Estimating Nominal Interest Rate Expectations: Overnight Indexed Swaps and the Term Structure*

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Abstract

Financial market participants and policymakers closely monitor the evolution of interest rate expectations. At any given time, the term structure of interest rates contains information regarding these expectations. No-arbitrage dynamic term structure models have regularly been used to estimate interest rate expectations, but daily frequency estimates of these models fail to accurately capture the evolution of interest rate expectations implied by surveys and financial market instruments. I propose the augmentation of no-arbitrage Gaussian affine dynamic term structure models with overnight indexed swap (OIS) rates in order to better estimate the evolution of interest rate expectations across the whole term structure. Drawing on Lloyd (2016a), I provide evidence that the OIS rates that I augment the model with have statistically insignificant excess returns and so provide valid information with which to identify interest rate expectations. The OIS-augmented model that I propose, estimated between January 2002 and December 2015 for the US, generates estimates of the expected path of short-term interest rates that closely correspond to those implied by federal funds futures rates and survey expectations, and accurately depict their daily frequency evolution. Against these metrics, the interest rate expectation estimates from OIS-augmented models are superior to estimates from existing Gaussian affine dynamic term structure models.

JEL Codes: C32, C58, E43, E47, G12.

Key Words: Term Structure of Interest Rates; Overnight Indexed Swaps; Monetary Policy Expectations; Dynamic Term Structure Model.

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1 Introduction

Financial market participants, researchers and policymakers closely monitor the *daily frequency* evolution of interest rate expectations. To achieve this, they consider a wide range of different financial market instruments and prices. For researchers and policymakers, it is important to attain an accurate measure of the evolution of these expectations in order to form judgements about the appropriateness of real-time policy decisions and to evaluate the effectiveness of existing policies.[1] For investors, understanding future interest rate expectations is important for discounting cash flows, valuing investment opportunities and engaging in profitable trade.

At any given time, the term structure of interest rates contains information regarding these expectations. For this reason, dynamic term structure models have increasingly been used to estimate and separately identify the dynamic evolution of (i) the expected path of future short-term interest rates and (ii) term premia,[2] two components of nominal government bond yields. By imposing no-arbitrage, these models provide estimates of the evolution of interest rate expectations that are consistent across the term structure. However, a popular class of these models — Gaussian affine dynamic term structure models (GADTSMs) — suffer from an identification problem that results in estimates of interest rate expectations that are spuriously stable (see, for example, Bauer, Rudebusch, and Wu, 2012; Kim and Orphanides, 2012; Guimarães, 2014).

Central to the identification problem is an informational insufficiency. Unaugmented GADTSMs use bond yield data as their sole input. These yields provide information of direct relevance to the estimation of the fitted bond yields. Absent additional information, estimates of interest rate expectations are poorly identified as they must also be derived from information contained within the actual bond yields. To do this, maximum likelihood or ordinary least squares estimates of, *inter alia*, the persistence of the (pricing factors derived from the) actual yields must be attained. However, as a symptom of the identification problem, a 'finite sample' bias will arise in these persistence parameters when there is insufficient information and a limited number of interest rate cycles in the observed yield data.[3] Finite sample bias will result in persistence parameters that are spuriously estimated to be less persistent than they really are and estimates of future short-term interest rates that are spuriously stable.[4] Because bond yields are highly persistent, the finite sample bias can be severe. Moreover, the severity of the bias is increasing in the persistence of the actual yield data. For daily frequency yields, which display greater persistence than lower-frequency data, the problem is particularly pertinent.

[1] See, for example, the broad literature evaluating the impact and transmission channels of various unconventional monetary policies enacted by central banks since 2007/8, which uses daily frequency changes in interest rate components to decompose the relative importance of the various channels (for more details, see: Lloyd, 2016b).

[2] The term premium represents the compensation investors receive for, *inter alia*, default risk, interest rate risk and illiquidity.

[3] Kim and Orphanides (2012, p. 242) state that "in a term structure sample spanning 5 to 15 years, one may not observe a sufficient number of 'mean reversions'."

[4] This 'finite sample' bias is well documented for ordinary least squares estimation of a univariate autoregressive process, where estimates of the autoregressive parameter will be biased downwards, implying less persistence than the true process (Stock and Watson, 2011). Within GADTSMs, the finite sample bias is a multivariate generalisation of this.

In this paper, I propose the augmentation of GADTSMs with overnight indexed swap (OIS) rates as an additional input to estimation to improve the identification of interest rate expectations from term premia in GADTSMs. OIS contracts are over-the-counter traded interest rate derivatives in which two counterparties exchange fixed and floating interest rate payments over its term. A counterparty will enter into an OIS agreement if they expect the payments they swap to exceed those they take on. Thus, OIS rates should reflect the average of investors' expectations of future short-term interest rates. I show that, by providing information for the separate identification of interest rate expectations, OIS-augmentation of GADTSMs does tackle the informational insufficiency at the center of the GADTSM identification problem.

Before estimating the OIS-augmented model for the US, I first verify that OIS rates do indeed provide accurate information about investors' expectations of the future short-term interest rate. Drawing on work by Lloyd (2016a), I verify that, between January 2002 and December 2015, the mean *ex post* realised excess return on 3-24-month OIS rates were insignificantly different from zero.[5]

I then present the OIS-augmented GADTSM, deriving expressions for the OIS pricing factor loadings that explicitly account for the geometric payoff structure in OIS contracts. I estimate the OIS-augmented model using maximum likelihood via the Kalman filter. To the extent that excess returns on OIS rates can vary on a day-to-day basis, I admit measurement error in the OIS excess returns over time in my OIS-augmented GADTSM. The Kalman filter maximum likelihood setup I use is well suited to account for this.

This is not the first paper to propose a solution to the identification problem in GADTSMs. Kim and Orphanides (2012) suggest the augmentation of GADTSMs with survey expectations of future short-term interest rates for the same purpose. Kim and Orphanides (2012) document that, between 1990 and 2003, the survey-augmented model does produce sensible estimates of interest rate expectations. Guimarães (2014) shows that, relative to an unaugmented GADTSM, the survey-augmented model provides estimates of interest rate expectations that both better correspond with survey expectations of future interest rates *and* deliver gains in the precision of interest rate expectation estimates. However, I show that estimated interest rate expectations from the OIS-augmented model are superior to the survey-augmented model for the 2002-15 period.

Bauer et al. (2012) propose an alternative solution, focused on resolving the finite sample bias problem: formal bias-correction of GADTSMs. They document that their bias-corrected estimates of interest rate expectations "are more plausible from a macro-finance perspective" (Bauer et al., 2012, p. 454) than those from an unaugmented GADTSM. However, as Wright (2014) states: the fact that bias-correction has notable effects on GADTSM-estimated interest rate expectations is merely a *symptom* of the identification problem. Bias-correction does not directly address the identification problem at the heart of GADTSM estimation: the informa-

[5]Formally, the 2-year OIS rate exhibits *ex post* excess returns that are statistically significant at the 10% level. However, further study of this in Lloyd (2016a) indicates that this marginally significant result is driven by the *ex ante* unexpected nature of the 2007/8 financial crisis and associated loosening of monetary policy, as opposed to risk premia within the 2-year contract. At all other maturities that I test, from 3 to 21 months, OIS rates exhibit statistically insignificant average *ex post* excess returns for the whole 2002-15 sample.

tional insufficiency. Moreover, Wright (2014) argues that the bias-corrected estimates of future interest rate expectations are "far too volatile" (Wright, 2014, p. 339). I find that estimated interest rate expectations from the OIS-augmented model are superior to bias-corrected estimates for the 2002-15 period.

The OIS-augmentation that I present is closest in philosophy to the former of these proposals: survey-augmentation. The GADTSM is augmented with additional information to better identify the evolution of interest rate expectations. However, OIS-augmentation differs in a number of important respects, which help to explain its superior performance *vis-à-vis* survey-augmentation. Primarily, although survey forecasts do help to address the informational insufficiency problem, they are ill-equipped for the estimation of daily frequency expectations. Survey forecasts of future interest rates are only available at a low frequency: quarterly or monthly, at best. Thus, survey forecasts are unlikely to provide sufficient information to accurately identify the daily frequency evolution of interest rate expectations. Moreover, the survey forecasts used by Kim and Orphanides (2012) and Guimarães (2014) correspond to the expectations of professional forecasters and not necessarily those of financial market participants.

OIS rates offer significant advantages over survey expectations for the daily frequency estimation of GADTSMs. Most importantly, OIS rates are available at a daily frequency, so provide information at the same frequency at which interest rate expectations are estimated. Secondly, OIS rates are formed as a result of actions by financial market participants, so can be expected to better reflect their expectations of future short-term interest rates. Third, the information in survey forecasts is limited in comparison to the expectational information contained in OIS rates. Survey forecasts typically provide information about expected future short-term interest rates for a short time period in the future.[6] In contrast, there exists a term structure of OIS contracts that can be used to infer the evolution of investors' interest rate expectations from now until a specified future date. The horizon of these OIS contracts corresponds exactly to the horizon of nominal government bonds.

Away from the GADTSM-literature, OIS rates are increasingly being used by academics to infer investors' expectations of future monetary policy absent a model structure (see, notably: Christensen and Rudebusch, 2012; Woodford, 2012; Bauer and Rudebusch, 2014; Lloyd, 2016b). These authors consider daily changes in OIS rates, attributing them to changes in investors' expectations of future short-term interest rates. Lloyd (2016a) formally studies the empirical performance of OIS rates as financial market-based measures of investors' interest rate expectations. He first compares the *ex post* excess returns on US OIS contracts with comparable-maturity federal funds futures contracts, widely used market-based measures of monetary policy expectations. Gürkaynak, Sack, and Swanson (2007b) document that federal funds futures dominate a range of other financial market instruments in forecasting the future path of short-term interest rates at horizons out to six months.[7] Lloyd (2016a) finds

[6] For example, the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia provide expectations of the average 3-month T-Bill rate during the current quarter, and the first, second, third and fourth quarters ahead.

[7] Gürkaynak et al. (2007b) compare the predictive power of federal funds futures to term federal funds loans, term eurodollar deposits, eurodollar futures, Treasury bills and commercial paper of comparable maturities.

that 3-11-month US OIS rates provide measures of monetary policy expectations as good as comparable-maturity federal funds futures rates for the 2002-15 period.[8] Lloyd (2016a) also assesses the empirical performance of OIS rates in the US, at longer maturities, and the UK, Japan and the Eurozone. Lloyd (2016a) concludes that UK, Japanese and, to a lesser extent, Eurozone OIS rates provide similarly good measures of interest rate expectations, implying that the method proposed in this paper is widely applicable in other countries.

OIS rates offer a further advantage over federal funds futures as a measure of interest rate expectations in a GADTSM-setting. The horizon of OIS contracts corresponds exactly to the horizon of the zero-coupon nominal government bond yield data used in GADTSMs. The horizon of a federal funds futures contract is a single month in the future, beginning on the first and ending on the last day of a specified month. Thus, OIS contracts provide a richer source of information with which to identify expected future short-term interest rates in the term structure of nominal government bond yields.

I document that the OIS-augmented model accurately captures investors' expectations of future short-term interest rates. The in-sample model estimates of interest rate expectations co-move closely with federal funds futures rates and survey expectations of future short-term interest rates. In these dimensions, the OIS-augmented model is superior to three other GADTSMs: (i) the unaugmented model, which only uses bond yield data to estimate both actual yields and interest rate expectations; (ii) the bias-corrected model of Bauer et al. (2012); and (iii) the survey-augmented model.[9] The OIS-augmented model is also best able to capture qualitative daily frequency movements in interest rate expectations implied by financial market instruments. Moreover, unlike the other models, the interest rate expectations implied by the OIS-augmented model obey the zero lower bound, despite the fact additional restrictions are not imposed to achieve this. This represents an important computational contribution in the light of recent computationally burdensome proposals for term structure modelling at the zero lower bound (see, for example Christensen and Rudebusch, 2013a,b).

The remainder of this paper is structured as follows. Section 2 describes the features of an OIS contract, defines its *ex post* realised excess return and illustrates the statistical insignificance of average *ex post* excess returns at a range of horizons. Section 3 lays out the unaugmented arbitrage-free GADTSM before describing the identification problem and 'finite sample' bias with direct reference to the model parameters. In section 4, I present the OIS-augmented model, which directly accounts for the payment structure of OIS contracts. In section 5, I document the data used and the estimation methodology. Section 6 presents the results of term structure estimation, documenting the superiority of the OIS-augmented model as a measure of interest rate expectations. Section 7 concludes.

[8] At very short-term horizons — 1-2 months — federal funds futures rates accurately reflect interest rate expectations (Hamilton, 2009).

[9] For the most direct comparison to the OIS-augmented model I propose in this paper, I estimate the survey-augmented model using the algorithm of Guimarães (2014) which uses the same Joslin et al. (2011) identification restrictions as my OIS-augmented model, as opposed to the Kim and Wright (2005) survey-augmented model that directly applies the Kim and Orphanides (2012) identification algorithm. Lloyd (2016b) shows that the Kim and Wright (2005) model performs worse than the OIS-augmented decomposition.

2 Overnight Indexed Swaps

An overnight index swap (OIS) is an over-the-counter traded interest rate derivative with two participating agents who agree to exchange fixed and floating interest rate payments over a *notional* principal for the life of the contract. The floating leg of the contract is constructed by calculating the accrued interest payments from a strategy of investing the notional principal in the overnight reference rate and repeating this on an overnight basis, investing principal plus interest each time. The reference rate for US OIS contracts is the effective federal funds rate. The 'OIS rate' represents the fixed leg of the contract. For vanilla US OIS contracts with a maturity of one year or less, money is only exchanged at the conclusion of the OIS contract. Upon settlement, only the net cash flow is exchanged between the parties.[10] That is, if the accrued fixed interest rate payment exceeds the floating interest payment, the agent who took on the former payments must pay the other at settlement. Importantly, there is no exchange of principal at any time for OIS contracts of all maturities.

Given its features, changes in OIS rates can reasonably be associated with changes in investors' expectations of future overnight interest rates over the horizon of the contract (Michaud and Upper, 2008). OIS contracts should contain only very small excess returns. Notably, because OIS contracts do not involve any initial cash flow, their liquidity premia will be small. Additionally, because OIS contracts do not involve an exchange of principal, their associated counterparty risk is small. Because many OIS trades are collateralised, credit risk is also minimised (Tabb and Grundfest, 2013, pp. 244-245). Unlike many LIBOR-based instruments, OIS contracts have increased in popularity amongst investors following the 2007/8 financial crisis (Cheng, Dorji, and Lantz, 2010).

2.1 Excess Returns on Overnight Indexed Swaps

To assess the magnitude of the excess returns within OIS rates, I present a mathematical expression for this quantity. Let $i^{OIS}_{t,t+n}$ denote the annualised n-month OIS rate, the fixed interest rate in the swap, quoted in month t. Let $i^{FLT}_{t,t+n}$ denote the annualised *ex post* realised value of the floating leg of the same swap contract. From the perspective of an agent who swaps fixed interest rate payments for the floating rate over the notional principal x, the net cash flow received is $\left(i^{OIS}_{t,t+n} - i^{FLT}_{t,t+n}\right) \times x$.

The floating leg of the contract $i^{FLT}_{t,t+n}$ is calculated by considering a strategy in which an investor borrows the swap's notional principal x, invests in the overnight reference rate and repeats the transaction on an overnight basis, investing principal plus interest each time. Let the contract trade day be denoted t_{1-s}, where s denotes the 'spot lag' of the contract in days. US OIS contracts have a two day spot lag $s = 2$, so the trade date is denoted t_{-1}.[11] Suppose that the n-month contract matures on the day t_N in the calendar month $t+n$. Then, the floating leg

[10] For vanilla OIS contracts with maturity in excess of one year, the exchange of net cash flows occurs at the end of every year (OpenGamma, 2013).

[11] That is, calculation of the payments to be made under the floating and fixed legs of the swap does not commence until two days after the contract was agreed.

is calculated based on the realised effective federal funds rate — the floating overnight reference rate for US OIS contracts — on days t_1, t_2, to t_N, where the effective federal funds rate on the day t_i is denoted: ffr_i. Following market convention, the mathematical expression for the floating leg of an n-month OIS contract, purchased on day t_{-1} in month t is:[12]

$$i^{FLT}_{t,t+n} = \left(\left[\prod_{i=1}^{N} (1 + \gamma_i ffr_i) \right] - 1 \right) \times \frac{360}{N} \tag{1}$$

where γ_i is the accrual factor of the form $\gamma_i = D_i/360$, where D_i is the day count between the business days t_i and t_{i+1}.[13] To compare this floating leg to the fixed leg $i^{OIS}_{t,t+n}$, which is reported on an annualised basis, $i^{FLT}_{t,t+n}$ is a multiple of $360/N$ in equation (1).[14]

From the perspective of the agent who swaps fixed for floating interest rate payments, $\left(i^{OIS}_{t,t+n} - i^{FLT}_{t,t+n}\right) \times x$ represents the payoff of a zero-cost portfolio.[15] Thus, in accordance with the terminology of Piazzesi and Swanson (2008), the '*ex post* realised excess return' on the n-month OIS contract purchased in month t is:

$$rx^{ois}_{t,t+n} = i^{OIS}_{t,t+n} - i^{FLT}_{t,t+n} \tag{2}$$

Throughout this section, I report *ex post* excess returns in basis points (i.e., $100 \times rx^{ois}_{t,t+n}$).

Under the expectations hypothesis, the fixed leg of the OIS contract must equal the *ex ante* expectation of the floating leg:

$$i^{OIS}_{t,t+n} = \mathbb{E}_t \left[i^{FLT}_{t,t+n} \right] \tag{3}$$

Thus, if the *ex post* realised excess return in equation (2) has zero mean, the *ex ante* forecasting error under the expectations hypothesis also has zero mean, supporting the proposition that the n-month OIS rate provides an accurate measure of investors' expectations of future short-term interest rates. In constructing the OIS-augmented GADTSM, I assume that the included OIS tenors satisfy the relationship in equation (3), motivating the subsequent estimation of *ex post* realised excess returns on OIS contracts of various maturities to test this assumption.

2.2 Estimated Average Excess Returns on OIS Contracts

The results presented in this sub-section are from Lloyd (2016a). To attain these results, I calculate the *ex post* realised floating leg of the n-month OIS contract using equation (1), accounting for the two-day spot lag, and the *ex post* realised excess return using equation (2)

[12] See both Cheng et al. (2010) and OpenGamma (2013).

[13] For example, on a week with no public holidays, the day count D_i will be set to 1 on Monday to Thursday, 3 on Friday, and 0 on Saturday and Sunday. The day count is divided by 360, and not 365, in accordance with the quoting convention of the US market, which uses a 30-day month and 360-day year. For additional details, see OpenGamma (2013).

[14] This, again, is in accordance with the US market quoting conventions. The fixed and floating legs of US OIS contracts are quoted according to the *Actual 360* market convention (OpenGamma, 2013, page 6).

[15] Formally, this portfolio involves borrowing x at the floating overnight index rate at day t_1 and rolling-over the borrowing to day t_N (resulting in the total floating rate payment $i^{FLT}_{t,t+n}$), while investing the x borrowed on day t_1 in the fixed interest rate $i^{OIS}_{t,t+n}$ for N-days.

Table 1: Average *Ex Post* Excess Returns on US OIS Contracts at Daily Frequency

Maturity in Months	**1**	**2**	**3**	**4**	**5**	**6**
$\widehat{\alpha}^{(n)}$	−3.31***	0.29	2.46	4.39	6.25	8.12
[t-statistic]	[−2.98]	[0.18]	[1.07]	[1.37]	[1.42]	[1.38]
Maturity in Months	**7**	**8**	**9**	**10**	**11**	**12**
$\widehat{\alpha}^{(n)}$	9.64	11.41	14.13	15.33	17.45	21.33
[t-statistic]	[1.25]	[1.19]	[1.26]	[1.14]	[1.13]	[1.25]
Maturity in Months	**15**	**18**	**21**	**24**	**36**	
$\widehat{\alpha}^{(n)}$	28.16	36.36	45.38	54.97*	91.43***	
[t-statistic]	[1.27]	[1.37]	[1.54]	[1.82]	[5.70]	

Note: Results from regression (4) for US OIS contracts. Sample: January 2002 to December 2015, Daily Frequency. Robust Hansen and Hodrick (1980) t-statistics are reported in square brackets. An excess return is significantly different from zero at a 1%, 5% and 10% significance level when the absolute value of the t-statistic exceeds 2.33, 1.96 and 1.645 respectively. These are denoted with asterisks ***, ** and * for the 1%, 5% and 10% significance levels respectively. All figures are reported in basis points to two decimal places.

at a daily frequency. I then run the following regression:

$$rx^{ois}_{t,t+n} = \alpha^{(n)} + \varepsilon_{t,t+n} \tag{4}$$

for the following maturities: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11 months; 1 year; 15, 18, 21 months; 2 and 3 years.

The results of this are presented in table 1. All regressions are reported using the January 2002 to December 2015 sample period,[16] the baseline period for all estimations in this paper. Because contract horizons at adjacent time periods overlap, I compute robust Hansen and Hodrick (1980) standard errors to account for the serial correlation this induces. I report t-statistics based on these standard errors.

Average *ex post* excess returns on 3-21-month OIS contracts are all insignificantly positive, ranging from 2.46 to 45.38 basis points. The average excess return on OIS contracts is broadly increasing in the maturity of the contract, consistent with the view that OIS rates will contain some, albeit statistically insignificant, term premia.

The average *ex post* excess returns on the 2-year OIS contract is 54.97 basis points, significant at the 10% significance level. At first sight, this finding undermines the claim the 2-year OIS contract provides accurate measures of investors' interest rate expectations. However, Lloyd (2016a) demonstrates that this result reflects the *ex ante* unexpected nature of the 2007/8 financial crisis and associated loosening of monetary policy, rather than risk premia within the contract. Lloyd (2016a) finds that by adding an additional '2008 dummy', set equal to one on dates where the OIS contract matures between January 22, 2008 to December 16, 2008 to account for the unexpected nature of US monetary policy loosening in 2008,[17] the average *ex*

[16]January 2002 is the first month in which daily OIS rate data at these maturities is regularly available on Bloomberg. See Lloyd (2016a, Appendix A) for a detailed description of OIS rate availability.

[17]January 22, 2008 confers to first US policy rate cut in 2008. On December 16, 2008, the federal funds rate target fell to 0-0.25 basis points.

post excess return on the 2-year OIS contract is 19.98 basis points and insignificantly different from zero.

The 1-month and 3-year OIS contracts both have statistically significant average excess returns. The average excess return on the 1-month OIS contract is -3.31 basis points and the 3-year contract has a positive average excess return of 91.43 basis points. Lloyd (2016a) conducts sensitivity analysis to investigate these results and concludes that: (i) the significantly negative average *ex post* excess return on the 1-month contract, most likely, reflects a lack of liquidity at this tenor, between 2002 and 2007 especially;[18] and (ii) the significantly positive average *ex post* excess return on the 3-year contract reflects risk premia in longer-horizon OIS contracts that blur their use as market-based measures of monetary policy expectations.[19]

Overall, the results in Lloyd (2016a) support the conclusion that, on average, 3-24-month OIS rates provide accurate measures of investors' interest rate expectations. Specifically, the results indicate that OIS contracts of these tenors conform to the expectations hypothesis, as stated in equation (3), containing statistically insignificant *ex ante* forecasting errors under the hypothesis. This verifies an important identifying assumption for the OIS-augmented GADTSM presented in section 4 below.

2.3 OIS Rates and Survey Expectations

To add further illustrative evidence to the proposition that OIS rates provide accurate information about investors' expectations of the future short-term interest rate path, I consider the relationship between OIS and another measure of investors' expectations, namely survey expectations.

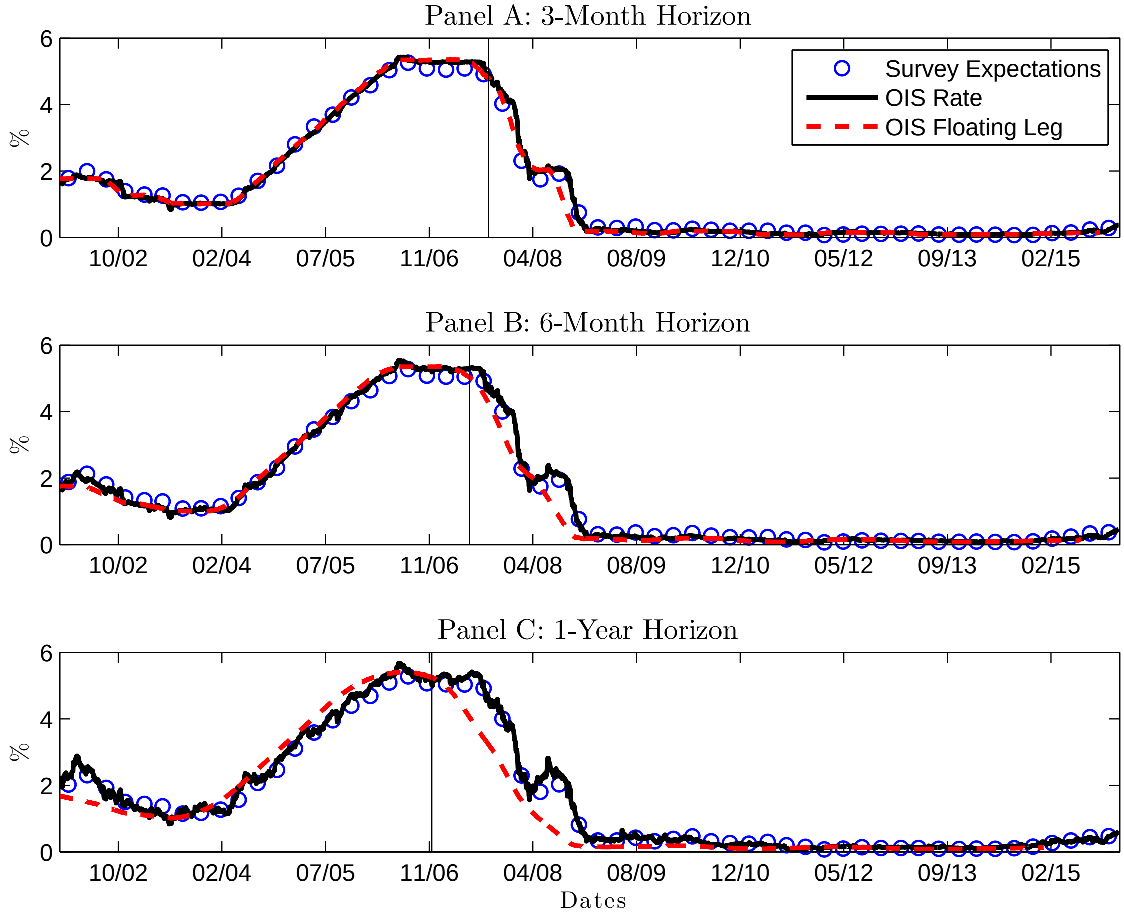
In figure 1, I plot the daily 3, 6 and 12-month OIS rates between January 2002 and December 2014 against both the daily frequency *ex post* realised floating leg of the swap and the quarterly frequency survey expectations of the future short-term nominal interest rate over the corresponding horizon. I construct approximations of survey forecasts for the average 3-month T-bill rate for each of the horizons using data from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia.[20] The survey is published every quarter and reports the mean forecasters' expectations of the average 3-month T-Bill rate over a specified time period in: the current quarter $\bar{i}_{t|t}^{3m,sur}$; and the first $\bar{i}_{t+1|t}^{3m,sur}$, second $\bar{i}_{t+2|t}^{3m,sur}$, third $\bar{i}_{t+3|t}^{3m,sur}$ and fourth $\bar{i}_{t+4|t}^{3m,sur}$ quarters subsequent to the current one, where t here denotes the current quarter. To construct the survey forecast approximations plotted in figure 1, I first calculate the implied expectations of the average 3-month T-Bill rate over the remainder of the current quarter using both the realised 3-month T-Bill rate over the current quarter and the survey expectation for the

[18]Although there is no direct evidence for this claim, anecdotal evidence is not unsupportive. Fleming et al. (2012, p. 14, table 7) find that the 3, 6 and 12-month tenors were the most liquid and commonly traded US OIS contracts during 2010.

[19]From this sensitivity analysis, Lloyd (2016a) also concludes that the 2-month OIS rate contains negative statistically significant *ex post* excess returns for the 2002-15 period, excluding 2008. As with the 1-month rate, Lloyd (2016a) concludes that this finding, most likely, reflects a lack of liquidity at this tenor between 2002 and 2007 especially.

[20]See Appendix A for a detailed specification of data sources. Guimarães (2014) uses survey forecasts from the *Survey of Professional Forecasters* in his estimation of the US term structure of interest rates.

Figure 1: Survey Expectations, OIS Rates and their *Ex Post* Realised Floating Leg



The daily OIS rates are from Bloomberg. See appendix A for detailed data source information. The daily *ex post* realised floating legs of the swaps are calculated in the manner described in section 2.1 and, specifically, in equation (1). The survey expectations are from the *Survey of Professional Forecasters*. The survey forecast, at each horizon, is attained by constructing the geometric weighted average of the mean response of forecasters relating to their expectation of the average 3-month T-Bill rate over the different relevant periods, in the manner described in section 2.3. The survey expectations are plotted on the forecast submission deadline date for each quarter. The vertical lines in each panel are plotted at the beginning of the time period 3, 6 and 12 months prior to November 2007 respectively, the official start date of the US recession according to the NBER dating.

average 3-month T-Bill rate for the current quarter. Using this and the longer-horizon survey expectations, I then calculate geometric weighted averages of survey forecasts from the *Survey of Professional Forecasters*. I use a geometric weighting scheme to replicate the geometric payoff structure embedded within OIS contracts, allowing comparison of the survey and OIS-implied expectations.

Although the *Survey of Professional Forecasters* data is available once a quarter, the deadline date for submission of the forecasts lies approximately halfway through the quarter.[21] In figure 1, I assume that the reported 3-month T-Bill expectations reflect forecasters' expectations on the deadline day, so plot these expectations on this date, rather than the end of the quarter. The weighting scheme used for the approximations in figure 1 is made possible because the

[21] For example, the deadline date for the 2013 Q1 survey was February 11th 2013.

survey deadline date lies approximately halfway through the 'current' quarter. Further details of the survey forecast approximation, including specific mathematical expressions, are presented in appendix B. The plots, in figure 1, demonstrate that survey and OIS-implied interest rate expectations have closely qualitatively and quantitatively co-moved between 2002 and 2015.

In figure 1, the difference between the OIS rate (solid black line) and the *ex post* realised floating leg (dashed red line) graphically depicts the excess return defined in equation (2). Visual inspection of figure 1 confirms the formal results from section 2.2: the OIS rate closely co-moves with the *ex post* realised path of the floating leg of the contract. The most notable deviation of the two quantities occurs in 2007-8, consistent with the financial turmoil that erupted in this period.[22] As many of the events during 2007-8 — Federal Reserve policy easing most importantly — were unanticipated, there is no reason to expect them to be reflected in *ex ante* expectations of future interest rates, explaining the difference in the quantities at this time.

3 Term Structure Model

In this section, I present the discrete-time GADTSM that is commonplace in the literature (see, for example Ang and Piazzesi, 2003; Kim and Wright, 2005). I then describe the identification problem, which arises from the estimation of unaugmented GADTSMs, with direct reference to the model's parameters. Since the focus of this paper is on the identification of short-term interest rate expectations at a daily frequency, hereafter I refer to t as a *daily* time index.[23]

3.1 Unaugmented Model Specification

The discrete-time GADTSM builds on three key foundations. First, there are K pricing factors $\mathbf{x}_t$ (a $K \times 1$ vector), which follow a first-order vector autoregressive process under the actual probability measure $\mathbb{P}$:

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1} \tag{5}$$

where $\boldsymbol{\varepsilon}_{t+1}$ is a stochastic disturbance with the conditional distribution $\boldsymbol{\varepsilon}_{t+1}|\mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$; $\mathbf{0}_K$ is a $K \times 1$ vector of zeros; and $\mathbf{I}_K$ is a $K \times K$ identity matrix. $\boldsymbol{\mu}$ is a $K \times 1$ vector and $\boldsymbol{\Phi}$ is a $K \times K$ matrix of parameters. $\boldsymbol{\Sigma}$ is a $K \times K$ lower triangular matrix, which is invariant to the probability measure.

Second, the one-period short-term nominal interest rate i_t is assumed to be an affine function of all the pricing factors:

$$i_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t \tag{6}$$

where δ_0 is a scalar and $\boldsymbol{\delta}_1$ is a $K \times 1$ vector of parameters.

Third, no-arbitrage is imposed. Following Duffee (2002), the pricing kernel M_{t+1} that prices

[22] The vertical lines in figure 1 denote the time period 3, 6 and 12 months prior to the official start of the US recession in November 2007, according to the NBER dating.

[23] However, the model can be estimated at lower frequencies (e.g. monthly), with the label for t changing correspondingly (e.g. a month). The results from monthly estimation are qualitatively similar to those from daily estimation, so are presented in appendix F.3.

all assets when there is no-arbitrage is of the following form:

$$M_{t+1} = \exp\left(-i_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\varepsilon}_{t+1}\right) \tag{7}$$

where $\boldsymbol{\lambda}_t$ represents a $K \times 1$ vector of time-varying market prices of risk, which are affine in the pricing factors:

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \tag{8}$$

where $\boldsymbol{\lambda}_0$ is a $K \times 1$ vector and $\boldsymbol{\Lambda}_1$ is a $K \times K$ matrix of parameters.

The assumption of no-arbitrage guarantees the existence of a risk-adjusted probability measure $\mathbb{Q}$, under which the bonds are priced (Harrison and Kreps, 1979).[24] Given the choice of market prices of risk in equation (8), the pricing factors $\mathbf{x_t}$ also follow a first-order vector autoregressive process under the $\mathbb{Q}$ probability measure:

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} \tag{9}$$

where:[25]

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \qquad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1.$$

and $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}$ is a stochastic disturbance with the conditional distribution $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}|\mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$.

Bond Pricing Since M_{t+1} is the nominal pricing kernel, which prices all nominal assets in the economy, the gross one-period return R_{t+1} on any nominal asset must satisfy:

$$\mathbb{E}_t\left[M_{t+1}R_{t+1}\right] = 1 \tag{10}$$

Let $P_{t,n}$ denote the price of an n-day zero-coupon bond at time t. Then, using $R_{t+1} = P_{t+1,n-1}/P_{t,n}$, equation (10) implies that the bond price is recursively defined:

$$P_{t,n} = \mathbb{E}_t\left[M_{t+1}P_{t+1,n-1}\right] \tag{11}$$

Alternatively, with no-arbitrage, the price of an n-period zero-coupon bond must also satisfy the following relation under the risk-adjusted probability measure $\mathbb{Q}$:

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-i_t)P_{t+1,n-1}\right] \tag{12}$$

By combining the dynamics of the pricing factors (equation (9)) and the short-term interest rate (equation (6)) with equation (12), the bond prices can be shown to be an exponentially

[24] The risk-adjusted probability measure $\mathbb{Q}$ is defined such that the price V_t of any asset that does not pay any dividends at time $t+1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-i_t)V_{t+1}\right]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the $\mathbb{Q}$ probability measure.

[25] See appendix C.2 for a formal derivation of these expressions.

affine function of the pricing factors:

$$P_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t\right) \tag{13}$$

where the scalar $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$, a $1 \times K$ vector, are recursively defined loadings:

$$\begin{aligned}
\mathcal{A}_n &= -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} \\
\mathcal{B}_n &= -\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}}
\end{aligned}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}_K'$ ensuring that the price of a 'zero period' bond is one.[26]

The continuously compounded yield on an n-day zero-coupon bond at time t, $y_{t,n} = -\frac{1}{n}\ln\left(P_{t,n}\right)$, is given by:

$$y_{t,n} = A_n + B_n \mathbf{x}_t \tag{14}$$

where $A_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$.

The risk-neutral yield on an n-day bond reflects the expectation of the average short-term interest rate over the n-day life of the bond, corresponding to the yields that would prevail if investors were risk-neutral.[27] That is, the yields that would arise under the expectations hypothesis of the yield curve. The risk-neutral yields can be calculated using:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t \tag{15}$$

where $\widetilde{A}_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.[28] Note that, because no-arbitrage is assumed, the bonds are priced under the risk-adjusted measure $\mathbb{Q}$, so the fitted yields are attained using parameters from the risk-adjusted probability measure $\mathbb{Q}$ — specifically $\left\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\right\}$. The risk-neutral yields are attained using parameters from the actual probability measure $\mathbb{P}$, $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$.

The spot term premium on an n-day bond is defined as the difference between the fitted yield (equation (14)) and the risk-neutral yield (equation (15)):

$$tp_{t,n} = y_{t,n} - \widetilde{y}_{t,n} \tag{16}$$

3.2 Unaugmented GADTSMs and the Identification Problem

Numerous studies have documented the problems in separately identifying expectations of future short-term interest rates (the risk-neutral yields) from term premia (see, for example: Bauer et al., 2012; Duffee and Stanton, 2012; Kim and Orphanides, 2012; Guimarães, 2014). The underlying source of difficulty is an informational insufficiency, which gives rise to finite sample

[26] See appendix C.1 for a formal derivation of these expressions.

[27] There is a small difference between risk-neutral yields and expected yields due to a convexity effect. In the homoskedastic model considered here, these effects are constant for each maturity and, in practice, small, corresponding to the $\frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}'$ term in the recursive expression for $\mathcal{B}_n$ above.

[28] See appendix C.3 for a formal derivation of these expressions.

bias as its symptom.

The unaugmented model uses data on zero-coupon bond yields as its sole input. This data provides a complete set of information about the dynamic evolution of the cross-section of yields — the yield curve. This provides sufficient information to accurately identify the risk-adjusted $\mathbb{Q}$ dynamics — specifically, the parameters $\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\}$ in equation (9) — which equation (14) shows are of direct relevance to estimating *actual* yields. However, if there is no additional information and the sample of yields contains too few interest rate cycles,[29] this data is not sufficient for the identification of the actual $\mathbb{P}$ dynamics — specifically, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ in equation (5) — which equation (15) illustrates are of relevance to the estimation of *risk-neutral* yields. Estimates of $\boldsymbol{\Phi}$ in equation (5) will suffer from finite sample bias. In particular, the persistent yields will have persistent pricing factors, so maximum likelihood or ordinary least squares estimates of the persistence parameters of the vector autoregressive process in equation (5) $\boldsymbol{\Phi}$ will be biased downwards.[30] That is, the estimated $\widehat{\boldsymbol{\Phi}}$ will understate the true persistence of the pricing factors, implying a spuriously fast mean reversion of future short-term interest rates.[31] Because, in the model, agents form expectations of future short-term interest rates based on estimates of pricing factor mean reversion in $\widehat{\boldsymbol{\Phi}}$, their estimates of the future short-term interest rate path will mean revert spuriously quickly too. Consequently, the estimated risk-neutral yields, which summarise the average of the expected path of future short-term interest rates, will vary little and will not accurately reflect the evolution of interest rate expectations.

The magnitude of the finite sample bias is increasing in the persistence of the data. So for daily frequency yield data, which is highly persistent, the bias will be more severe. This not only motivates the augmentation of the GADTSM with additional data, but motivates the use of additional *daily frequency* data, namely: OIS rates.

4 The OIS-Augmented Model

To augment the model with OIS rates, I employ Kalman filter-based maximum likelihood estimation. The Kalman filtering approach is particularly convenient for the augmentation of GADTSMs, as it can handle mixed-frequency data. Specifically, for OIS-augmentation, this allows estimation of the GADTSM for periods extending beyond that for which OIS rates are available.[32] Moreover, Duffee and Stanton (2012) find that Kalman filtering methods, which assume all bond yields are priced with error, provide better parameter estimates than for mod-

[29] Kim and Orphanides (2012, p. 242) state that a term structure samples spanning 5 to 15 years may contain too few interest rate cycles.

[30] This is a multivariate generalisation of the downward bias in the estimation of autoregressive parameters by ordinary least squares in the univariate case.

[31] Bauer et al. (2012) highlight the persistence of the pricing factors by considering, *inter alia*, the maximum eigenvalue of $\widehat{\boldsymbol{\Phi}}$.

[32] In this paper, I use daily US OIS rates from 2002, the first date for which these rates are consistently available at all the relevant tenors on Bloomberg. I estimate the GADTSMs from this date to directly isolate the effect of OIS rates on GADTSM. However, Lloyd (2016b) applies an OIS-augmented GADTSM estimated from July 1990 to December 2015, with OIS rates from 2002, to assess the relative efficacy of various interest rate channels of unconventional monetary policy.

els in which only a subset of yields are priced with error. Guimarães (2014) applies Kalman filter-based estimation to a survey-augmented GADTSM and concludes that, when parameter estimates from the unaugmented model, identified with the Joslin, Singleton, and Zhu (2011) normalisation scheme, are used as initial values for the survey-augmented model, the optimisation routine converges to the optimum very quickly.

To implement the Kalman filter-based estimation, I use equation (5), the vector autoregression for the latent pricing factors under the actual $\mathbb{P}$ probability measure, as the transition equation.

The observation equation differs depending on whether or not OIS rates are observed on day t or not. On days when the OIS rates are *not* observed (i.e. days prior to January 2002), the observation equation is formed by stacking the N yield maturities in equation (14) to form:

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\mathbf{x}_t + \mathbf{\Sigma}_Y \mathbf{u}_t \tag{17}$$

where: $\mathbf{y}_t = [y_{t,n_1}, ..., y_{t,n_N}]'$ is the $N \times 1$ vector of bond yields; $\mathbf{A} = [A_{n_1}, ..., A_{n_N}]'$ is an $N \times 1$ vector and $\mathbf{B} = \left[B'_{n_1}, ..., B'_{n_N}\right]'$ is an $N \times K$ matrix of bond-specific loadings; $A_{n_\iota} = -\frac{1}{n_\iota}\mathcal{A}_{n_\iota}\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \mathbf{\Phi}^{\mathbb{Q}}, \mathbf{\Sigma}; \mathcal{A}_{n_{\iota-1}}, \mathcal{B}_{n_{\iota-1}}\right)$ and $B_{n_\iota} = -\frac{1}{n_\iota}\mathcal{B}_{n_\iota}\left(\boldsymbol{\delta}_1, \mathbf{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n_{\iota-1}}\right)$ are the bond-specific loadings; and $\iota = 1, 2..., N$ such that n_ι denotes the maturity of bond ι in days. The $N \times 1$ vector $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}_N, \mathbf{I}_N)$ denotes the yield measurement error, where $\mathbf{0}_N$ is an N-vector of zeros and $\mathbf{I}_N$ is an $N \times N$ identity matrix. Here, like much of the existing literature,[33] I impose a homoskedastic form for the yield measurement error, such that $\mathbf{\Sigma}_Y$ is an $N \times N$ diagonal matrix with common diagonal element σ_e, the standard deviation of the yield measurement error. The homoskedastic error is characterised by a single parameter σ_e, maintaining computational feasibility for an already high-dimensional maximum likelihood routine.

On days when OIS rates are observed, the observation equation for the Kalman filter is augmented with OIS rates. The following proposition illustrates that OIS rates can (approximately) be written as an affine function of the pricing factors with loadings A_j^{ois} and B_j^{ois} for J different OIS maturities, where $j = j_1, j_2, ..., j_J$ denote the J OIS horizons in days. The loadings presented in this proposition are calculated by assuming that the expectations hypothesis (equation (3)) holds for the OIS tenors included in the model, an assumption that was verified in section 2 for the maturities used here. Moreover, the loadings explicitly account for the compounding involved in calculating the floating leg of the OIS contract. It is in this respect that the technical setup of the OIS-augmented GADTSM most clearly differs from the survey-augmented model. In the latter, the loadings are based on an arithmetic expectational structure.

Proposition The j-day OIS rate $i_{t,t+j}^{ois}$, where $j = j_1, j_2, ...j_J$, can be (approximately) written as an affine function of the pricing factors $\mathbf{x}_t$:

$$i_{t,t+j}^{ois} = A_j^{ois} + B_j^{ois}\mathbf{x}_t \tag{18}$$

[33] See, for example, Guimarães (2014).

where $A_j^{ois} \equiv \frac{1}{j}\mathcal{A}_j^{ois}\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{j-1}^{ois}, \mathcal{B}_{j-1}^{ois}\right)$ and $B_j^{ois} \equiv \frac{1}{j}\mathcal{B}_j^{ois}\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{j-1}^{ois}\right)$ are recursively defined as:

$$\begin{aligned}
\mathcal{A}_j^{ois} &= \delta_0 + \boldsymbol{\delta}_1'\boldsymbol{\mu} + \mathcal{A}_{j-1}^{ois} + \mathcal{B}_{j-1}^{ois}\boldsymbol{\mu} \\
\mathcal{B}_j^{ois} &= \boldsymbol{\delta}_1'\boldsymbol{\Phi} + \mathcal{B}_{j-1}^{ois}\boldsymbol{\Phi}
\end{aligned}$$

where $\mathcal{A}_0^{ois} = 0$ and $\mathcal{B}_0^{ois} = \mathbf{0}_K'$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

Proof: See appendix D.

Given this, the observation equation of the Kalman filter on the days OIS rates are observed is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{ois} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{ois} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{ois} \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \boldsymbol{\Sigma}_Y & \\ & \boldsymbol{\Sigma}_O \end{bmatrix} \mathbf{u}_t^{ois} \tag{19}$$

where, in addition to the definitions of $\mathbf{A}$, $\mathbf{B}$ and $\boldsymbol{\Sigma}_Y$ above: $\mathbf{i}_t^{ois} = \left[i_{t,j_1}^{ois}, ..., i_{t,j_J}^{ois}\right]'$ is the $J \times 1$ vector of OIS rates; $\mathbf{A}^{ois} = \left[A_{j_1}^{ois}, ..., A_{j_J}^{ois}\right]'$ is a $J \times 1$ vector and $\mathbf{B}^{ois} = \left[B_{j_1}^{ois\prime}, ..., B_{j_J}^{ois\prime}\right]'$ is a $J \times K$ matrix of OIS-specific loadings; and $\mathbf{u}_t^{ois} \sim \mathcal{N}\left(\mathbf{0}_{N+J}, \mathbf{I}_{N+J}\right)$ denotes the yield and OIS measurement error, where $\mathbf{0}_{N+J}$ is an $N+J$-vector of zeros and $\mathbf{I}_{N+J}$ is an $(N+J) \times (N+J)$ identity matrix. The inclusion of the measurement error permits non-zero OIS forecast errors and imposes that this forecast error is zero *on average*. I tested two parameterisations of the volatility matrix $\boldsymbol{\Sigma}_O$ a $J \times J$ diagonal matrix, for OIS measurement errors: one with distinct volatilities for each OIS maturity and another with common volatilities. Both include independent errors — diagonal $\boldsymbol{\Sigma}_O$. After testing the two, I impose a homoskedastic form for the OIS measurement error, such that $\boldsymbol{\Sigma}_O$ has common diagonal element σ_o, the standard deviation of the OIS measurement error.[34] The imposition of homoskedastic OIS measurement errors also provides computational benefits, as there are fewer parameters to estimate than if a more general measurement error covariance structure was permitted.[35]

5 Methodology

5.1 Data

In all models, bond yields $\mathbf{y}_t$ of the following maturities are used: 3 and 6 months, 1 year, 18 months, 2 years, 30 months, 3 years, 42 months, 4 years, 54 months, 5, 7 and 10 years.[36] For the 3 and 6-month yields, I use US T-Bill rates — made available by the Federal Reserve — in accordance with much of the existing dynamic term structure literature and evidence from

[34] The risk-neutral yields from the homoskedastic model provide a superior fit for interest rate expectations, *vis-à-vis* other survey and market-based measures of interest rate expectations, relative to the heteroskedastic model.

[35] Kim and Orphanides (2012) and Guimarães (2014) impose homoskedasticity on the survey measurement errors in their Kalman filter setup for this reason.

[36] These yield maturities correspond to those used by Adrian, Crump, and Moench (2013).

Greenwood, Hanson, and Stein (2015), who document a marked wedge between 1-26-week T-Bill rates and corresponding maturity fitted zero-coupon bond yields.[37] The remaining rates are from the continuously compounded zero-coupon yields of Gürkaynak, Sack, and Wright (2007a). This data is constructed from daily-frequency fitted Nelson-Siegel-Svensson yield curves. Using the parameters of these curves, which are published along with the estimated zero-coupon yield curve, I back out the cross-section of yields for the 11 maturities from 1 to 10-years.[38]

OIS rates are from *Bloomberg*. I use combinations of 3, 6, 12 and 24-month OIS rates in the OIS-augmented models. The choice of these maturities is motivated by evidence in section 2 and Lloyd (2016a). Since US OIS rates are only consistently available from January 2002, my baseline estimation sample period runs from January 2002 to December 2015 to isolate the effect of OIS augmentation on GADTSM estimation.

5.2 Estimation

The OIS-augmented model relies on Kalman filter-based maximum likelihood estimation, for which the pricing factors $\mathbf{x}_t$ are latent. Normalisation restrictions must be imposed on the parameters to achieve identification. For this, I appeal to the normalisation scheme of Joslin et al. (2011), which "allows for computationally efficient estimation of G[A]DTSMs" (Joslin et al., 2011, p. 928) and fosters faster convergence to the global optimum of the model's likelihood function than other normalisation schemes (see, for example: Dai and Singleton, 2000).[39] This permits a two-stage approach to estimating the OIS-augmented model.

To benefit fully from the computational efficiency of the Joslin et al. (2011) normalisation scheme, I *first* estimate the unaugmented GADTSM (hereafter, labelled the OLS/ML model), presented in section 3.1, assuming that K portfolios of yields are priced without error, to attain initial values for the Kalman filter used in the second estimation stage. In particular, these K yield 'portfolios', $\mathbf{x}_t$, correspond to the first K estimated principal components of the bond yields. Under the Joslin et al. (2011) normalisation, this itself enables a two sub-stage estimation: first the $\mathbb{P}$ parameters are estimated by OLS on equation (5) using the K estimated principal components in the vector $\mathbf{x}_t$; second the $\mathbb{Q}$ parameters are estimated by maximum likelihood (see appendix E for details).

Having attained these OLS/ML parameter estimates, I *second* estimate the OIS-augmented model — which assumes all yields are observed with error — using the OLS/ML parameter estimates as initial values for the Kalman filter-based maximum likelihood routine.

[37]To foster comparison with zero-coupon bond yields, the T-Bill rates are converted from their discount basis to the yield basis.

[38]The Nelson-Siegel-Svensson yield curve equation used to back out the cross-section of yields at a daily frequency is reported in equation (22) of Gürkaynak, Sack, and Wright (2006), an earlier working paper version of Gürkaynak et al. (2007a).

[39]The computational benefits of the Joslin et al. (2011) normalisation scheme arise because it only imposes restrictions on the short-rate and the factors $\mathbf{x}_t$ under the $\mathbb{Q}$ probability measure. Consequently, the $\mathbb{P}$ and $\mathbb{Q}$ dynamics of the model do not exhibit strong dependence. Under the Dai and Singleton (2000) scheme, restrictions on the volatility matrix $\mathbf{\Sigma}$, which influences both the $\mathbb{P}$ and $\mathbb{Q}$ evolution of the factors (see equations (5) and (9)), create a strong dependence between the parameters under the two probability measures, engendering greater computational complexity in the estimation.

6 Term Structure Results

In this section, I provide estimation results for the following GADTSMs, each estimated at a daily frequency: (1) an unaugmented OLS/ML model, estimated using the Joslin et al. (2011) identification scheme where K portfolios of yields are observed without error and are measured with the first K estimated principal components of the bond yields; (2) the Bauer et al. (2012) bias-corrected model; (3) a survey-augmented model, using expectations of future short-term interest rates for the subsequent four quarters as an additional input, estimated with the Kalman filter using the algorithm of Guimarães (2014) (see appendix E for details);[40] and (4) the OIS-augmented model, described above. I estimate three variants of the OIS-augmented model. The first, baseline setup, includes the 3, 6, 12 and 24-month OIS rates (4-OIS-Augmented model). The second and third models include the 3, 6 and 12-month (3-OIS-Augmented model) *and* 3 and 6-month (2-OIS-Augmented model) tenors respectively.[41] Of the three OIS-augmented models, I find that the 4-OIS-Augmented model provides risk-neutral yields that best fit the evolution of interest rate expectations, in and out-of-sample.

In accordance with the well-rehearsed evidence of Litterman and Scheinkman (1991), that the first three principal components of bond yields explain well over 95% of their variation, I estimate the models with three pricing factors ($K = 3$).[42] By using the three-factor specification, for which the pricing factors have a well-understood economic meaning (the level, slope and curvature of the yield curve respectively), I am able to isolate and explain the economic mechanisms through which the OIS-augmented model provides superior estimates of expectations of future short-term interest rates *vis-à-vis* the unaugmented, bias-corrected and survey-augmented models.

6.1 Model Fit

In this sub-section, I discuss four aspects of model fit: estimated bond yields, estimated OIS rates, estimated pricing factors and parameter estimates.

6.1.1 Fitted Bond Yields

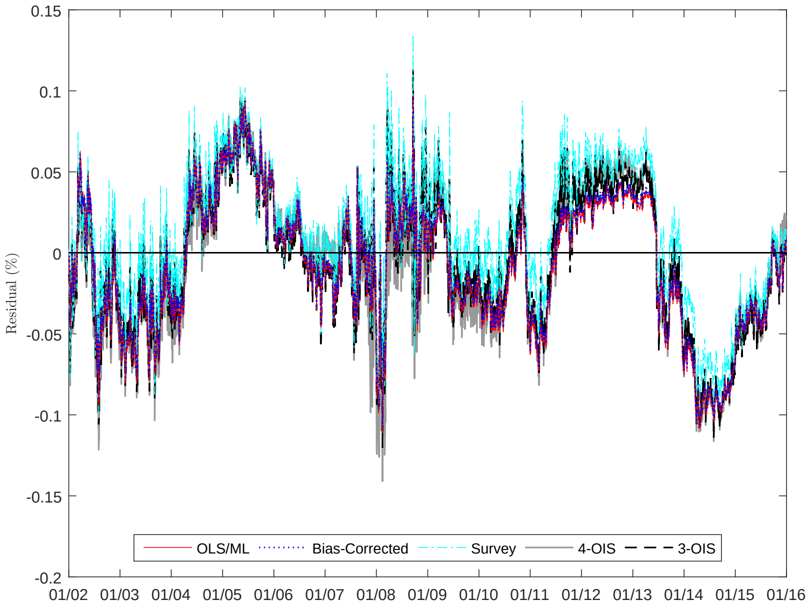
Importantly, the augmentation of the GADTSM with OIS rates does not compromise the overall fit of the model with respect to actual bond yields. The overall actual yield fit is strikingly similar across all the models estimated. Figure 2 provides illustrative evidence of this, plotting

[40]For direct comparison to my OIS-augmented model, I estimate the survey-augmented model by applying the algorithm of Guimarães (2014), who also uses the same Joslin et al. (2011) identification scheme as me. Kim and Orphanides (2012) implement a different identification scheme in the estimation of their survey-augmented model. Like Guimarães (2014), I use survey expectations from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia, including forecasts of the 3-month T-Bill rate for the remainder of the current quarter and the first, second, third and fourth quarters ahead.

[41]Because the results from the 2-OIS-augmented model are inferior to those from the 4 and 3-OIS-augmented models, I present results for the 2-OIS-augmented model in appendix F.

[42]I also estimate a four factor specification in the light of evidence by Cochrane and Piazzesi (2005, 2008) and Duffee (2011) who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields. These results are reported in appendix F.2.

Figure 2: Residual of the 2-Year Fitted Yield from GADTSMs



Note: Residuals of the 2-year fitted yield from five GADTSMs: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

the residuals of the 2-year fitted yield from the OLS/ML, bias-corrected, survey-augmented, 4-OIS-augmented and 3-OIS-augmented models. The residuals from each model follow similar qualitative and quantitative paths.[43]

The similar actual yield fit of the models is intuitive. I augment the GADTSM with OIS rates to provide additional information with which to better estimate parameters under the actual probability measure $\mathbb{P}$ $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$, which directly influences estimates of the risk-neutral yields. Estimates of the fitted yield depend upon the risk-adjusted measure $\mathbb{Q}$ parameters $\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\}$, which are not directly influenced by the OIS rates in the model, and are well-identified with bond yield data that provides information on the dynamic evolution of the cross-section of yields.

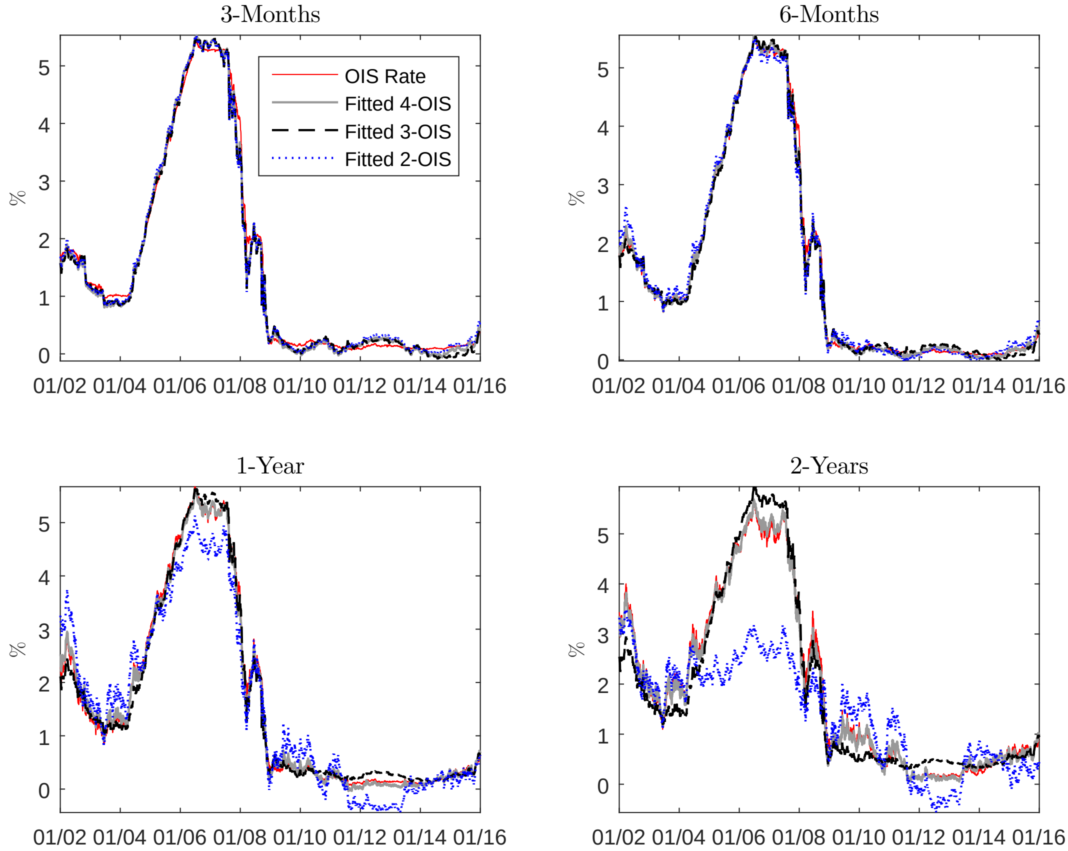
6.1.2 Fitted OIS Rates

Alongside estimates of the actual bond yield, the OIS-augmented models also provide fitted values for OIS rates. Figure 3 presents evidence that the OIS-augmented models provide accurate estimates of actual OIS rates, by plotting the 3, 6, 12 and 24-month OIS rates against the corresponding-maturity fitted-OIS rates from the 4, 3 and 2-OIS-augmented models.[44] The plots illustrate that the 4-OIS-augmented model best fits the 3, 6, 12 and 24-month OIS rates.

[43] Table 8, in appendix F.1, provides more detailed evidence of the similar actual yield fit of the models, documenting the root mean square error (RMSE) for each model at each yield maturity.

[44] Table 9, in appendix F.1, provides more detailed numerical evidence to support the claim.

Figure 3: Fitted OIS Rates from the OIS-Augmented Models

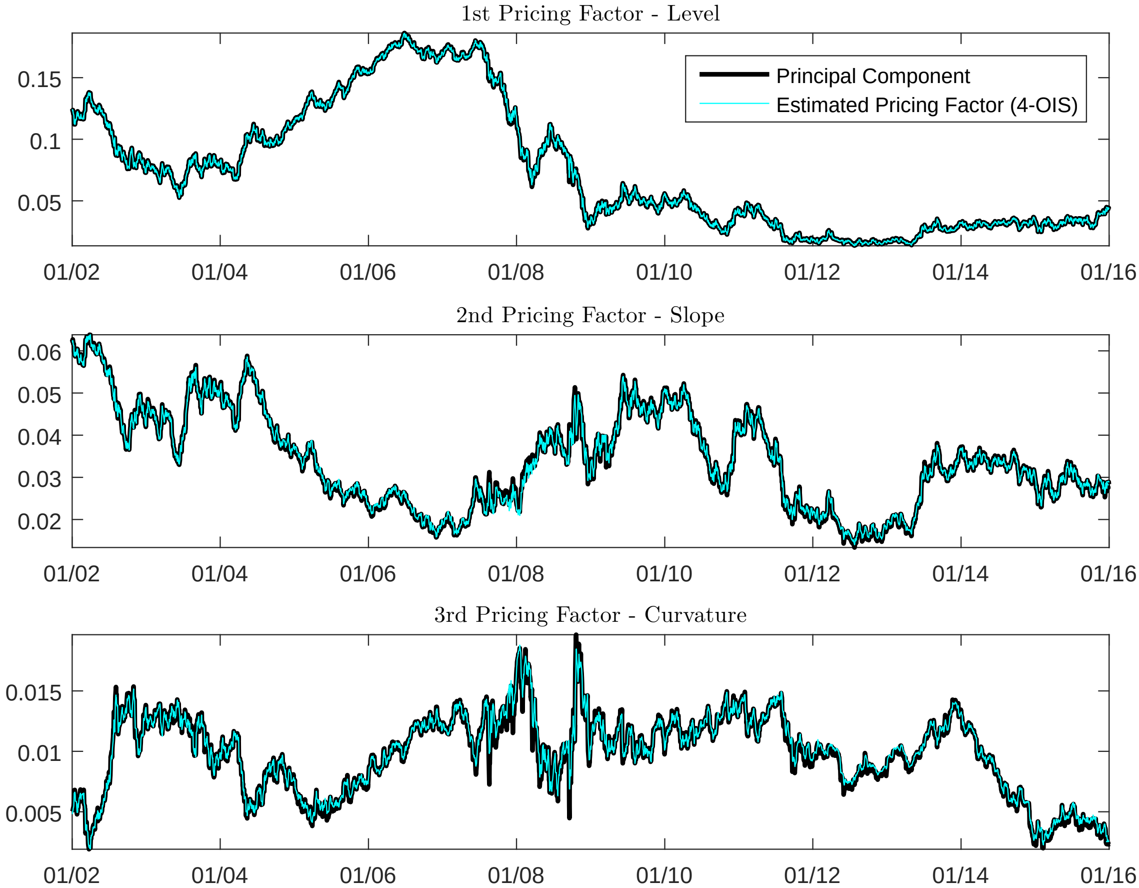


Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with three pricing factors using daily data from January 2002 to December 2015. All figures are in annualised percentage points.

This is unsurprising, as these four OIS rates are observed variables in the 4-OIS-augmented model. The 3-OIS-augmented model fits the 3, 6 and 12-month OIS rates well too. Moreover, the 3-OIS-augmented model captures the major qualitative patterns in the 2-year OIS rate — with the exception of the 2011-3 period — despite the 2-year OIS rate not being observed in the model. The 2-OIS-augmented model fits OIS rates least well. This is unsurprising, as it uses the fewest OIS rates as observables.

The fact the OIS-augmented models do not fit OIS rates as well as they fit bond yields — the quantitative value of OIS-RMSE (approximately 10 basis points) is almost double that of the bond yield-RMSE (approximately 5 basis points) — is neither worrying nor surprising. The GADTSM uses thirteen bond yields as inputs to estimate the cross-section of fitted yields in every time period, whereas only four OIS rates are used to fit the cross-section of OIS rates. Moreover, adding additional OIS rates is not warranted given that they are included to improve the fit of model-implied interest rate expectations and that longer-maturity OIS rates contain significant term premia (Lloyd, 2016a).

Figure 4: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model, implied by the Kalman filter.

6.1.3 Pricing Factors

Of additional interest for the OIS-augmented model is whether the inclusion of OIS rates affects the model's pricing factors $\mathbf{x}_t$. To investigate this, I compare the estimated principal components of the bond yields — used as pricing factors in the OLS/ML model — to the estimated model-implied pricing factors from Kalman filter estimation of the OIS-augmented models. Figure 4 plots the time series of the first three principal components, estimated from the panel of bond yields, against the estimated pricing factors from the 4-OIS-augmented model. For all three factors, the Kalman filter-implied pricing factors are nearly identical to the estimated principal components.[45] This implies that OIS rates do not include any additional information, over and above that in bond yields, of value in fitting the actual yields. This, again, is intuitive: OIS rates are included in the GADTSM to provide information useful for the identification of the risk-neutral yields, not the fitted yields.

[45]Table 10, in appendix F.1, demonstrates that the summary statistics of the estimated principal components and pricing factors are almost identical too.

6.1.4 Parameter Estimates

Recall, from section 3.2, that informational insufficiency in GADTSMs gives rise to finite sample bias. Persistent yields will have persistent pricing factors, resulting in estimates of the persistence parameters $\widehat{\mathbf{\Phi}}$ that are biased downwards. Following Bauer et al. (2012), I numerically assess the extent to which OIS-augmentation reduces finite sample bias by reporting the maximum eigenvalues of the estimated persistence parameters $\widehat{\mathbf{\Phi}}$. The higher the maximum eigenvalue, the more persistent the estimated process.

As a benchmark, the maximum absolute eigenvalue of $\widehat{\mathbf{\Phi}}$ for the unaugmented OLS/ML model is 0.9987. For both the survey and 4-OIS-augmented models, the maximum absolute eigenvalue of $\widehat{\mathbf{\Phi}}$ is 0.9993, indicating that, in comparison to the unaugmented model, augmentation with additional information does serve to mitigate finite sample bias.[46] This indicates that OIS-augmentation does help to resolve the informational insufficiency in GADTSMs, and its associated symptoms. However, to assess this more thoroughly, a comparison of model-implied interest rate expectations is necessary. A well-identified model should accurately reflect the evolution of interest rate expectations.

6.2 Model-Implied Interest Rate Expectations

The central focus of this paper is the identification and estimation of interest rate expectations within GADTSMs — the risk-neutral yields. Figure 5 plots the 2-year risk-neutral yields and term premia from the GADTSMs estimated between January 2002 and December 2015 in panels A and B, respectively.

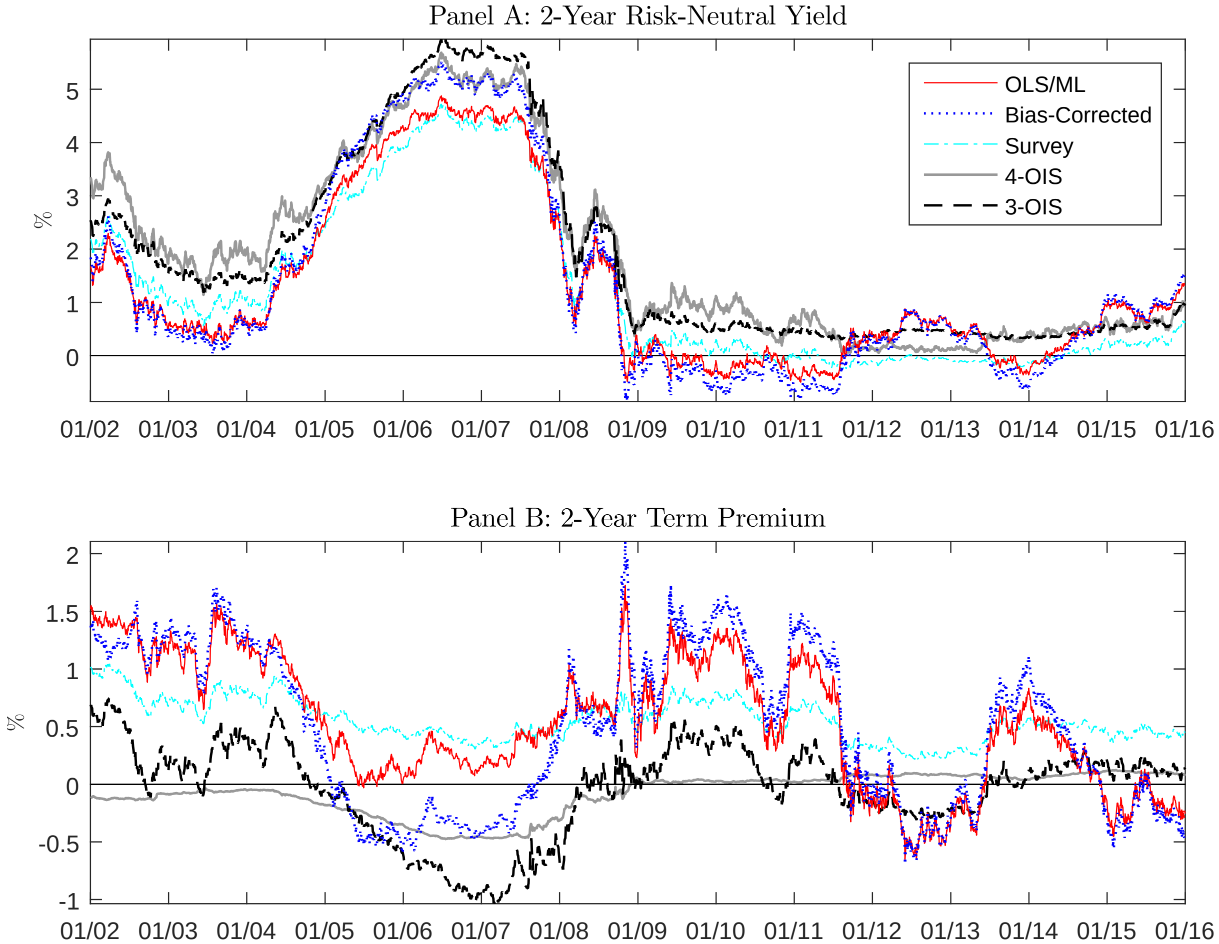
Panel A of figure 5 offers illustrative evidence of the effect of OIS-augmentation on the GADTSM estimates of expected future short-term interest rates. For the majority of the 2002-15 sample period, the five models exhibit similar qualitative patterns: rising to peaks and falling to troughs at similar times. However, there are also a number of notable differences between the series that help to illustrate the benefits of OIS-augmentation.

For the majority of the 2002-15 sample period, the OIS-augmented models generate risk-neutral yields that exceed those from the OLS/ML and bias-corrected models.

Moreover, marked differences exist in the evolution risk-neutral yield estimates from the models from late-2008 onwards. These differences have contradictory and counterfactual implications for the efficacy of monetary policy at this time. First, from late-2009 to late-2011, the risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying counterfactual expectations of negative interest rates. In contrast, unlike the other models, the risk-neutral yields implied by the OIS-augmented model obey the zero lower bound — i.e. estimated interest rate expectations never fall negative — despite the fact additional restrictions are not imposed to achieve this. This represents an important computational contribution in the light of recent computationally burdensome proposals for term structure modelling

[46] The corresponding statistic for the bias-corrected model, which performs bias-correction directly on the $\mathbf{\Phi}$, is 1.0000 (to four decimal places). However, as the 'true' persistence of the pricing factors is unknown, a comparison of the bias-corrected model with survey and OIS-augmented models is not possible on these grounds.

Figure 5: Estimated Yield Curve Decomposition



Note: In panels A and B, I plot the estimated risk-neutral yields and term premia from each of five GADTSMs, respectively. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. All figures are in annualised percentage points.

at the zero lower bound (see, for example Christensen and Rudebusch, 2013a,b).

Second, between late-2011 and 2013, the 2-year risk neutral yields from the OLS/ML and bias-corrected models rise to a peak during 2012, indicating an increase in expected future short-term interest rates over a 2 year horizon. In contrast, during the 2011-13 period, the 2-year risk-neutral yield estimates from the 3-OIS-augmented model remain broadly stable, while the corresponding estimates from the 4-OIS-augmented model fall slightly to a trough. Between late-2011 and 2013, the Federal Reserve engaged in forward guidance designed to influence investors' expectations of future short-term interest rates, signalling that interest rates would be kept at a low level for an extended period of time. For instance, on January 25, 2012, the Federal Open Market Committee (FOMC) stated that "economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through late-2014," while on September 13, 2012 this time-dependent guidance was altered to signal a more delayed rate rise: "low levels for the federal funds rate are likely to be warranted at least through mid-2015." These statements imply that policymakers' were seeking to defer investors' expectations of future rate rises between late-2011 and 2013. In this respect, the finding that expectations of future short-term interest rates over the coming 2 years rose during this period, as implied by the OLS/ML and bias-corrected models, appears counter-intuitive. These models predict that investors began to expect rate rises sooner rather than later. Subsequent quantitative analysis demonstrates that the OIS-augmented models provide superior estimates of interest rate expectations during this period. The OIS-augmented models imply that investors were expecting rate rises no sooner, and possibly slightly later, than they had in previous period.

6.2.1 Risk-Neutral Yields and Federal Funds Futures

I begin the quantitative evaluation of GADTSM-implied risk-neutral yields by comparing the model-implied interest rate expectations to federal funds futures rates. A federal funds futures contract pays out at maturity based on the average effective federal funds rate realised during the calendar month specified in the contract.

Federal funds futures rates have long been used as measures of investors' expectations of future short-term interest rates (Lao and Mirza, 2015) and many authors have assessed the quantitative accuracy of federal funds futures rates as predictors of future monetary policy. Gürkaynak et al. (2007b) conclude that, in comparison to a range of other financial market-based measures of interest rate expectations, federal funds futures rates provide the superior forecasts of future monetary policy out to 6 months. Lloyd (2016a) finds that, at a monthly frequency between 2002 and 2015, the average *ex post* realised excess returns on 1-11-month federal funds futures contracts are insignificantly different from zero. Motivated by this evidence, I compare estimated risk-neutral yields from each of the GADTSMs to corresponding-horizon 1-11-month federal funds futures contracts.

To facilitate this comparison, I first calculate 1, 2, ..., 11-month risk-neutral yields using the estimated model parameters from each GADTSM. I then calculate risk-neutral 1-month instantaneous *forward* yields using the estimated risk-neutral yields.[47] Like federal funds futures contracts, the risk-neutral 1-month forward rates settle based on outcomes during a 1 month period in the future. However, because of the settlement structure of federal funds futures contracts, I compare risk-neutral forward yields and federal funds futures rates on the final day of each calendar month.[48] I find that the risk-neutral forward yields from the 4-OIS-augmented

[47] To calculate the risk-neutral instantaneous forward rate $\widetilde{f}_{t_1,t_2}$, I use the following formula:

$$\widetilde{f}_{t_1,t_2} = \frac{1}{d_2 - d_1}\left(\frac{1+\widetilde{y}_2 d_2}{1+\widetilde{y}_1 d_1} - 1\right)$$

where $\widetilde{y}_1$ ($\widetilde{y}_2$) is the risk-neutral yield for the time period $(0, t_1)$ ($(0, t_2)$) and d_1 (d_2) is the time length between time 0 and time t_1 (t_2) in years.

[48] See Lloyd (2016a) for a detailed description of the settlement structure of federal funds futures contracts. The salient point here is that an n-month federal funds futures contract traded on day t_i of the calendar month t has the same settlement period as an n-month contract traded on a different day t_j in the same calendar month t. For this reason, the horizon of a federal funds futures contract and the risk-neutral forward yield only align on the final calendar day of each month.

Table 2: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the Risk-Neutral 1-Month Forward Yields *vis-à-vis* Corresponding Horizon Federal Funds Futures Rates

Sample: January 2002 to December 2015					
Maturity	OLS/ML	BC	Survey	4-OIS	3-OIS
$0 \to 1$ Months	0.2304	0.2270	0.2044	**0.1912**	0.2034
$1 \to 2$ Months	0.2258	0.2165	0.1840	**0.1359**	0.1474
$2 \to 3$ Months	0.2483	0.2333	0.1901	**0.0956**	0.1110
$3 \to 4$ Months	0.2940	0.2745	0.2240	**0.0916**	0.1076
$4 \to 5$ Months	0.3407	0.3178	0.2595	**0.1013**	0.1143
$5 \to 6$ Months	0.3905	0.3657	0.2972	**0.1135**	0.1284
$6 \to 7$ Months	0.4489	0.4226	0.3452	**0.1343**	0.1399
$7 \to 8$ Months	0.5091	0.4829	0.3944	**0.1470**	0.1583
$8 \to 9$ Months	0.5735	0.5491	0.4516	**0.1507**	0.1902
$9 \to 10$ Months	1.0458	1.0263	0.9323	**0.6881**	0.7439
$10 \to 11$ Months	2.4287	2.5077	2.2411	**2.0179**	2.0797

Note: RMSE of the risk-neutral 1-month forward yields from each of the five GADTSMs in comparison to corresponding-horizon federal funds futures rates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. The risk-neutral forward yields and the federal funds futures rates are compared on the final day of each calendar month. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

model most closely align with corresponding-horizon federal funds futures rate, implying that the 4-OIS-augmented model provides superior estimates of investors' expected future short-term interest rates.

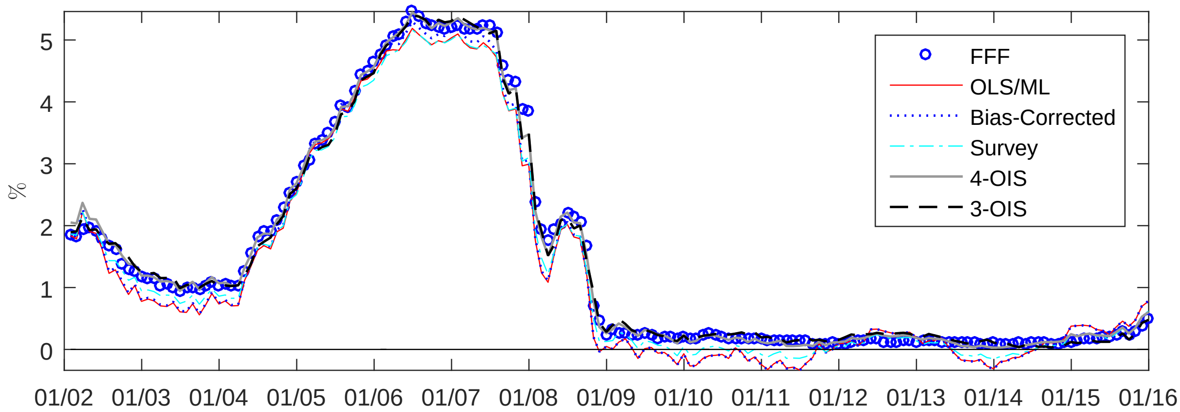
Table 2 provides formal evidence in support of this conclusion, presenting the RMSE of risk-neutral 1-month forward yields from different GADTSMs and corresponding-horizon federal funds futures rates. On a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates, as measured by federal funds futures rates, at every horizon. Moreover, of the OIS-augmented models, the 4-OIS-augmented model best fits federal funds future-implied interest rate expectations at each horizon. Even at extremely short horizons, the benefits of OIS-augmentation are large: the RMSE fit of the unaugmented OLS/ML and bias-corrected models at the $3 \to 4$ month horizon is over three times larger than that of the 4-OIS-augmented model.

Despite fitting federal funds futures-implied interest rate expectations worse than the OIS-augmented models, the survey-augmented model does perform better than the unaugmented OLS/ML and bias-corrected models in this regard. This supports the claim that, while survey-augmentation does help to reduce the informational insufficiency problem in GADTSMs, quarterly frequency survey expectations are not sufficient for the accurate identification of interest rate expectations at higher frequencies.

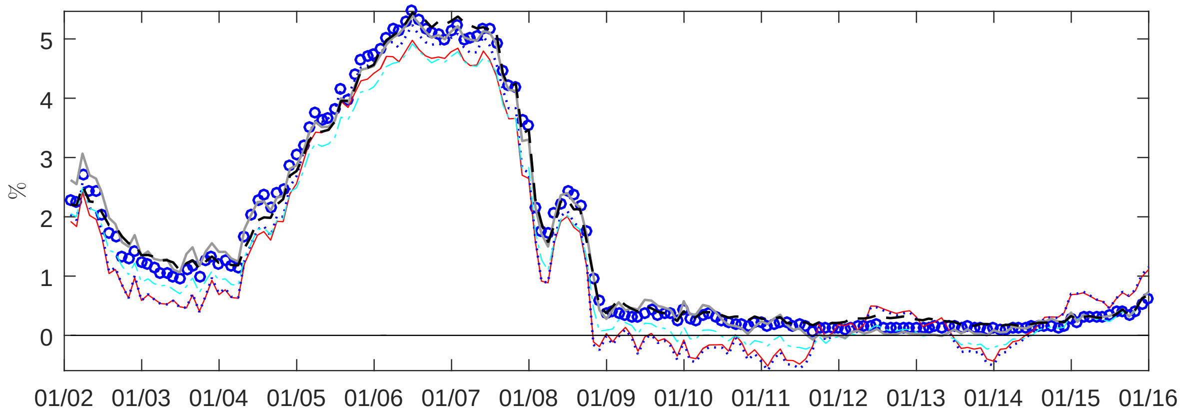
Figure 6 provides visual comparison of the risk-neutral 1-month forward yields and corresponding-horizon federal funds futures rates. Here, I plot the risk-neutral 1-month forward yields from

Figure 6: Estimated Risk-Neutral 1-Month Forward Yields and Comparable-Horizon Federal Funds Futures (FFF) Rates

3 → 4 Month Ahead Risk-Neutral Forward and Federal Funds Futures Rates



6 → 7 Month Ahead Risk-Neutral Forward and Federal Funds Futures Rates



Note: I plot estimated 3 to 4-month and 6 to 7-month ahead risk-neutral forward yields from each of five GADTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates, plotted on the final day of each calendar month. All figures are in annualised percentage points.

the unaugmented OLS/ML, bias-corrected, survey-augmented, 4-OIS-augmented and 3-OIS-augmented GADTSMs against corresponding horizon federal funds futures rates. The plot highlights the causes of the difference in fit highlighted by table 2. Three important observations follow.

First, between 2002 and 2005, the OLS/ML, bias-corrected and survey-augmented GADTSMs generate estimated risk-neutral forward yields that persistently fall below the corresponding horizon federal funds futures rate. In contrast, the estimated risk-neutral forward yields from the OIS-augmented models align more closely with federal funds futures rates during this period, especially at the 3 to 4-month horizon.

Second, between early-2006 and mid-2007, the risk-neutral forward yields from the OLS/ML and survey-augmented models fall substantially below the interest rate expectations implied by federal funds futures rates. The risk-neutral forward yields from the bias-corrected model also fall below the interest rate expectations implied by federal funds futures rates, albeit to a lesser extent. In contrast, the risk-neutral forward yields from the OIS-augmented models closely align with federal funds futures during this period.

Third, from 2009 onwards, the risk-neutral forward yields from the OLS/ML, bias-corrected and survey-augmented models differ greatly from the corresponding-horizon federal funds futures rates. Moreover, these models offer counterfactual predictions for the evolution of interest rate expectations during this period. In particular, from early-2009 to late-2011, the risk-neutral forward yields from the OLS/ML and bias-corrected models are persistently negative, implying that investors expected future short-term interest rates to fall negative. Moreover, from late-2011 to mid-2012, the risk-neutral forward yields from the OLS/ML and bias-corrected models rise to a peak. Not only is this contrary to the policy narrative at the time — policymakers were engaging in time-dependent forward guidance that sought to push back the date investors expected policy rates to lift-off from their zero lower bound — it is also counterfactual with respect to market-implied interest rate expectation. In contrast, the OIS-augmented models align closely with federal funds futures-implied expectations from December 2008 onwards.

Overall, the comparison of risk-neutral forward yields and federal funds rates supports the claim that OIS-augmentation of GADTSMs serves to improve the identification of interest rate expectations.

6.2.2 Risk-Neutral Yields and Short-Term Survey Expectations

As further evidence in support of this claim, I compare the model-implied interest rate expectations to short-term survey expectations. The preferred GADTSM(s) should also be able to reasonably capture the qualitative and quantitative evolution of comparable-horizon survey expectations. Against this metric, I find that the 4-OIS-augmented model provides superior overall estimates of short-term interest rate expectations, in comparison to all other models.

I compare the estimated 3, 6 and 12-month risk-neutral yields to corresponding-horizon survey expectations. I calculate approximate short-term interest rate expectations using data from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. I construct the weighted arithmetic average of the mean expectation of the 3-month T-Bill rate in the current quarter and the first, second, third and fourth quarters ahead. A complete description of how these expectations are approximated is presented in appendix B. To compare the estimated risk-neutral yields to these survey expectations, I calculate the RMSE of the risk-neutral yields *vis-à-vis* the corresponding horizon survey expectation on survey submission deadline dates.

Table 3 presents the results of this analysis. On a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates at each horizon. By this metric, the OLS/ML and bias-corrected models provide the most inferior estimates of future short-term interest rate at all three horizons.

At the 6 and 12-month horizons the 4-OIS-augmented model provides the superior fit of

Table 3: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 3, 6-Month and 1-Year Survey Expectations

	Sample: January 2002 to December 2015		
Model	RMSE vs. 3-Month Survey Expectation	RMSE vs. 6-Month Survey Expectation	RMSE vs. 1-Year Survey Expectation
OLS/ML	0.2252	0.2856	0.4748
Bias-Corrected	0.2243	0.2865	0.4864
Survey	0.1776	0.2023	0.3487
4-OIS	0.1756	**0.1509**	**0.1677**
3-OIS	**0.1719**	0.1650	0.2203

Note: RMSE of the risk-neutral yields from each of the five GADTSMs in comparison to approximated survey expectations. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. The construction of the survey expectation approximations is described in appendix B. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

survey expectations. Strikingly, at the 1-year horizon, the RMSE fit of the OLS/ML and bias-corrected models are almost three times that of the 4-OIS-augmented model. Although the 3-OIS-augmented model provides the lowest RMSE fit for the 3-month survey expectation, the RMSE fit of the 4-OIS-augmented model is only 0.37 basis points higher at this horizon. In contrast, at the 1-year horizon the RMSE fit of the 4-OIS-augmented model is 5.26 basis points lower than the 3-OIS-augmented model.

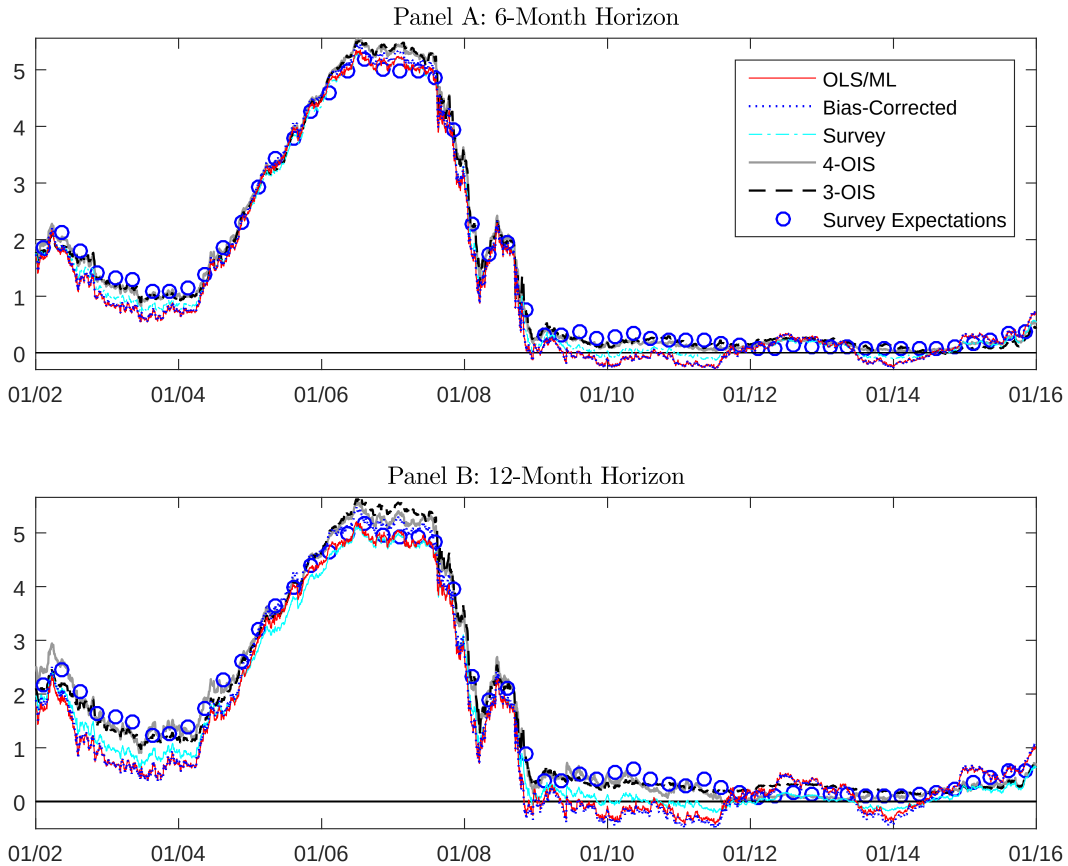
Surprisingly, the survey-augmented model, which uses the same survey expectations as an input to estimation, does not provide a superior fit for these expectations at any horizon *vis-à-vis* the OIS-augmented models. This supports the claim that quarterly frequency survey expectations are not sufficient for the accurate identification of higher frequency interest rate expectations within a GADTSM framework. Nevertheless, the RMSE fit of the survey-augmented model is superior to the fit of both the OLS/ML and bias-corrected models at all horizons, supporting the claim that augmentation of GADTSMs with additional information can aid the identification of risk-neutral yields.

Figure 7 provides a graphical illustration of the evolution of estimated risk-neutral yields and the approximated survey expectations at the 6 and 12-month horizons. Three observations follow.

First, between 2002 and 2005, the OLS/ML, bias-corrected and survey-augmented GADTSMs generated estimated risk-neutral yields that persistently fall below the corresponding horizon survey expectation. In contrast, the estimated risk-neutral yields from the OIS-augmented models closely co-move with the approximated survey expectations during this period. This corroborates with the comparison of risk-neutral forward yields and federal funds future-implied interest rate expectations in section 6.2.1.

Second, between early-2006 and mid-2007, the risk-neutral yields from the OIS-augmented

Figure 7: Short-Term Interest Rate Expectations



Note: I plot estimated 6-month and 1-year risk-neutral yields from each of five GADTSMs in panels A and B, respectively. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. I compare the estimated risk-neutral yields to approximated survey expectations of future short-term interest rates over the same horizon. The construction of the survey expectation approximations is described in appendix B. All figures are in annualised percentage points.

models exceed interest rate expectations implied by surveys. During this short period, the risk-neutral yields from the OLS/ML, bias-corrected and survey-augmented models more closely align with survey expectations. However, this difference appears is related to the information in survey expectations, rather than the risk-neutral yields implied by the GADTSMs. Recall that in figure 6, I find that the risk-neutral forward yields from the OIS-augmented models closely align with federal funds futures-implied interest rate expectations during this period.

Third, as in section 6.2.1, the GADTSMs offer markedly different estimates of interest rate expectations from December 2008 onwards. From early-2009 to late-2011, the 6 and 12-month risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying, counter-factually, that investors expected future short-term interest rates to fall negative. From late-2011 to mid-2012, the risk-neutral yields from the OLS/ML and bias-corrected models rise to peak. Again, this is both contrary to the policy narrative and the survey expectations at the time. In contrast, the OIS-augmented models — the 4-OIS-augmented model especially

— align closely with survey expectations from December 2008 onwards. The risk-neutral yield from the 4-OIS-augmented model closely tracks the survey expectations during 2012 especially.

Overall, the results further support the claim that the OIS-augmentation of GADTSMs serves to improve the identification of interest rate expectations. At short-term horizons, OIS-augmented models — the 4-OIS-augmented model especially — provide superior estimates of investors' expectations of future short-term interest rates.

6.2.3 Long-Term Interest Rate Expectations

The expectational horizons considered in the previous sub-section are short-term. However, GADTSMs provide estimates of risk-neutral yields for the whole term structure, at horizons further into the future. This is the an important motive for using GADTSMs to estimate interest rate expectations, instead of market-based financial measures; market-based financial measures seldom provide accurate measures of investors' interest rate expectations at horizons in excess of 2 years (see Lloyd, 2016a).

Within a GADTSM, the 10-year risk-neutral yield on date t provides an estimate for the expected average short-term interest rate for the 10 year period following date t. In general, survey data on these longer-term interest rate expectations are not readily available, making it difficult to systematically test the long-horizon interest rate expectations attained from GADTSMs. However, in recent years, the New York Federal Reserve's *Survey of Primary Dealers* have asked respondents an increasing number of questions regarding their longer-term interest rate expectations.[49] Specifically, since October 2013, respondents have been asked to: "provide your estimate of the longer run target federal funds rate and your expectation for the average federal funds rate over the next 10 years".[50] The latter of these requests corresponds to the information contained within the 10-year risk-neutral yields attained from the GADTSMs: the expectation of the average of the short-term interest rate over a 10 year horizon.

To quantitatively assess the longer-term interest rate expectations implied by the GADTSMs, I compare the estimated 10-year risk-neutral yield to the median "expectations for the average federal funds rate over the next 10 years" of survey respondents on the survey deadline dates. Again, I calculate the RMSE fit of the risk-neutral yields *vis-à-vis* the survey expectations.

Table 4 presents the results from this analysis. However, because the sample of long-term survey expectations is relative short, the RMSE analysis in table 4 is not as rigorous as in tables 2 and 3. Nevertheless, the results support the primary conclusion of this paper: that the OIS-augmented models provide unambiguously superior estimates of future short-term interest rate expectations over a 10-year horizon. The RMSE fit of the OLS/ML model is over three times higher than the RMSE fit of the 4-OIS-augmented model.[51]

[49] The questions and results of these surveys are publicly available from: `www.newyorkfed.org/markets/primarydealer_survey_questions.html`.

[50] In the surveys, the question preceding this was: "provide your estimate of the most likely outcome (i.e., the mode) for the target federal funds rate or range at the end of each half-year period".

[51] Although, for this sample, the RMSE fit of the 3-OIS-augmented model is lowest, the 19.99 basis point difference is small in comparison to other models over the same 10-year horizon. With so few observations, a difference this small is not sufficient to statistically distinguish the 3 and 4-OIS-augmented models.

Table 4: GADTSM-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 10-Year Survey Expectation

Model	Sample: October 2013 to December 2015 RMSE vs. 10-Year Expectation, Survey of Primary Dealers
OLS/ML	1.6073
Bias-Corrected	1.9478
Survey	2.4433
4-OIS	0.5375
3-OIS	**0.3376**

Note: RMSE of the risk-neutral yields from each of the five GADTSMs in comparison to the 10-year survey expectation. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015. The survey expectation is from the Survey of Primary Dealers, New York Federal Reserve. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

6.2.4 Daily Changes in GADTSM-Implied Interest Rate Expectations

In this paper, I have emphasised the benefits that OIS-augmentation offers for the identification and estimation of interest rate expectations from GADTSMs at a *daily frequency*. As OIS rates are available at a daily frequency, they offer potentially large benefits when estimating the daily frequency evolution of interest rate expectations. To illustrate these benefits, I directly analyse the daily changes in GADTSM-implied risk-neutral yields.

The analysis of daily changes in interest rate expectations is an integral part of historical monetary policy analysis. Most recently, a number of authors have used daily changes in interest rate expectations and term premia to assess the relative efficacy of various interest rate channels of unconventional monetary policies (see Lloyd, 2016b, and the references within). For the OIS-augmented GADTSM to be well-suited to historical policy analysis of this sort, it is important that the risk-neutral yields provide an accurate depiction of the daily frequency evolution of interest rate expectations. Specifically, for the GADTSM-implied interest rate expectations to reasonably reflect the expectations of investors over a comparable horizon at a daily frequency, they should, at the very least, qualitatively match numerical measures of investors' interest rate expectations. To test this, I compare the sign of daily changes in 3, 6 12 and 24-month risk-neutral yields to the sign of daily changes in comparable-maturity OIS rates.[52] For the GADTSM to reasonably reflect investors' expectations, the sign of the daily change in the risk-neutral yield should correspond to the sign of the daily change in the comparable horizon OIS rate. I record the proportion of positive and negative daily changes in OIS rates that are

[52]I use the sign of daily changes in OIS rates because their horizon corresponds exactly to that of the nominal government bond yields I use. Although it may seem somewhat tautological to compare an OIS-augmented GADTSM to OIS rates, previous results indicate that this may not be the case. In table 3, the survey-augmented model does not provide the best fit for the survey-expectations which are used as an input to its estimation.

Table 5: Proportion of Daily Changes in OIS Rates Matched in Sign by the Daily Changes in In-Sample GADTSM Risk-Neutral Yields

Sample: January 2002 to December 2015				
	Maturity			
Model	3-Months	6-Months	1-Year	2-Years
Proportion of Positive Daily Changes Matched				
OLS/ML	84.33%	92.72%	94.44%	93.41%
Bias-Corrected	84.33%	92.72%	94.44%	93.12%
Survey	82.84%	91.75%	93.46%	96.85%
4-OIS	**84.34%**	**94.17%**	**95.75%**	**97.99%**
3-OIS	82.84%	90.78%	92.81%	95.70%
Proportion of Negative Daily Changes Matched				
OLS/ML	87.21%	91.85%	94.63%	91.86%
Bias-Corrected	87.21%	91.85%	94.03%	91.86%
Survey	87.79%	92.70%	95.82%	98.43%
4-OIS	**89.53%**	**93.99%**	**96.42%**	**98.69%**
3-OIS	86.05%	90.56%	93.73%	98.16%

Note: Proportion of daily changes in 3, 6, 12 and 24-month OIS rates (in excess of one standard deviation of their daily change in absolute value) that are matched in sign by the daily changed in the corresponding maturity GADTSM risk-neutral yield. All proportions are expressed as a percentage to two decimal places. Four GADTSMs are compared: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); and (iv) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2015.

matched in sign by the change in the corresponding-horizon risk-neutral yields. To focus on significant changes in OIS rates, I omit days on which OIS rates changed by less, in absolute value, than one standard deviation of the daily changes in the OIS rate over the whole sample. The results are presented in table 5.

The results indicate that the 4-OIS-augmented model unambiguously provides the best qualitative match for the sign of daily changes in 3, 6, 12 and 24-month OIS rates for both positive and negative changes. For example, the 4-OIS-augmented model is the only to match over 95% of positive daily changes in 1-year OIS rates. Moreover, at the 2-year horizon, the sign of daily changes in the risk-neutral yield from the 4-OIS-augmented model matches 97.99% (98.69%) of positive (negative) OIS rate, almost 5% (7%) more than the OLS/ML and bias-corrected models match.

Overall, the results in table 5 are consistent with the claim that the 4-OIS-augmented model best reflects the daily frequency evolution of short-term interest rate expectations.

6.3 Explaining the Benefits of OIS-Augmentation

The preceding discussion highlights that, for the OIS-augmented models provide estimates of expected future short-term interest rates that are superior to the OLS/ML, bias-corrected and survey-augmented models. Moreover, within the class of OIS-augmented models considered, the 4-OIS-augmented model, on balance, outperforms the 2 and 3-OIS-augmented models.

Figure 7 highlights that there are differences between the risk-neutral yields from OIS-augmented models and the OLS/ML and bias-corrected models for the whole 2002-15 sample period. In particular, since 2009, the risk-neutral yields from the models offer distinctly different qualitative and quantitative predictions for estimated interest rate expectations.

To understand the economic reasons behind these differences, it is informative to draw on the canonical description of the first three principal components of bond yields as the level, slope and curvature of the yield curve respectively together with the model-implied loadings on these factors.[53] Figure 8 plots these loadings for both the calculation of fitted yields $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$ (top row) and the risk-neutral yields $\tilde{B}_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$ (bottom row) for the 3-month to 10-year maturities. To refine discussion, loadings are presented for the two most inferior models — OLS/ML and bias-corrected — and the most superior — 4-OIS-augmented models. These loadings illustrate the extent to which the fitted and risk-neutral yields react to a one unit shock to a pricing factor at a given maturity, keeping all other pricing factors constant.

Unsurprisingly, the fitted yield loadings from the OLS/ML, bias-corrected and 4-OIS-augmented models are almost identical for all maturities. This reinforces the similarities between fitted yields for different GADTSMs. The benefits of OIS-augmentation arise from the separate identification of interest rate expectations from term premia, rather than the fitting of actual yields.

However, the risk-neutral yield loadings from the models differ at all horizons. These differences help to explain why the 4-OIS-augmented model is superior as a measure of interest rate expectations, and provide economic reasons for the qualitative and quantitative differences in risk-neutral yields from 2009 onwards.

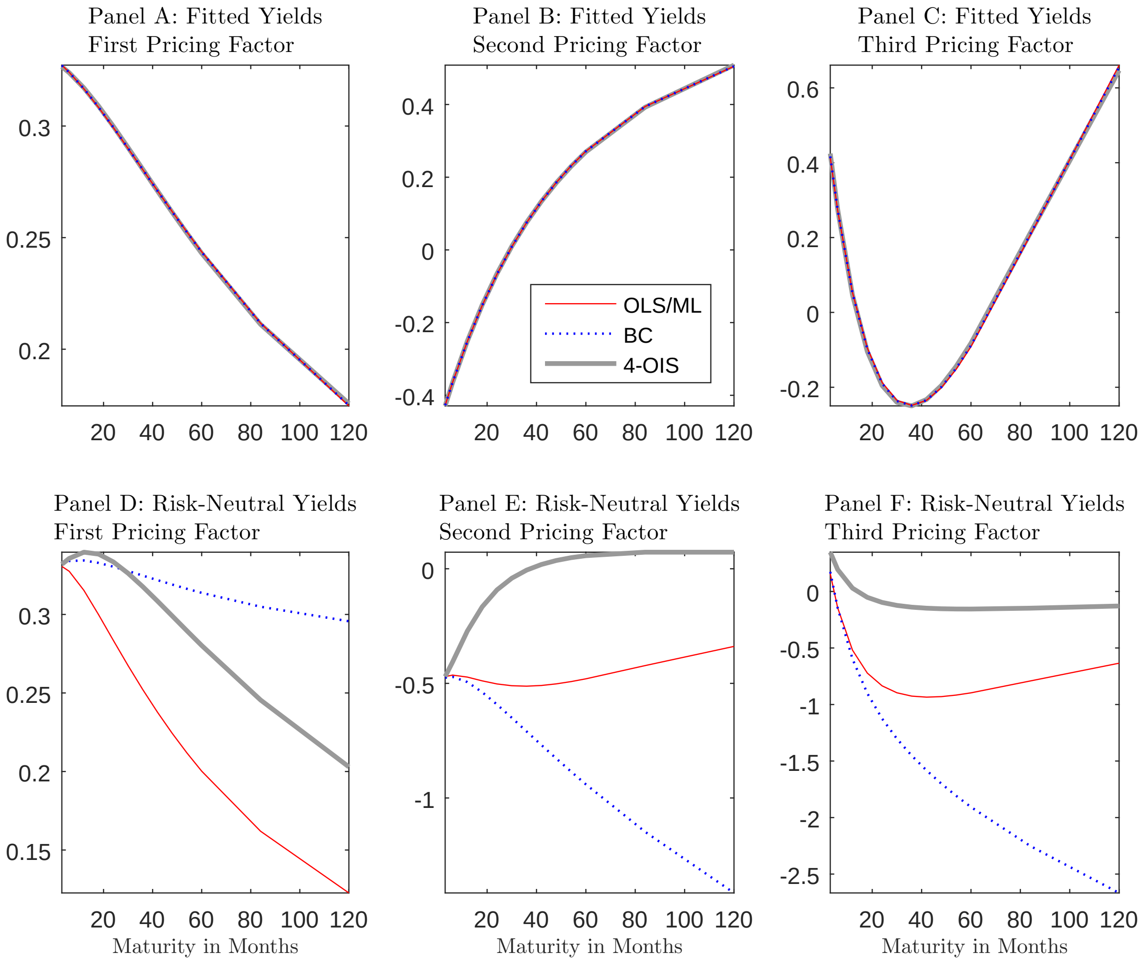
From late-2011 to 2013, the risk-neutral yields from the OLS/ML and bias-corrected models rise to a peak during 2012, falling back below zero for a short period from late-2013 to early-2014. The risk-neutral yields from the OIS-augmented models do not exhibit such a significant spike during 2012. This period was characterised by two notable phenomena. First, the target federal funds rate was at its zero lower bound. Having been set at this level in December 2008, the FOMC were signalling that it would be kept at this rate into the future through forward guidance. Second, the Eurozone sovereign debt crisis elevated Eurozone government bond yields. This was associated with a reduction in yields on, comparatively safe, longer-term US government bonds.[54] During the 2011-13 period therefore, the US yield curve was characterised by a reduction in its *slope*, with no change in its *level*.

Panel E of figure 8 illustrates that an decrease in the slope of the yield curve places upward pressure on estimated risk-neutral yields at all maturities in the OLS/ML, bias-corrected and 4-OIS-augmented models. That is, decreases in the yield curve slope, for a given level and

[53]Importantly, because the estimated pricing factors from the three-factor OIS-augmented models almost exactly correspond with the estimated principal components (see figure 4), this economic intuition is valid for these models.

[54]Formally, the sovereign debt crisis began in 2008, as longer-term interest rates in affected countries began to increase. Nevertheless, longer-term Eurozone interest rates peaked in 2011/12 (Corsetti, Kuester, Meier, and Müller, 2013, 2014).

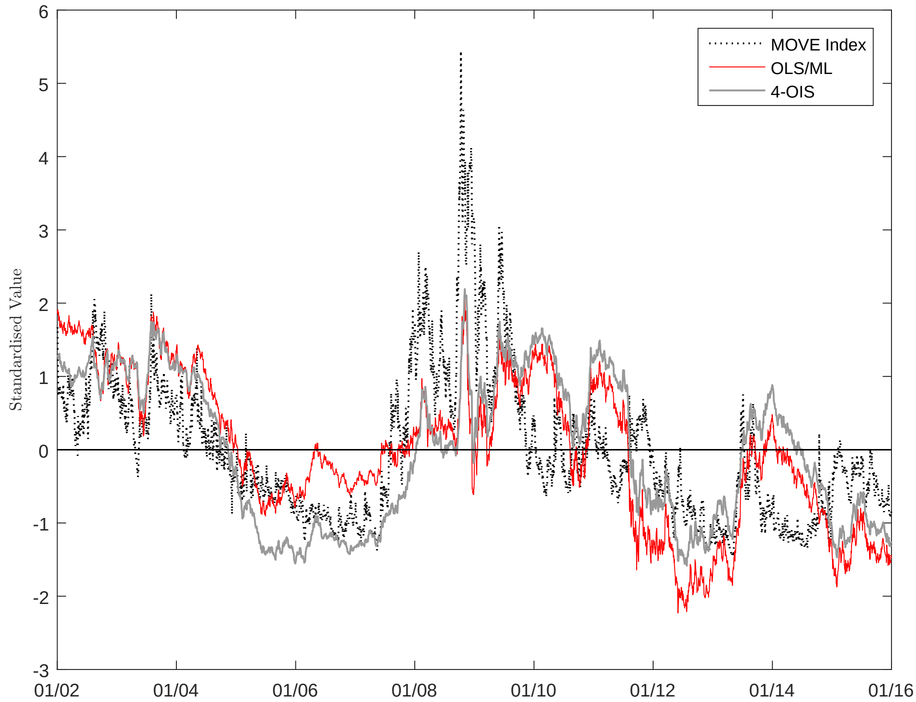
Figure 8: Fitted and Risk-Neutral Yield Factor Loadings



Note: I plot the estimated yield loadings B_n for the fitted and risk-neutral yields, for each the three pricing factors, from the OLS/ML, bias corrected and 4-OIS-augmented models estimated with three factors from January 2002 to December 2015. These coefficients can be interpreted as the *ceteris paribus* response of the fitted and risk-neutral bond yields at a given maturity to a contemporaneous shock to the respective pricing factor. The horizontal axis labels denote the maturity, in months. The three models are denoted by: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); and (iii) the 4-OIS-augmented model (4-OIS).

curvature, tend to be associated with diminished term premia. However, the risk-neutral yields from the 4-OIS-augmented model react less strongly to a change in the yield curve slope. This is most visible at longer-horizons, where the difference in risk-neutral yield loadings for the 4-OIS-augmented and bias-corrected models widen considerably. This helps to explain why the risk-neutral yields from the 4-OIS-augmented do not rise to a peak in mid-2012, while those from the OLS/ML and bias-correct models do. The 4-OIS-augmented model does not exhibit the same peak, because the inclusion of OIS rates in the estimation alters the loading on that pricing factor. This constellation of factor loadings helps to attain risk-neutral yields that align more closely with survey and market-implied expectations of future short-term interest rates during the zero lower bound period with the 4-OIS-augmented model.

Figure 9: Standardised 10-Year Term Premia and Merrill Lynch Option Volatility Estimate (MOVE) Index)



Note: I plot the standardised one-month Merrill Lynch Option Volatility Estimate (MOVE) index against the standardarised estimates of term premia from (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML), and (ii) the 4-OIS-augmented model (4-OIS). The models are estimated with three factors from January 2002 to December 2015.

6.4 Model-Implied Term Premia

Alongside estimates of expectations of future short-term interest rates, GADTSMs provide estimates for the daily evolution of term premia. Although there is no direct metric against which to compare estimated term premia, Adrian et al. (2013) compare a standardised version of their estimated daily ten-year term premium to a standardised version of the one-month Merrill Lynch Option Volatility Estimate (MOVE) index. This latter index is a measure of implied volatility from option contracts written on US Treasuring bonds.[55] Thus, variation in MOVE reflects changes in the risk of holding US Treasuries.

Like Adrian et al. (2013), in figure 9 I plot the standardised estimates of the 10-year term premium from the OLS/ML and 4-OIS-augmented models against the standardised one-month MOVE index. The time series exhibit a strong positive correlation. The correlation coefficient between the standardised 10-year term premium estimate from the 4-OIS-augmented model and the standardised MOVE index is 0.60, marginally higher than the corresponding statistic of 0.58 for the OLS/ML model. This demonstrates that the estimated term premia from the 4-OIS-augmented model do reflect the risk of holding Treasury bonds.

[55]Formally, the series used here (and in Adrian et al. (2013)) is defined as a yield curve weighted index of the normalised implied volatility on 1-month Treasury options. It is the weighted average of volatilities on 2, 5, 10 and 30-year bond yields.

7 Conclusion

Financial market participants and policymakers closely monitor the evolution of interest rate expectations using a wide range of financial market instruments. In this paper, I investigate the informational content in OIS rates for this purpose and document how OIS rates can be used to improve estimates of nominal interest rate expectations attained from the term structure of nominal government bond yields.

I report that OIS rates provide an accurate measure of investors' future interest rate expectations. Drawing on Lloyd (2016a), I find that 3-24-month OIS rates contain small and statistically insignificant average excess returns. I then present an OIS-augmented GADTSM for estimating the daily frequency evolution of interest rate expectations that explicitly accounts for the geometric payoff structure in OIS contracts. Most existing arbitrage-free GADTSMs use information on the term structure of nominal government bond yields to identify both expectations of the future path of short-term interest rates and term premia. Numerous authors have drawn attention to an informational insufficiency in the estimation of these models. Kim and Orphanides (2012) propose survey-augmented GADTSMs as a solution to this problem. However, survey forecasts of future interest rates are only available at a low frequency (quarterly or monthly, at best) and reflect investors' expectations of future short-term interest rates for a window of time in the future (e.g. one, two, three or four quarters ahead). OIS rates, on the other hand, are available at a daily frequency and have a horizon that align exactly with those of the zero-coupon nominal government bonds used in the estimation of GADTSMs. The term structure of OIS rates can therefore be readily augmented to a GADTSM for nominal government bond yields. I show that augmenting the GADTSM with OIS rates provides additional information, specifically related to future short-term interest rate expectations, that can help better identify the evolution of these expectations. Using OIS rates in an arbitrage-free GADTSM enables the estimation of future short-term interest rate expectations for the whole term structure — from 3 months to 10 years — in a manner that is consistent along the cross-section. Estimates of interest rate expectations from OIS-augmented GADTSMs are superior to those from existing GADTSMs. In particular, short and long-horizon in-sample OIS-augmented risk-neutral yields accurately depict quantitative patterns in federal funds futures rates and survey expectations. These time series also match qualitative daily patterns exhibited by financial market instruments. This implies that OIS-augmented GADTSMs are well suited for daily frequency policy analysis. Thus, OIS-augmented GADTSMs provide reliable and policy-relevant estimates of interest rate expectations along the whole term structure.

Additionally, this paper highlights the need to test the performance of GADTSMs in a range of dimensions — for example accuracy of fitted yields, risk-neutral yields and term premia — before applying them to analysis of monetary policy. This paper proposes a battery of such tests for future research.

The contribution of this paper extends beyond the GADTSM-literature. For example, the OIS-augmented GADTSM can be applied to better understand the transmission of monetary policy. In related work, I use estimates of interest rate expectations from the OIS-augmented model to assess the effect of US unconventional monetary policy — large-scale asset purchases and forward guidance — on longer-term interest rates (Lloyd, 2016b). I compare the implications from the OIS-augmented model to the bias-corrected and survey-augmented models, and demonstrate that the use of more accurate estimates of interest rate expectations (from the OIS-augmented model) can have dramatic implications for resulting conclusions, overturning existing results. In Lloyd (2016b), I find that US longer-term interest rates did fall on US large-scale asset purchase and forward guidance announcement days, with falls in interest rate expectations, not term premia, explaining the majority of this.

To conclude, OIS rates accurately reflect investors' future short-term interest rate expectations, providing useful information for improved identification of interest rate expectations in arbitrage-free GADTSMs.

Appendix

A Data Sources

The data in section 2 was from the following sources:

Table 6: Data Sources - Average Excess Return Regressions - Section 2

Data Series	Description and Source
US OIS Rates	*Bloomberg*, with codes: `USSOA` 1 month; `USSOB` 2 months; ... ; `USSOK` 11 months; `USSO1` 1 year; `USSO1C` 15 months; `USSO1F` 18 months; `USSO1I` 21 months; `USSO2` 2 years; and `USSO3` 3 years.
US Effective Federal Funds Rate	Federal Reserve Statistical Release H.15: `www.federalreserve.gov/releases/h15/data.htm`.
US Survey Forecasts of the 3-Month T-Bill Rate	*Survey of Professional Forecasters*, available from the Federal Reserve Bank of Philadelphia website here: `www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files/TBILL/`. The survey deadline dates are available here: `www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/spf-release-dates.txt`.

The data use for the GADTSM was from the following sources:

Table 7: Data Sources - GADTSM - Section 6

Data Series	Description and Source
US Treasury Bill Rates	Federal Reserve Statistical Release H.15: `www.federalreserve.gov/releases/h15/data.htm`.
US Zero-Coupon Treasury Yields	Gürkaynak, Sack, and Wright (2007a), the updated data from which is available here: `www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`.
US Federal Funds Futures Rates	*Bloomberg* with codes: `FF2`, which settles based on the 1st full calendar month in the future; `FF3`, which settles based on the 2nd full calendar month in the future; ...; `FF12`, which settles based on the 11th calendar month in the future. And `www.quandl.com/data/OFDP/FUTURE_FFX`, where `X` should be replaced by the horizon of the contract in months.
US OIS Rates	*Bloomberg*. See table 6 for detailed source information.
US Survey Forecasts of the 3-Month T-Bill Rate	See table 6 above.
Merrill Lynch Option Volatility Estimate (MOVE)	*Bloomberg*, with the code `MOVE Index`. This is a yield curve weighted index of the normalised implied volatility on one month Treasury options. It is a weighted average of volatilities on the current US 2, 5, 10 and 30 year government notes.

B Approximated Survey Forecasts

B.1 Geometric Weighting Scheme

In this section, I present the formal details underlying the survey forecast approximation I construct for figure 1 using data from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. The survey is published every quarter and reports the mean forecasters' expectations of the average 3-month T-Bill rate over a specified time period in: the current quarter $\bar{i}^{3m,sur}_{t|t}$; and the first $\bar{i}^{3m,sur}_{t+1|t}$, second $\bar{i}^{3m,sur}_{t+2|t}$, third $\bar{i}^{3m,sur}_{t+3|t}$ and fourth $\bar{i}^{3m,sur}_{t+4|t}$ quarters subsequent to the current one, where t denotes the current quarter. All quantities are plotted on the survey submission deadline dates, which lie approximately halfway through each quarter.

To construct a geometric approximation for the average expectation of the 3-month T-Bill rate over the 3-months following the deadline date, I construct an equally weighted geometric average of the expectation of the 3-month rate for the current and the subsequent quarter. An equal weighting is made possible because the survey deadline date lies approximately halfway through the 'current' quarter. I use a geometric average to replicate the floating leg of an OIS contract, which equation (1) shows to have a geometric structure. This facilitates direct comparison of the survey and OIS-implied expectations.

To achieve this, I first use the survey expectation for the average 3-month T-Bill rate over the current quarter $\bar{i}^{3m,sur}_{t|t}$ and the realised average of the 3-month T-Bill rate for the first-half of the quarter prior to the deadline date $\bar{i}^{3m,real}_{t}$ to approximate the survey expectations for the average 3-month T-Bill rate for the remainder of the current quarter, denoted $\bar{i}^{3m,sur}_{t_+|t}$. This is calculated from the following expression:

$$\bar{i}^{3m,sur}_{t|t} = \frac{1}{2}\bar{i}^{3m,real}_{t} + \frac{1}{2}\bar{i}^{3m,sur}_{t_+|t}$$

Then, to calculate the average survey expectation of the 3-month T-Bill rate over the three months from the deadline date t, $i^{3m,sur}_{t|t}$, I use the approximation:

$$i^{3m,sur}_{t|t} = \left[\left(1 + \frac{\bar{i}^{3m,sur}_{t_+|t}}{100}\right)^{\frac{1}{2}} \times \left(1 + \frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{1}{2}} - 1\right] \times 100$$

where $i^{3m,sur}_{t|t}$, $\bar{i}^{3m,sur}_{t_+|t}$ and $\bar{i}^{3m,sur}_{t+1|t}$ are all reported in percentage points.

The average expectation of the 3-month T-Bill rate over the six-months following the deadline date t, $i^{6m,sur}_{t|t}$, is approximated using a similar geometric *weighted* average procedure: the expectation of the 3-month rate for the remainder of the current quarter and second quarter ahead are both given weights of 1/4; and the first quarter ahead expectation has weight 1/2.

Mathematically, this is written as:

$$i^{6m,sur}_{t|t} = \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{1}{2}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{1}{4}} - 1\right] \times 100$$

The average expectation of the 3-month T-Bill rate over the year following the submission date t, $i^{1y,sur}_{t|t}$, is approximated by a geometric weighted average of the remainder of the current quarter and first, second, third and fourth quarter ahead expectations, of the form:

$$\begin{aligned} i^{1y,sur}_{t|t} = \Bigg[&\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{8}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{1}{4}} \\ &\times \left(1+\frac{\bar{i}^{3m,sur}_{t+3|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+4|t}}{100}\right)^{\frac{1}{8}} - 1\Bigg] \times 100 \end{aligned}$$

B.2 Arithmetic Weighting Scheme

In this section, I present the formal details underlying the survey forecast approximation I construct for risk-neutral yield comparison in section 6.2.2 using the same data from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. All RMSE calculations are made by comparing risk-neutral yields to the approximated survey expectations on the survey submission deadline dates.

To construct an arithmetic approximation for the average expectation of the 3-month T-Bill rate over the 3-months following the deadline date, I construct a weighted arithmetic average of the expectation of the 3-month rate for the current and subsequent quarters. The weighting is made possible because the survey deadline date lies approximately halfway through the 'current' quarter. This facilitates direct comparison of the survey and risk-neutral yield-implied expectations. Mathematically, this average survey expectation of the 3-month T-Bill rate over the three months from the deadline date t, $\tilde{i}^{3m,sur}_{t|t}$, is approximated by:

$$\tilde{i}^{3m,sur}_{t|t} = \frac{1}{2}\bar{i}^{3m,sur}_{t+|t} + \frac{1}{2}\bar{i}^{3m,sur}_{t+1|t}$$

The arithmetic average expectation of the 3-month T-Bill rate over the 6-months and 12-months following the deadline date, $\tilde{i}^{6m,sur}_{t|t}$ and $\tilde{i}^{1y,sur}_{t|t}$, are respectively approximated by:

$$\begin{aligned} \tilde{i}^{6m,sur}_{t|t} &= \frac{1}{4}\bar{i}^{3m,sur}_{t+|t} + \frac{1}{2}\bar{i}^{3m,sur}_{t+1|t} + \frac{1}{4}\bar{i}^{3m,sur}_{t+2|t} \\ \tilde{i}^{1y,sur}_{t|t} &= \frac{1}{8}\bar{i}^{3m,sur}_{t+|t} + \frac{1}{4}\bar{i}^{3m,sur}_{t+1|t} + \frac{1}{4}\bar{i}^{3m,sur}_{t+2|t} + \frac{1}{4}\bar{i}^{3m,sur}_{t+3|t} + \frac{1}{8}\bar{i}^{3m,sur}_{t+4|t} \end{aligned}$$

C Baseline Gaussian Affine Dynamic Term Structure Model

C.1 Bond Pricing Using the Risk-Adjusted Probability Measure $\mathbb{Q}$

To guarantee the existence of a risk-adjusted probability measure $\mathbb{Q}$, under which the bonds are priced, no-arbitrage is imposed (Harrison and Kreps, 1979). The risk-adjusted probability measure $\mathbb{Q}$ is defined such that the price V_t of any asset that does not pay any dividends at time $t+1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)V_{t+1}]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the $\mathbb{Q}$ probability measure. Thus, with no-arbitrage, the price of an n-day zero-coupon bond must satisfy the following relation:

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \tag{20}$$

Using this, it is possible to show that the nominal bond price is an exponentially affine function of the pricing factors:

$$P_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t\right) \tag{21}$$

such that the corresponding continuously compounded yield $y_{t,n}$ is affine in the pricing factors:

$$y_{t,n} = -\frac{1}{n}\ln\left(P_{t,n}\right) = A_n + B_n \mathbf{x}_t \tag{22}$$

where $A_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$.

To attain recursive expressions for $\mathcal{A}_n$ and $\mathcal{B}_n$:

$$\begin{aligned}
\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t &= \ln P_{t,n} \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp\left(-i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1}\right)] \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\delta_0 - \boldsymbol{\delta}_1'\mathbf{x}_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right]\right)\right] \\
&= -\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t\right] + \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right] \\
&= -\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t\right] + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' \\
&= \left\{-\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}}\right\} + \left\{-\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}}\right\}\mathbf{x}_t
\end{aligned}$$

using (21) in the third line, (6) and (9) in the fourth line, and using the property of the log-normal distribution in conjunction with the fact that $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}|\mathbf{x}_t \sim \mathcal{N}\left(\mathbf{0}_K, \mathbf{I}_K\right)$ to write the expression $\ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right]$ as $\frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}'$ in the sixth line

By the method of undetermined coefficients, the recursive definitions for the scalar $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and the $1 \times K$ vector $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$ follow from the final line:

$$\begin{aligned}\mathcal{A}_n &= -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} && (23)\\ \mathcal{B}_n &= -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} && (24)\end{aligned}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}'_K$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

C.2 Bond Pricing Using the Pricing Kernel and the Actual Probability Measure $\mathbb{P}$

Under the actual $\mathbb{P}$ probability measure, the bond price is given by equation (11):

$$P_{t,n} = \mathbb{E}_t\left[M_{t+1}P_{t+1,n-1}\right]$$

where this expectation is taken under the $\mathbb{P}$ measure.

Using this, it is also possible to show that the nominal bond price is an exponentially affine function of the pricing factors, as in equation (21). To attain recursive expressions for $\mathcal{A}_n$ and $\mathcal{B}_n$:

$$\begin{aligned}\mathcal{A}_n + \mathcal{B}_n\mathbf{x}_t &= \ln P_{t,n}\\ &= \ln \mathbb{E}_t\left[M_{t+1}P_{t+1,n-1}\right]\\ &= \ln \mathbb{E}_t\left[\exp\left(-i_t - \frac{1}{2}\boldsymbol{\lambda}'_t\boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t\boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1}\right)\right]\\ &= \ln \mathbb{E}_t\left[\exp\left(\begin{array}{c} -\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t - \frac{1}{2}\left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t\right)'\left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t\right)\\ -\left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t\right)'\boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right)\end{array}\right)\right]\\ &= -\delta_0 - \boldsymbol{\delta}'_1\mathbf{x}_t + \mathcal{A}_{n-1} - \mathcal{B}_{n-1}\boldsymbol{\Sigma}\left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t\right)\\ &\quad + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}\left(\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t\right)\end{aligned}$$

using (7) and (13) in the third line, and (5), (6) and (8) in the fourth line.

By the method of undetermined coefficients, the recursive definitions for the scalar $\mathcal{A}_n$ and the $1 \times K$ vector $\mathcal{B}_n$ follow from the final line:

$$\begin{aligned}\mathcal{A}_n &= -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}'_{n-1} + \mathcal{B}_{n-1}\left(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0\right) && (25)\\ \mathcal{B}_n &= -\boldsymbol{\delta}'_1 + \mathcal{B}_{n-1}\left(\boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\right) && (26)\end{aligned}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}'_K$, where $\mathbf{0}'_K$ is a $K \times 1$ vector of zeros.

Comparing (23) and (24) with (25) and (26) yields the relationship between $\mathbb{P}$ and $\mathbb{Q}$ parameters:

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \qquad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1.$$

C.3 Risk-Neutral Yields

The risk-neutral yield on an n-day bond reflects the yield that would prevail if investors were risk-neutral. That is, the risk-neutral yield corresponds to that which would arise under the actual probability measure $\mathbb{P}$.

The risk-neutral bond price $\widetilde{P}_{t,n}$ is of the form:

$$\widetilde{P}_{t,n} = \mathbb{E}_t \left[\exp(-i_t) \widetilde{P}_{t+1,n-1} \right] \tag{27}$$

and can be shown to be an exponentially affine function of the pricing factors:

$$\widetilde{P}_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t\right) \tag{28}$$

where $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$. Thus, the risk-neutral yield is affine in the pricing factors:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t \tag{29}$$

where $\widetilde{A}_n = -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n = -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.

To attain the recursive expressions for $\widetilde{A}_n$ and $\widetilde{B}_n$, note that from equation (20):

$$\begin{aligned}
\tilde{y}_{t,n} &= -\frac{1}{n} \ln \mathbb{E}_t \left[\exp\left\{-i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1}\right\}\right] \\
\widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t &= -\frac{1}{n} \ln \mathbb{E}_t \left[\exp\left\{-\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right]\right\}\right] \\
&= -\frac{1}{n}\left[\left\{-\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}\right\} + \left\{-\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}\right\}\mathbf{x}_t\right]
\end{aligned}$$

using (27) and (28) in the first line, and (6) and (5) in the second line. The expectation is taken under the actual probability measure $\mathbb{P}$. By the method of undetermined coefficients, it follows that:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t$$

where $\widetilde{A}_n = -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n = -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.

D Overnight Indexed Swap Augmentation

To calculate the loadings for the OIS observation equation, first note that the *annualised* floating leg of a j-day OIS contract, with starting day $t+1$, is given by:

$$i^{flt}_{t,t+j} = \left(\left[\prod_{i=1}^{j} (1 + \gamma_{t+i} i_{t+i}) \right] - 1 \right) \times \frac{T_{yr}}{j}$$

where γ_{t+i} is an accrual factor, which is set to $1/T_{yr}$ for all time periods, and $T_{yr} = 252$ is the number of trading days in a year.[56] i_t is the one-period short-term floating interest rate (equation (6)) used as the reference rate for the swap — the effective federal funds futures rate.

Therefore, under the expectations hypothesis, the OIS rate $i^{ois}_{t,t+n}$ will be:

$$i^{ois}_{t,t+j} = \left(\mathbb{E}_t \left[\prod_{i=1}^{j} (1 + \gamma_{t+i} i_{t+i}) \right] - 1 \right) \times \frac{T_{yr}}{j} \tag{30}$$

Since this refers to the expected path for the nominal short-term interest rate, and not just the risk-neutral path, I set $\boldsymbol{\Sigma} = \mathbf{0}_K$ in what follows.

For a one period OIS contract, $j = 1$:

$$\begin{aligned} i^{ois}_{t,t+1} &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_{t+1} \right) \right] \times T_{yr} \\ &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \left[\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t \right] \right) \right] \times T_{yr} \\ &= \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) \end{aligned}$$

For a two period OIS contract, $j = 2$, the expectations hypothesis requires that:

$$\begin{aligned} i^{ois}_{t,t+2} &= \left(\mathbb{E}_t \left[\left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) \right) \left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_{t+1} \right) \right) \right] - 1 \right) \times (T_{yr}/2) \\ &= \left(\left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) \right) \left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \left[\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t \right] \right) \right) - 1 \right) \times (T_{yr}/2) \end{aligned}$$

The expectations operator is removed in line 2 of the above, since everything in this final line is known to an individual at time t. Rearranging this:

$$1 + \frac{2}{T_{yr}} i^{ois}_{t,t+2} = \left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) \right) \left(1 + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi}^2 \mathbf{x}_t \right) \right)$$

[56] For the term structure model, the accrual and annualisation factors use the convention that there are 252 business trading days in a year, as opposed to the market quoting convention of 360 days used in section 2. Given that daily yield data is only available on 252 days per year, I adopt this convention to ensure that the *horizon* for each OIS rate corresponds to their actual maturity date and that of a corresponding maturity zero-coupon bond. This convention is also adopted for daily frequency term structure estimation by, amongst others, Bauer and Rudebusch (2014).

and, by using $\ln(1+x) \approx x$ for small $|x|$, then:

$$\begin{aligned}
\frac{2}{T_{yr}} i^{ois}_{t,t+2} &\approx \gamma\left(2\delta_0 + 2\boldsymbol{\delta}_1'\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^2\mathbf{x}_t\right) \\
i^{ois}_{t,t+2} &\approx \frac{1}{2}\left(2\delta_0 + 2\boldsymbol{\delta}_1'\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^2\mathbf{x}_t\right)
\end{aligned}$$

For a three period OIS contract, $j = 3$, the same steps as above yield the following expression:

$$i^{ois}_{t,t+3} \approx \frac{1}{3}\left(3\delta_0 + 3\boldsymbol{\delta}_1'\boldsymbol{\mu} + 2\boldsymbol{\delta}_1'\boldsymbol{\Phi}\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^2\boldsymbol{\mu} + \boldsymbol{\delta}_1'\boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^2\mathbf{x}_t + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^3\mathbf{x}_t\right)$$

The continued iteration can be summarised by the following expressions:

$$i^{ois}_{t,t+j} = A^{ois}_j + B^{ois}_j\mathbf{x}_t \tag{31}$$

where $A^{ois}_j \equiv \frac{1}{j}\mathcal{A}^{ois}_j\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}^{ois}_{j-1}, \mathcal{B}^{ois}_{j-1}\right)$ and $B^{ois}_j = \dfrac{1}{j}\mathcal{B}^{ois}_j\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}^{ois}_{j-1}\right)$ are recursively defined as:

$$\begin{aligned}
\mathcal{A}^{ois}_j &= \delta_0 + \boldsymbol{\delta}_1'\boldsymbol{\mu} + \mathcal{A}^{ois}_{j-1} + \mathcal{B}^{ois}_{j-1}\boldsymbol{\mu} \\
\mathcal{B}^{ois}_j &= \boldsymbol{\delta}_1'\boldsymbol{\Phi} + \mathcal{B}^{ois}_{j-1}\boldsymbol{\Phi}
\end{aligned}$$

where $\mathcal{A}^{ois}_0 = 0$ and $\mathcal{B}^{ois}_0 = \mathbf{0}_K'$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

E Estimation Procedure

To identify the unaugmented model described in section 3, I use the normalisation scheme proposed by Joslin et al. (2011). The Joslin et al. (2011) normalisation fosters faster convergence to the global optimum of the model's likelihood function than other identification schemes for two reasons. First, this normalisation allows for the (near) separation of the $\mathbb{P}$ and $\mathbb{Q}$ probability measure likelihood functions, the product of which comprises the overall model likelihood function. Moreover, the Joslin et al. (2011) normalisation reduces the dimensionality of the parameter space. In the baseline, unaugmented model, the parameters governing bond pricing are:

$$\boldsymbol{\Theta} = \left\{\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}\right\}$$

The Joslin et al. (2011) normalisation scheme uniquely maps these parameters to a smaller set:

$$\left\{i_{\infty}^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\Sigma}\right\}$$

where: (i) $i_{\infty}^{\mathbb{Q}}$ is the risk-neutral expectation of the long-run short-term nominal interest rate; (ii) $\boldsymbol{\lambda}^{\mathbb{Q}}$ is a $K \times 1$ of the eigenvalues of $\boldsymbol{\Phi}^{\mathbb{Q}}$; and (iii) $\boldsymbol{\Sigma}$ is a lower triangular matrix with positive diagonal entries.

For all the term structure models estimated in this paper I use the convention that there are 252 business days in a year, corresponding to the number of days for which bond yield data exists per year.[57] To ensure that the *horizon* for each bond corresponds to their actual maturity date, I set the daily horizons of the 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 month yields to $n =$ 63, 126, 252, 378, 504, 630, 756, 882, 1008, 1134, 1260, 1764, 2520 days ($N = 13$). Similarly, the 3, 6, 12 and 24 month OIS rates have the following horizon in days: $j = 63, 126, 252, 504$ ($J = 4$).

E.1 OLS/ML Estimation of the Baseline, Unaugmented GADTSM

Assuming that K portfolios of bonds are priced without error, then the Joslin et al. (2011) normalisation permits the complete separation of the $\mathbb{P}$ and $\mathbb{Q}$ likelihood functions. In this paper, as in many others, I use the first K principal components of the observed bond yields as the set of K portfolios that are priced perfectly (see, for example: Joslin et al., 2011). Defining these portfolios $\mathcal{P}_t \equiv W y_t = W y_t^{obs} \equiv \mathcal{P}_t^{obs}$, where W is the principal component weighting matrix and y_t^{obs} is the vector of observed yields, then Joslin et al. (2011) show that the likelihood function for the unaugmented model laid out in section 3.1 is:

$$\mathcal{L}\left(y_t^{obs}|y_{t-1}^{obs}; \boldsymbol{\Theta}\right) = \mathcal{L}\left(y_t^{obs}|\mathcal{P}_t; \boldsymbol{\lambda}^{\mathbb{Q}}, i_{\infty}^{\mathbb{Q}}, \boldsymbol{\Sigma}, \sigma_u\right) \times \mathcal{L}\left(\mathcal{P}_t|\mathcal{P}_{t-1}; \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}\right)$$

where σ_u is the standard deviation of the measurement error of the N observed yields.

This normalisation admits a two-stage estimation process. First, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ are

[57] This convention is also adopted for daily frequency term structure estimation by, amongst others, Bauer and Rudebusch (2014).

directly estimable by running OLS on the VAR in equation (5), where $\mathbf{x}_t \equiv \mathcal{P}_t$. Moreover, this provides initial values for the maximum likelihood estimation of the lower triangular elements of the matrix $\mathbf{\Sigma}$. Second, taking $\left\{\widehat{\boldsymbol{\mu}}, \widehat{\mathbf{\Phi}}\right\}$ as given, the parameters $\left\{i_{\infty}^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \mathbf{\Sigma}, \sigma_u\right\}$ can be estimated by maximum likelihood.

E.2 Bias-Corrected Estimation

To estimate the bias-corrected decomposition, I rely entirely on the methodology of Bauer et al. (2012, Section 4). The MATLAB code for this is available here: `faculty.chicagobooth.edu/jing.wu/research/zip/brw_table1.zip`.

E.3 Survey-Augmentation

To augment the model with survey expectations of future interest rates, I employ Kalman filter-based maximum likelihood estimation. This estimation methodology, using survey expectations, draws most directly on Guimarães (2014).

Like Guimarães (2014), I use survey expectations from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. I use forecasts for the 3 month T-Bill 1, 2, 3 and 4 quarters ahead, available at a quarterly frequency. I augment the model with the survey expectations on the survey submission deadline day.[58]

The survey-augmented Kalman filter has a similar form to the OIS-augmented setup presented in section 3. The transition equation of the Kalman filter is equation (5), the vector autoregression for the latent pricing factors under the actual $\mathbb{P}$ probability measure.

On days when the survey forecasts are *not* observed, the observation equation is given by equation (17). As with the OIS-augmented model, I maintain a homoskedastic form for the yield measurement error.

On days when the S survey forecasts, $s = s_1, s_2, ..., s_S$, are observed, the observation equation is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{sur} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{sur} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{sur} \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \mathbf{\Sigma}_Y \\ \mathbf{\Sigma}_S \end{bmatrix} \mathbf{u}_t^{sur} \tag{32}$$

where, in addition to the definitions of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{\Sigma}_Y$ above, $\mathbf{i}_t^{sur} = \left[i_{t,s_1}^{sur}, ..., i_{t,s_S}^{sur}\right]'$, $\mathbf{A}^{sur} = \left[A_{s_1}^{sur}, ..., A_{s_S}^{sur}\right]'$, $\mathbf{B}^{sur} = \left[B_{s_1}^{sur\prime}, ..., B_{s_S}^{sur\prime}\right]'$ and $\mathbf{u}_t^{sur} \sim \mathcal{N}\left(\mathbf{0}_{N+S}, \mathbf{I}_{N+S}\right)$ denotes the yield and survey measurement error, where $\mathbf{0}_{N+S}$ is an $N+S$-vector zeros and $\mathbf{I}_{N+S}$ is an $(N+S)\times(N+S)$ identity matrix. As with the yield measurement error, I impose a homoskedastic form for the survey measurement error, such that $\mathbf{\Sigma}_S$ is a $S \times S$ diagonal matrix with common diagonal element σ_s, the standard deviation of the survey measurement error. Appendix C of Guimarães (2014) presents the functional forms for A_s^{sur} and B_s^{sur}, which account for the arithmetic nature of survey expectations.

As with the OIS-augmented model, I estimate the survey-augmented model by using the OLS/ML parameter estimates as initial values for the Kalman filter.

[58]For survey submission dates that are not business days, I augment the model with survey data on the preceding business day.

E.4 OIS-Augmentation of the GADTSM and Kalman Filtering

When the Kalman filter is used, the assumption that K portfolios of yields are observed without error is no longer made. Instead, all yields (and portfolios thereof) can be observed with error. Consequently, the exact separation of the likelihood function described in section E.1 is no longer applicable. However, the parameter estimates attained from OLS/ML estimation of the unaugmented model do provide initial values for the Kalman filter-based optimisation routine.[59] Doing so, ensures that computational time is reasonably fast.

[59] Guimarães (2014) follows similar steps to estimate a survey-augmented GADTSM using the Joslin et al. (2011) normalisation scheme.

F Term Structure Results

In this section, I present additional results from my estimation of the GADTSMs.

F.1 Additional Results for the Three-Factor Specification

Model-Implied Fitted Yields Table 8 presents the root mean square error (RMSE) for the fitted yields from each of the term structure models. The RMSE is presented for each maturity, and the average over all maturities, for the sample period: January 2002 to December 2015. The fit, compared to the actual yield, is broadly similar across all six models. Specifically, the average RMSE for each of the models at all thirteen maturities differ by no more then 0.3 basis points. Thus, all models fit actual bond yields similarly well.

Table 8: GADTSM Fit: Root Mean Square Error (RMSE) of the Fitted Yields *vis-à-vis* the Actual Yields

Sample: January 2002 to December 2015						
Maturity	OLS/ML	BC	Survey	4-OIS	3-OIS	2-OIS
3-Months	0.0995	0.0999	0.1053	0.1093	0.1032	0.1017
6-Months	0.0531	0.0529	0.0544	0.0544	0.0560	0.0539
1-Year	0.0712	0.0714	0.0782	0.0735	0.0745	0.0773
18-Months	0.0579	0.0576	0.0610	0.0619	0.0599	0.0609
2-Years	0.0415	0.0409	0.0416	0.0450	0.0421	0.0417
30-Months	0.0246	0.0240	0.0247	0.0276	0.0254	0.0245
3-Years	0.0161	0.0159	0.0193	0.0186	0.0181	0.0183
42-Months	0.0226	0.0226	0.0266	0.0242	0.0241	0.0250
4-Years	0.0321	0.0319	0.0352	0.0337	0.0327	0.0332
54-Months	0.0390	0.0386	0.0410	0.0410	0.0389	0.0388
5-Years	0.0424	0.0418	0.0434	0.0452	0.0420	0.0411
7-Years	0.0280	0.0271	0.0282	0.0355	0.0320	0.0275
10-Years	0.0662	0.0653	0.0646	0.0594	0.0612	0.0572
Average	0.0457	0.0454	0.0480	0.0484	0.0469	0.0463

Note: RMSE of the fitted yields from each of the six three-factor GADTSMs, computed by comparing the model-implied fitted yield to the actual yield on each day. All figures are expressed in annualised percentage points. The six GADTSMs are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); (v) the 3-OIS-augmented model (3-OIS); and (vi) the 2-OIS-augmented model (2-OIS).

Model-Implied Fitted OIS Rates Table 9 presents the RMSE for the fitted OIS rates from each of the OIS-augmented term structure models. The RMSE is presented for each maturity for the sample period: January 2002 to December 2015. The results demonstrate that the 4-OIS-augmented model provides superior estimates of all four OIS rates, reaffirming the conclusion discussed with respect to figure 3. Intuitively, the 3-OIS-augmented model provides the second best fit of all four OIS rates, while the 2-OIS-augmented model provides the most inferior fit. Furthermore, the 3-OIS-augmented model is only marginally inferior to the 4-OIS-augmented model at the 3, 6 and 12-month maturities, for which the former uses these OIS rates as observables. However, at the 2-year horizon, the 4-OIS-augmented model provides a substantial improvement in fit *vis-à-vis* the 3-OIS-augmented model.

Table 9: GADTSM Fit: Root Mean Square Error (RMSE) of Fitted OIS Rates *vis-à-vis* the Actual OIS Rates

Sample: January 2002 to December 2015			
Maturity	4-OIS	3-OIS	2-OIS
3-Months	0.1383	0.1534	0.1591
6-Months	0.0930	0.1200	0.1524
1-Year	0.0942	0.1429	0.4049
2-Year	0.1048	0.3563	1.0597

Note: RMSE of the fitted OIS rates from each of the three OIS-augmented GADTSMs, computed by comparing the model-implied fitted OIS rate to the actual OIS rate on each day. All figures are expressed in annualised percentage points. The three GADTSMs are: (i) the 4-OIS-augmented model (4-OIS); (ii) the 3-OIS-augmented model (3-OIS); and (iii) the 2-OIS-augmented model (2-OIS).

Estimated Pricing Factors and Principal Components Table 10 presents summary statistics for the estimated principal components of the actual bond yields and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models for the sample period: January 2002 to December 2015. The results demonstrate that the principal components and estimated pricing factors evolve similarly, implying that OIS rates do not include any additional information, over and above that in bond yields, of value to the fitting of actual yields. In particular, the summary statistics of the estimated principal components and the estimated pricing factors from the 4-OIS-augmented models are similar.

Moreover, table 10 further demonstrates that the inclusion of different maturities of OIS rate in the term structure model does not appreciably alter estimates of actual bond yields. The summary statistics of the estimated pricing factors from the 4, 3 and 2-OIS-augmented models are all similar. Augmentation of GADTSMs with OIS rates only influences estimated parameters under the actual probability measure $\mathbb{P}$ and thus risk-neutral yields.

F.2 Four-Factor Specification

In the light of evidence by Cochrane and Piazzesi (2005, 2008) and Duffee (2011), who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields, I do estimate a four-factor specification of the OLS/ML, bias-corrected, survey-augmented and

Table 10: Estimated Principal Components and Estimated Pricing Factors: Summary Statistics

Summary Statistics	1st Factor	2nd Factor	3rd Factor
	Estimated Principal Components		
Mean	0.0746	0.0341	0.0098
Variance	0.0026	0.0001	0.0000
Skewness	0.6785	0.3591	-0.3081
Kurtosis	2.1498	2.3273	2.5543
	4-OIS: Estimated Pricing Factors		
Mean	0.0746	0.0342	0.0099
Variance	0.0026	0.0001	0.0000
Skewness	0.6728	0.3797	-0.3176
Kurtosis	2.1401	2.3400	2.5643
	3-OIS: Estimated Pricing Factors		
Mean	0.0746	0.0341	0.0098
Variance	0.0026	0.0001	0.0000
Skewness	0.6778	0.3595	-0.3193
Kurtosis	2.1490	2.3295	2.5520
	2-OIS: Estimated Pricing Factors		
Mean	0.0746	0.0341	0.0098
Variance	0.0026	0.0001	0.0000
Skewness	0.6769	0.3559	-0.3392
Kurtosis	2.1477	2.3175	2.5260

Note: Summary statistics for the first three estimated principal components from actual yield data and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models. All statistics are reported to four decimal places.

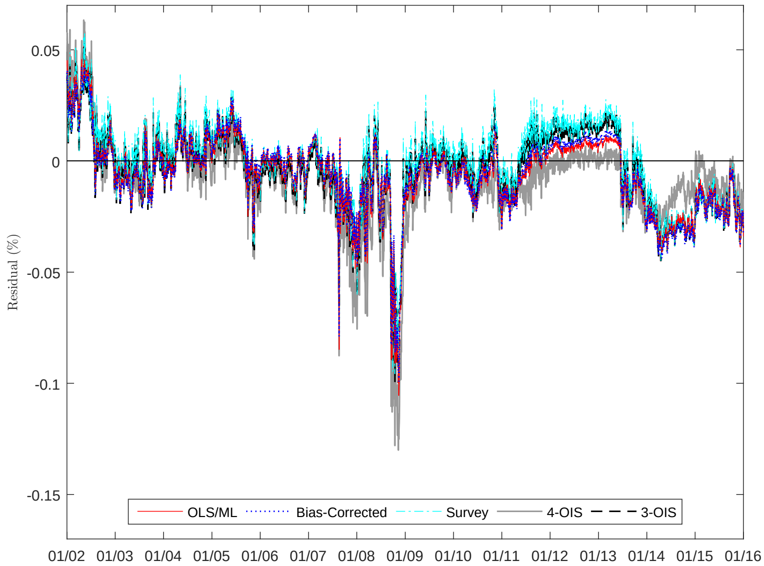
4-OIS-augmented GADTSMs. Although the four-factor model better fits actual bond yields for the 2002-15 sample, I do not present these results in the main body of the paper because the economic meaning of the pricing factors in a three-factor model is well understood (i.e., level, slope and curvature), while the economic interpretation the fourth factor is less well understood.

I estimate the four-factor model using the same underlying daily data as the three-factor model presented in the main body of the paper.

Fitted Yields Figure 10 demonstrates that the fitted yields from the four-factor GADTSMs do not differ markedly from one another. Here I plot the residual of the 2-year fitted yield from the four-factor model.

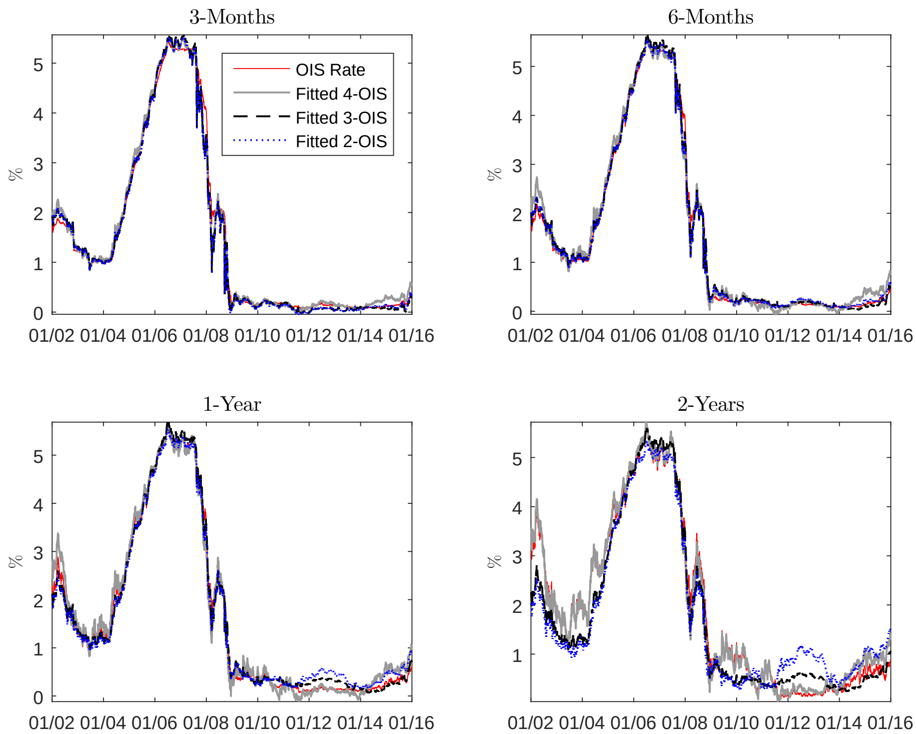
Fitted OIS Rates As with the three-factor models, the four-factor OIS-augmented models accurately fit OIS rates. Figure 11 demonstrates this, plotting the actual and fitted 3, 6, 12 and 24-month OIS rates.

Figure 10: Residual of the 2-Year Fitted Yield from Four-Factor GADTSMs



Note: Residuals of the 2-year fitted yield from five monthly frequency GADTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey; (iv) the 4-OIS-augmented model; and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2014. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

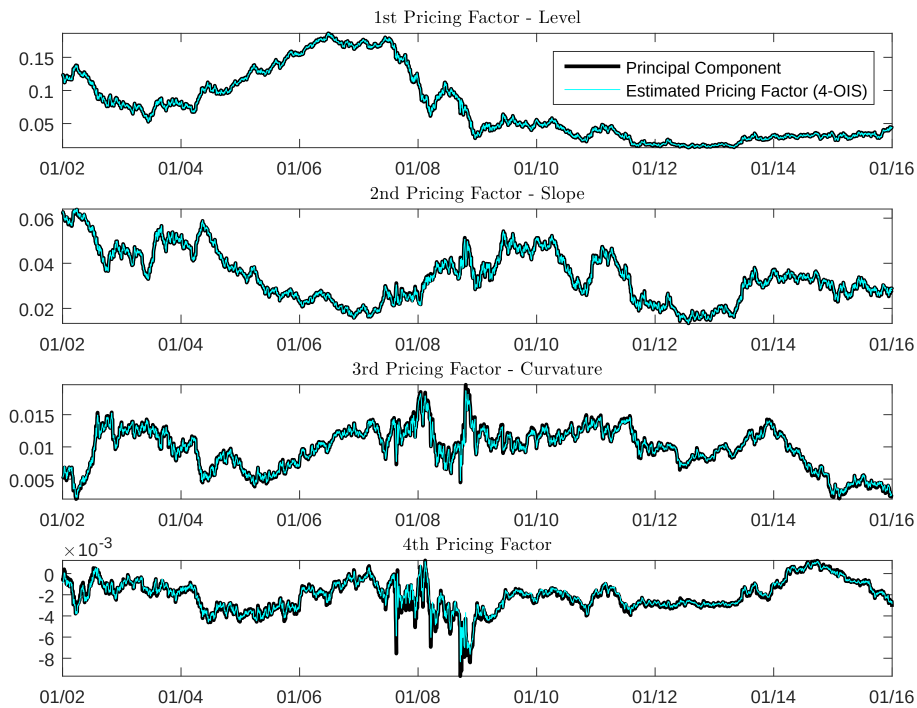
Figure 11: Fitted OIS Rates from the Four-Factor OIS-Augmented Models



Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with four pricing factors using daily data from January 2002 to December 2015. All figures are in annualised percentage points.

Pricing Factors Figure 12 plots the first four estimated principal components of the daily frequency bond yield data, and the four estimated pricing factors from the 4-OIS-augmented model. As with the three-factor model, the plot demonstrates that the inclusion of OIS rates in the estimation of GADTSMs does not significantly influence the bond pricing factors, as the quantities closely co-move.

Figure 12: First Four Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the Four-Factor 4-OIS-Augmented Model



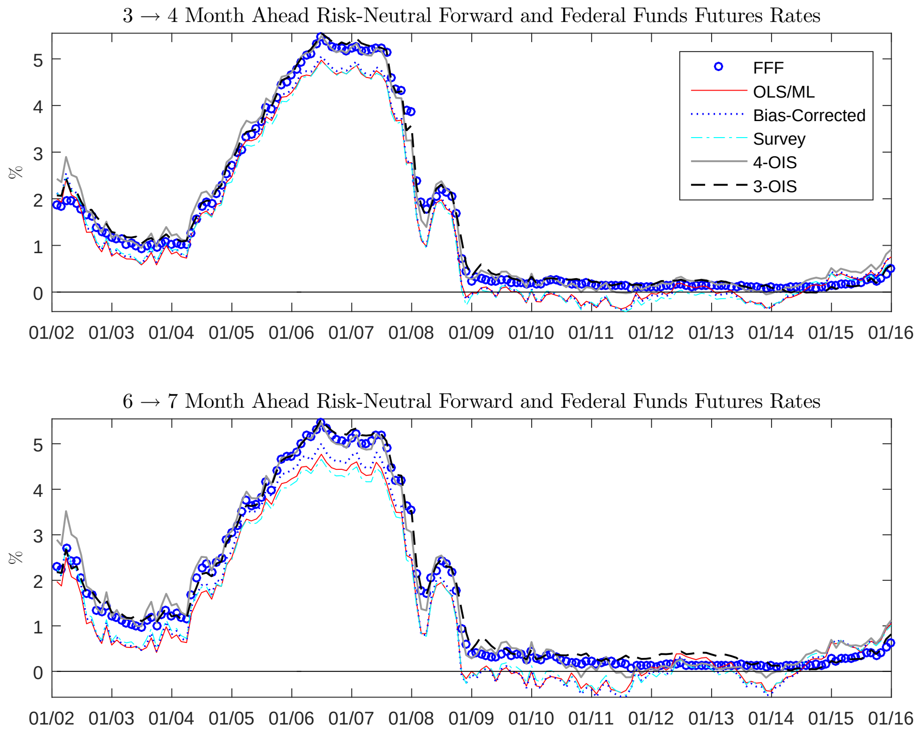
Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the four-factor 4-OIS-augmented model implied by the Kalman filter.

Interest Rate Expectations Figure 13 plots the $3 \rightarrow 4$ and $6 \rightarrow 7$ month ahead risk-neutral forward yields against comparable-horizon federal funds futures rates. The figure demonstrates that the OIS-augmented models provide the closest fit for market-based measures of interest rate expectations for the majority for the 2002-15 sample.

F.3 Monthly Frequency GADTSMs

For robustness, I also estimate the GADTSMs at a monthly frequency. The monthly frequency models have the same structure as described in the main body of the paper, with the time index t now representing a month, rather than a day. To estimate the model, I use bond yields and OIS rates from the final day of each calendar month. I estimate the monthly frequency models using the same 13 bond yields and 4 OIS rates for the January 2002 to December 2015. The headline conclusion is as follows: the benefits of OIS-augmentation for estimates of future short-term interest rate expectations carry over from daily frequency estimation to lower frequencies,

Figure 13: Estimated Risk-Neutral 1-Month Forward Yields from the Four-Factor Models and Comparable-Horizon Federal Funds Futures (FFF) Rates



Note: I plot estimated 3 to 4-month ahead and 6 to 7-month ahead risk-neutral forward yields from each of five GADTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2015. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates. All figures are in annualised percentage points.

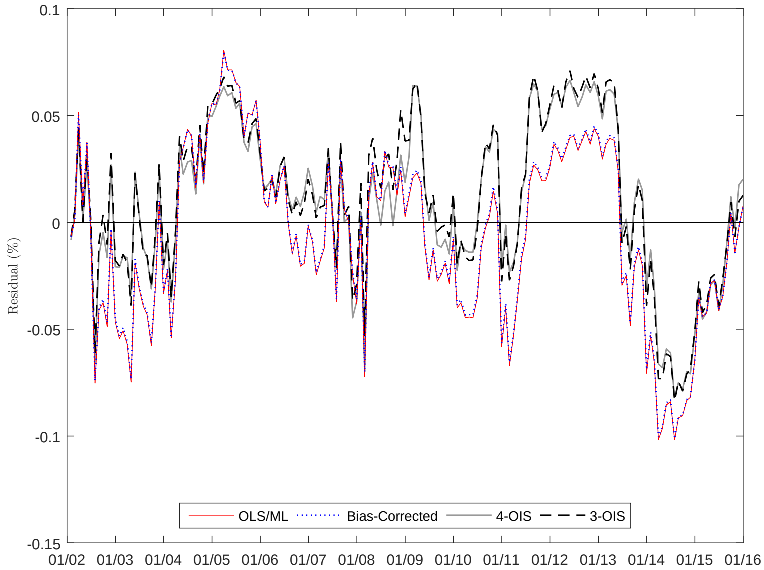
such as the monthly frequency.

Fitted Yields Figure 14 illustrates that the fitted yields from the monthly frequency GADTSMs do not differ markedly (i) from one another and (ii) in comparison to the daily frequency estimates presented in the main body of the paper. Here, I plot the residual of the 2-year fitted yield from the monthly frequency GADTSMs. They serve to illustrate that the models provide a similar fit for actual bond yields.

Fitted OIS Rates As with the daily frequency results, the monthly frequency OIS-augmented models accurately fit OIS rates. Figure 15 demonstrates, again, that the 4-OIS-augmented model accurately fits the 3, 6, 12 and 24-month OIS rates. Although the 4-OIS-augmented provides a visually superior fit of all four OIS rates, the 2 and 3-OIS-augmented models do provide estimates of OIS rates that fit actual OIS rates reasonably well.

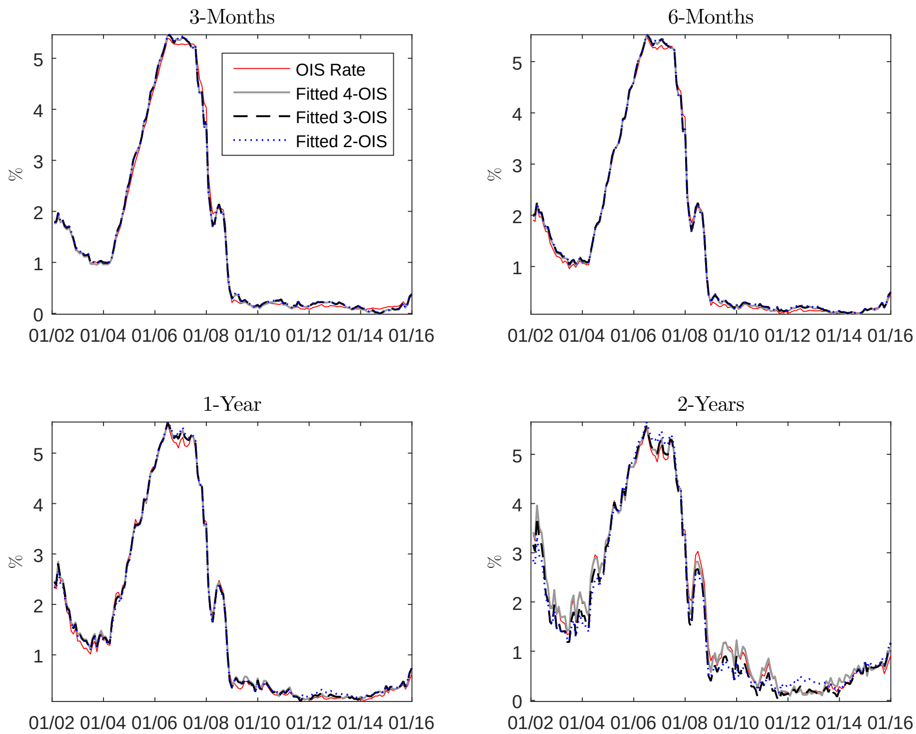
Pricing Factors Figure 16 plots the estimated principal components of the monthly frequency bond yield data and the estimated pricing factors from the monthly-frequency 4-OIS-augmented

Figure 14: Residual of the 2-Year Fitted Yield from Monthly Frequency GADTSMs



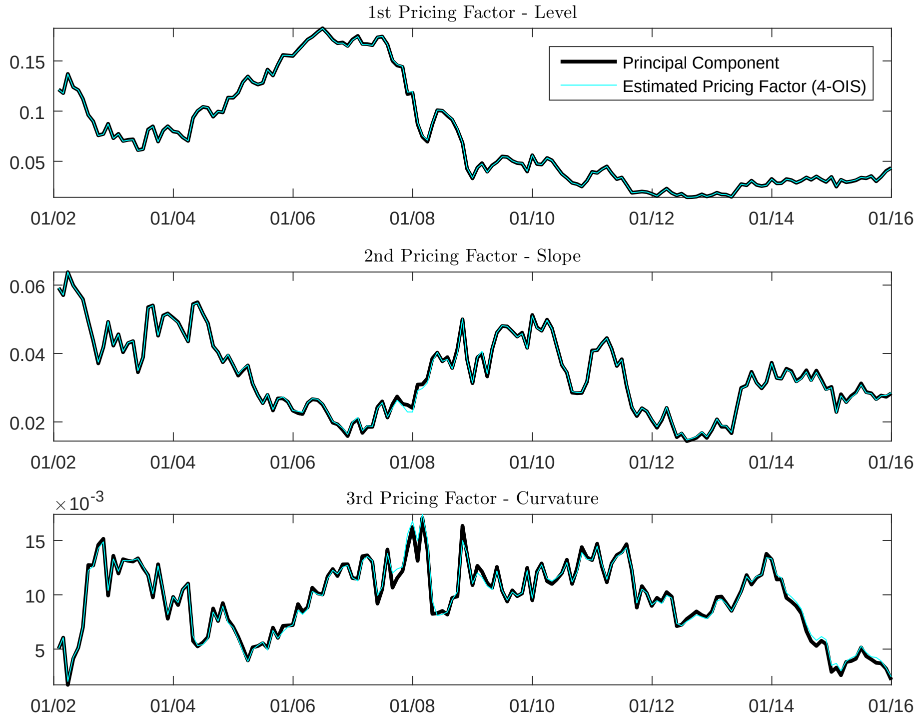
Note: Residuals of the 2-year fitted yield from four monthly frequency GADTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model; and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2014. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

Figure 15: Fitted OIS Rates from the Monthly Frequency OIS-Augmented Models



Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented GADTSMs. The models are estimated with three pricing factors using end of month data from January 2002 to December 2015. All figures are in annualised percentage points.

Figure 16: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model at a Monthly Frequency

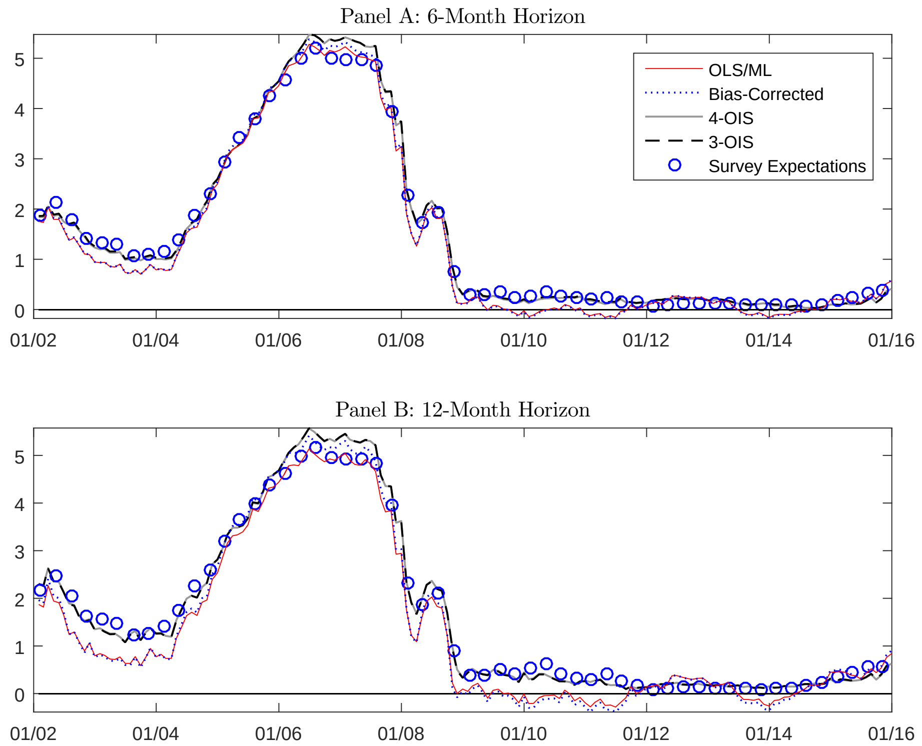


Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model implied by the Kalman filter.

model. As with the daily frequency model, the plot demonstrates that the inclusion of OIS rates in the estimation of GADTSMs does not significantly influence the bond pricing factors. The two quantities evolve almost identically.

Interest Rate Expectations Finally, in figure 17, I plot the 6-month and 1-year risk-neutral yields from the monthly frequency OLS/ML, bias-corrected, 4 and 3-OIS-augmented GADTSMs against comparable horizon survey expectations. The figure highlights that the monthly frequency models provide similar estimates for the level of interest rate expectations at a given time. The OIS-augmented models, again, visually provide the best fit of survey expectations.

Figure 17: Short-Term Interest Rate Expectations from the Monthly Frequency Models



Note: I plot estimated 6-month and 1-year risk-neutral yields from each of four GADTSMs in panels A and B, respectively. The four models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model (4-OIS); and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2015. I compare the estimated risk-neutral yields to approximated survey expectations of future short-term interest rates over the same horizon. The construction of survey expectation approximations is described in appendix B. All figures are in annualised percentage points.

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