.e. if the risky asset pays off its higher value in that state $(1 - \lambda_3)$. Or 0 if at the level the LRR is set and chosen risk level, banks can survive all states of the world.

We begin by showing that if the bank takes the maximum level of risk, the expected loss of deposit funds can be larger under an LRR. First see that if the bank takes the maximum risk, where the maximum risk the bank can take is $(1 - \omega_{max}) = k_{lev}\frac{\lambda_2}{\lambda_3}$, it can never survive a state $s_2$ shock. Banks never survive state $s_2$ when they take the maximum risk iff:

$$\omega_{max}(1 - \lambda_1) + (1 - \omega_{max})(1 - \lambda_3) < (1 - k_{lev})$$

Plugging in what we know to be $\omega_{max}$ and rearranging:

$$k_{lev} [\lambda_2 + (1 - \lambda_3) - (1 - \lambda_1)] < \lambda_1$$
If \( \lambda_2 + (1 - \lambda_3) - (1 - \lambda_1) \) < 0, then this clearly holds since \( \lambda_1 > 0 \). Suppose \( \lambda_2 + (1 - \lambda_3) - (1 - \lambda_1) \) > 0, then the LHS is maximised at \( k_{lev} = 1 \). Imposing this, the expression simplifies to:

\[
\lambda_2 < \lambda_3
\]

which is true by definition. Given this holds for \( k_{lev} = 1 \), the maximum of the function, it must be true for all \( k_{lev} < 1 \). So, if banks take the maximal risk, they can only survive state \( s_1 \).\(^{42}\)

Given this, let us now show that in the case where banks take the maximum risk and survive state \( s_1 \), but not state \( s_2 \), the expected loss of deposit funds can be larger under an LRR. We prove by contradiction. Suppose this is not the case and the expected loss of deposit funds are lower under an LRR when banks maximise their risk-taking, then it must be that:

\[
[(1 - \lambda_1) - (1 - \lambda_3) \pi] (\omega_{rw} - 1 + \frac{k_{lev}}{\lambda_2}) < k_{lev} - k(\omega)
\]

\[
[(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2] k_{lev} < k(\omega) [(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2]
\]

If \( [(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2] > 0 \),\(^{43}\)

\[
k_{lev} < k(\omega)
\]

which is a contradiction. So if the bank takes the maximal risk, the expected loss of deposit funds can be larger under an LRR.

Let us now prove the first statement of proposition 3. As discussed before, there are two cases which determine optimal risk-taking. First, the optimal risk level can be pinned down by the FOC. Second, if this is insufficient to satisfy the shareholders’ participation constraint, the shareholders’ participation constraint can pin down the risk level. Let us take each case in turn. Suppose the LRR is set just above the risk-based capital requirement and the FOC determines \( \omega \), then the expected loss of deposit funds will be lower under an LRR if:

\[
(1 - \mu) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1) - (1 - \omega_{lev}) (1 - \lambda_3) \pi]
\]

\(^{42}\)Since \( \omega_{max} (1 - \lambda_1) + (1 - \omega_{max}) (1 - \lambda_3) < (1 - k_{lev}) \), then it must also be that \( \omega_{max} (1 - \lambda_1) < (1 - k_{lev}) \), which confirms that banks can never survive all shocks if they take the maximum risk.

\(^{43}\)For the alternative assumption, clearly the expected loss of deposit funds would be lower since \( k_{lev} > k(\omega) \), hence we do not need to consider this case.
\[(1 - k(\omega)) - \omega_{rw}(1 - \lambda_1) - (1 - \omega_{rw})(1 - \lambda_3) \pi]\]

Plugging in the optimal values:
\[k_{lev} > k(\omega_{rw}) + [(1 - \lambda_1) - (1 - \lambda_3) \pi] \frac{\lambda_2 (\rho - \mu)}{c}\]

Since \(k_{lev}\) can be any value above \(k(\omega_{rw})\), there exists a region just above \(k(\omega_{rw})\) in which the expected loss of deposit funds can be higher. Consider all \(k\) above this level, since the solution \(\omega\) set by the FOC is either constant or increasing in \(k\), for any \(k\) above this level, so long as the solution is set by the FOC, the expected loss of deposit funds will decrease in \(k\), and thus the expected loss of deposit funds will be lower under an LRR. However, for this to be the case, the FOC must be feasible, namely the solution must be less than the maximum possible risk level. So we must impose this additional condition. Hence, for the expected loss of deposit funds to be lower under an LRR, we require an interior solution and for the LRR to be set higher than the level above. i.e. \(k_{lev} > k_0 \equiv \max\{(1 - \omega_{lev})\lambda_2, k(\omega_{rw}^*) + [(1 - \lambda_1) - (1 - \lambda_3) \pi] \frac{\lambda_2 (\rho - \mu)}{c}\}\). Therefore, if the risk level is set by the FOC, for all \(k > k_0\), the expected loss of deposit funds will be strictly lower under an LRR.

Now consider the second case in which the shareholders’ participation constraint determines \(\omega\). Define \(E_c(risky) = \mu \pi R_2^b + \mu(1 - \pi)(1 - \lambda_2)\) and \(E_c(safe) = \mu R_1\), this case will hold when:
\[\omega_{lev} E_c(safe) + (1 - \omega_{lev}) E_c(risky) - (1 - k) \Pr(survive) < \rho k\]

So \(\omega\) is:
\[\frac{E_c(risky) - \rho k - (1 - k) \Pr(survive)}{E_c(risky) - E_c(safe)} = \omega\]

This is decreasing in \(k\), so risk-taking is increasing in \(k\). We show that if \(\rho \leq \hat{\rho}\) (defined below), even if the shareholders’ participation constraint determines risk-taking, for all \(k > k_0\), the expected loss of deposit funds will be lower.

To yield worse outcomes, it must be that the shareholders’ participation constraint determines risk-taking, as otherwise, the previous result holds. Due to the discrete nature of the problem, at different levels of the LRR, the expected loss of deposit funds can jump. At first, banks may only survive state \(s_1\), but as the LRR rises (e.g. if \(k_{lev} \geq \lambda_3\)), banks may then be able to survive state \(s_2\) when the risky asset pays off its higher state \(s_2\) payoff, \((1 - \lambda_3)\). Depending on how fast risk increases as the LRR rises, as \(k_{lev}\) approaches 1, probability of default can approach
Consider the first region in which banks can only survive state $s_1$. To be detrimental, it must be that:

\[
(1 - \mu) \left[ (1 - k_{lev}) - \frac{\mu(R^2 \pi + (1 - \pi)(1 - \lambda_2)) - \rho k - (1 - k) \mu}{\mu(R^2 \pi + (1 - \pi)(1 - \lambda_2)) - \mu R_1} (1 - \lambda_1) - (1 - \frac{\mu(R^2 \pi + (1 - \pi)(1 - \lambda_2)) - \rho k - (1 - k) \mu}{\mu(R^2 \pi + (1 - \pi)(1 - \lambda_2)) - \mu R_1}) (1 - \lambda_3) \pi \right] \\
> (1 - \mu) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1) - (1 - \omega_{rw}) (1 - \lambda_3) \pi]
\]

Rearranging, we find that if $\rho > \mu + \frac{\mu(R^2 \pi + (1 - \pi)(1 - \lambda_2) - R_1)}{(1 - \lambda_1) - (1 - \lambda_3) \pi}$, we can write a condition on $k_{lev}$ such that above this, the expected loss of deposit funds is larger under an LRR. So, to be detrimental, it must be that:

\[
k_{lev} > \tilde{k}_0 \equiv \frac{[E_c(risky) - E_c(safe)] [EL_{rw}/(1 - \mu) + (1 - \lambda_3) \pi - 1] + [E_c(risky) - \mu][(1 - \lambda_1) - (1 - \lambda_3) \pi]}{\rho - \mu}[(1 - \lambda_1) - (1 - \lambda_3) \pi] - [E_c(risky) - E_c(safe)]
\]

The numerator is positive. This can be seen as follows:

\[
[E_c(risky) - E_c(safe)] [EL_{rw}/(1 - \mu) + (1 - \lambda_3) \pi - 1] + [E_c(risky) - \mu][(1 - \lambda_1) - (1 - \lambda_3) \pi]
\]

\[
> [E_c(risky) - E_c(safe)] [EL_{rw}/(1 - \mu) + (1 - \lambda_3) \pi - 1] + [(1 - \lambda_1) - (1 - \lambda_3) \pi]
\]

\[
= [E_c(risky) - E_c(safe)][(1 - \omega_{rw})][(1 - \lambda_1) - \lambda_2 - \pi(1 - \lambda_3)] > 0
\]

But the right hand side of $\tilde{k}_0$ must be less than 1, since $k_{lev} \leq 1$. This is only true if:

\[
\rho > \tilde{\rho}_0 \equiv E_c(risky) + \frac{EL_{rw}/(1 - \mu) + \pi(1 - \lambda_3)}{(1 - \lambda_1) - \pi(1 - \lambda_3)} [E_c(risky) - E_c(safe)]
\]

\[
\Leftrightarrow \rho > \tilde{\rho}_0 \equiv \frac{cE_c(risky)[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \pi(1 - \lambda_3)c[E_c(risky) - E_c(safe)]}{c[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3) \pi][E_c(risky) - E_c(safe)]}
\]

\[
+ [E_c(risky) - E_c(safe)] \frac{c(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3) \pi][\mu(\pi R^2_1 + (1 - \lambda_2) \pi - R_1) + \mu \lambda_2]}{c[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3) \pi][E_c(risky) - E_c(safe)]}
\]

So, in this region if $\rho < \tilde{\rho}_0$, even if the shareholders’ participation constraint determines $\omega$, risk-taking will not increase fast enough to lead to a detriment.

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44If $\rho$ is less than this, then the condition becomes $k_{lev} < \tilde{k}_0$ where $\tilde{k}_0 < 0$ (since the numerator is positive - see text), but then since $k_{lev} \geq 0$, by definition the expected loss of deposit funds will always be lower under an LRR.
Let us now consider the second region wherein banks can survive state $s_2$ with probability $\pi$. Finding a similar condition, the expected loss of deposit funds will be larger under an LRR if the participation constraint pins down the risk choice and

$$(1 - \mu) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1) - (1 - \omega_{rw})(1 - \lambda_3)\pi] = EL_{rw}$$

$$< (1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)]$$

Define $E_c(\text{risky}) = \mu \pi R^c_{\pi} + \mu (1 - \pi)(1 - \lambda_2) + (1 - \mu)\pi (1 - \lambda_3)$ and $E_c(\text{safe}) = \mu R_1 + (1 - \mu)\pi (1 - \lambda_1)$, then if $\rho > \frac{\mu \pi R^c_{\pi} + (1 - \pi)(1 - \lambda_2) + (1 - \mu)(1 - \lambda_3)\pi - [\mu R_1 + (1 - \mu)(1 - \lambda_1)\pi]}{1 - \lambda_1} + \mu + (1 - \mu)\pi$, this simplifies to:

$$k_{lev} > k_1 \equiv \frac{[E_c(\text{risky}) - E_c(\text{safe})][EL_{rw}/(1 - \mu)(1 - \pi) - 1] + [E_c(\text{risky}) - \mu](1 - \lambda_1)}{(\rho - \mu)(1 - \lambda_1) - [E_c(\text{risky}) - E_c(\text{safe})]}$$

Again, the RHS must be less than 1 since $k_{lev} \leq 1$, but this is only true if

$$\rho > \hat{\rho}_1 \equiv \frac{E_c(\text{risky})(1 - \pi)(1 - \lambda_1)c}{(1 - \lambda_1)(1 - \pi)c + [E_c(\text{risky}) - E_c(\text{safe})]\lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi]}$$

$$E_c(\text{risky}) - E_c(\text{safe})] \frac{(1 - \lambda_1)(1 - \pi)c + [E_c(\text{risky}) - E_c(\text{safe})]\lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi]}$$

So again, if $\rho \leq \hat{\rho}_1$, even if the shareholders’ participation constraint determines risk-taking, the expected loss of deposit funds will lower under an LRR. If attainable, the third region in which the probability of default falls to zero (and thus also the expected loss of deposit funds), clearly is lower than under a solely risk-based capital framework, and so, we can conclude that if $\rho < \hat{\rho} = \min\{\hat{\rho}_0, \hat{\rho}_1\}$, even if the shareholders’ participation constraint determines $\omega$, risk-taking will not increase fast enough to lead to a detriment. Hence, combining our result on the lower bound, we can state that if $\rho < \hat{\rho}$, the expected loss of deposit funds under an LRR will be lower for all $k_{lev} \geq k_0$. This proves the first statement.

Let us now show that these upper bound levels are strictly greater than the lower bound level derived earlier. The lower bound level was defined at the point

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45As before, if $\rho$ is less than this, since the numerator is positive ($[E_c(\text{risky}) - E_c(\text{safe})][EL_{rw}/(1 - \mu)(1 - \pi) - 1] + [E_c(\text{risky}) - \mu](1 - \lambda_1) > [E_c(\text{risky}) - E_c(\text{safe})][EL_{rw}/(1 - \mu) + (1 - \lambda_3)\pi - 1] + [E_c(\text{risky}) - \mu][(1 - \lambda_1) - (1 - \lambda_3)\pi] > 0$ as shown above), then the condition simplifies to $k < k_1$ where $k_1 < 0$, but since $k_{lev} \geq 0$, the expected loss of deposit funds will always be smaller.
We show that this is larger than the lower bound and thus, it must also be that

\[ k_{\text{lev}} = k(\omega_{\text{ru}}^* + \omega_{\text{ru}}^*(1 - \lambda_1) - \omega_{\text{lev}}^*(1 - \lambda_1) - (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)(1 - \lambda_3)\pi \]

where \( \omega_{\text{lev}}^* \) denotes optimal levels. The first upper bound level is defined at the point where

\[ k_{\text{lev}} = k(\omega_{\text{ru}}^* + \omega_{\text{ru}}^*(1 - \lambda_1) - \omega_{\text{lev}}^*(1 - \lambda_1) - (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)(1 - \lambda_3)\pi \]

where \( \omega_{\text{lev}}^* < \omega_{\text{ru}}^* \), it must be that this upper bound is strictly greater than the lower bound.

Equally, the upper bound is also strictly greater than the \( k \) required for an interior solution, i.e. \( k = (1 - \omega_{\text{lev}}^*)/\lambda_2 \). This will be true if and only if:

\[ k(\omega_{\text{ru}}^* + \omega_{\text{ru}}^*(1 - \lambda_1) - \omega_{\text{lev}}^*(1 - \lambda_1) - (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)(1 - \lambda_3)\pi > (1 - \omega_{\text{lev}}^*)/\lambda_2 \]

Rearranging, we find

\[ (\omega_{\text{lev}}^* - \omega_{\text{ru}}^*)/\lambda_2 + (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)[(1 - \lambda_1) - (1 - \lambda_3)\pi] > 0 \]

which must be true since \( \omega_{\text{ru}}^* > \omega_{\text{lev}}^* > \omega_{\text{lev}}^{pc} \) and \( [(1 - \lambda_1) - (1 - \lambda_3)\pi] > \lambda_2 \).

Consider the second upper bound now, this is defined at the point where

\[
1 + k \frac{(\rho - (\mu + (1 - \mu)\pi))(1 - \lambda_1) - 1}{E_c(\text{risky}) - E_c(\text{safe})} = \frac{E_{L_{\text{r}}}}{(1 - \mu)(1 - \pi)}
\]

Since \( \rho > \hat{\rho}_1 \), the LHS is increasing in \( k \). Hence, the \( k \) that solves this equation (i.e. \( \tilde{k}_1 \)), ceteris paribus, must be larger than the \( k \) that solves

\[
1 + k \frac{(\rho - (\mu + (1 - \mu)\pi))(1 - \lambda_1) - 1}{E_c(\text{risky}) - E_c(\text{safe})} = \frac{E_{L_{\text{r}}}}{(1 - \mu)}
\]

Since this is identical except the RHS is smaller. So, let us take the smaller \( k \).

We show that this is larger than the lower bound and thus, it must also be that \( \tilde{k}_1 \) is too. This lower \( k \) is defined at \[ k_{\text{lev}} = (1 - k_{\text{lev}} - \omega_{\text{lev}}^{pc}(1 - \lambda_1)] = E_{L_{\text{r}}}/(1 - \mu) =

\[
(1 - k(\omega_{\text{ru}}^* - \omega_{\text{ru}}^*(1 - \lambda_1) - (1 - \omega_{\text{ru}}^*)(1 - \lambda_3)\pi.\]

Again, the lower bound is defined at the point where \( k_{\text{lev}} = k(\omega_{\text{ru}}^* + \omega_{\text{ru}}^*(1 - \lambda_1) - \omega_{\text{lev}}^*(1 - \lambda_1) - (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)(1 - \lambda_3)\pi.\)

Since \( \omega_{\text{lev}}^{pc} < \omega_{\text{lev}}^* \), it must be that this upper bound is strictly greater than the lower bound. Equally, \( k > (1 - \omega_{\text{ru}}^*/\lambda_2 \).

This is true if:

\[
(1 - \omega_{\text{ru}}^*)/\lambda_2 + (\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)[(1 - \lambda_1) - (1 - \lambda_3)\pi] > (1 - \omega_{\text{lev}}^*)/\lambda_2\]

which we can rearrange as

\[-(\omega_{\text{ru}}^* - \omega_{\text{lev}}^*)/\lambda_2 + (1 - \lambda_1)(\omega_{\text{ru}}^* - \omega_{\text{lev}}^*) + (1 - \omega_{\text{ru}}^*)(1 - \lambda_3)\pi > 0\]

which is true since \( \omega_{\text{ru}}^* > \omega_{\text{lev}}^{pc} \) and \( (1 - \lambda_1) > \lambda_2 \).

We can conclude therefore that for all \( \rho \), if \( k_{\text{lev}} \in (k_0, \overline{k}_0) \), where \( \overline{k}_0 = min\{k_0, \tilde{k}_1\} \), the expected loss of deposit funds will be lower under an LRR. This proves the second statement.

Lastly, consider the third statement. We show that for \( \rho > \hat{\rho} \) and \( k > \overline{k}_0 \) it is possible that for all \( k \) above this level, the expected loss of deposit funds will be higher than \( E_{L_{\text{r}}} \). It is not possible to analytically prove a general statement in this region, so we illustrate numerically that the statement is possible. Suppose
the parameters are such that $\mu = 0.999$, $\pi = 0.9$, $R_1 = 1.02$, $R_\pi^2 = 1.2$, $c = 9^{46}$, $\lambda_1 = 0.02$, $\lambda_2 = 0.1$, $\lambda_3 = 0.8$. This gives $\tilde{\rho}_0 = 1.2026$ and $\tilde{\rho}_1 = 1.2596$. Suppose $\rho = 1.22$, this gives $\tilde{k}_0 = 0.6238$ and $\tilde{k}_1 = 1.5116$. So, for these parameter values, $\tilde{\rho}_0 < \tilde{\rho}_1$, and $\tilde{k}_0 < \tilde{k}_1$. We plot the expected loss of deposit funds under an LRR minus $EL_{rw}$ from $\tilde{k}_0$ to $k_{\text{max}}$, where $k_{\text{max}} \equiv \frac{E_r(risky) - \mu}{\rho - \mu}$. Figure 8 illustrates that for all levels of the LRR above $\tilde{k}_0$ (and $\rho > \hat{\rho}$) until it hits the point $k_{\text{max}}$ at which point the LRR cannot be set any higher, the expected loss of deposit funds under an LRR is larger. This proves the third statement.

Note: The graph shows the difference between the expected loss of deposit funds under an LRR minus the expected loss of deposit funds under a solely risk-based capital framework from $\tilde{k}_0$ to $k_{\text{max}}$.