External Imbalances and Sustainability*

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Abstract

We use vector error correction model to investigate a country's intertemporal solvency. Following the general approach developed by Pesaran (2008) we propose a formal long-run solvency condition and derive its testable implications. The focus is on country's stocks of assets and liabilities rather than flows. Allowing for differential rates of return on asset and liability side of the country balance sheet we are able to incorporate valuation effect and imperfect substitutability which seem to be important especially for the US. We find that the US has not run into potential insolvency yet despite its huge amount of debts accumulated. However, it becomes increasingly relying on excess return from assets over liabilities to finance its debts since the 1980s.

1 Introduction

We have seen considerable external imbalances around the globe in recent years, as a result of the rapid development in international trade and financial integration. In particular the US has experienced prolonged current account deficit since early 1980s, absorbing about three-fourths of the combined current account surpluses from the rest of the world in 2006. A number of industrial economies, such as France, Italy, Spain, Australia and Britain, have been running substantial deficits in their current accounts as well since mid 1990s. But the situation in Asia and oil-producing countries is different. After the late 1990s financial crisis many Asian economies has built up large stocks of foreign reserves mainly in the form of the US treasury bills and the dollar. The surge in oil prices has boosted revenues and the foreign reserves in oil-producing economies. Consequently there is a significant reallocation of wealth across countries. The US has transformed from a net creditor into a net debtor, and become increasingly dependent on central banks from the emerging economies in Asia and the oil-producing economies to finance its debts. An important question is whether such pattern of global external imbalances is sustainable. There is no free lunch - debts have to be paid off, and this requires an adjustment towards a new balance in the future. But when and what may trigger the adjustment; at what speed and by what means may the rebalancing process take place? All of these have important

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implications for the world economy. In this paper we focus on the first question and propose a method for assessing long-run solvency.

Central to many debates today is the emergence of substantial current account imbalances. By definition, current account is equal to the gap between domestic saving and investment. A current account deficit can thus be interpreted as the result of excess domestic absorption. This may be due to a shift of domestic preference towards foreign goods, or due to a presence of attractive investment opportunity in the home market. Early studies in the 1980s emphasize the internal forces from the domestic economy to explain current account dynamics. Such perspective may be summarized as a National Income and Product accounts identity as follow

$$\begin{array}{c}\text{domestic private}\\ \text{saving}\end{array} + \begin{array}{c}\text{trade}\\ \text{deficit}\end{array} = \begin{array}{c}\text{private}\\ \text{investment}\end{array} + \begin{array}{c}\text{government budget}\\ \text{deficit}\end{array},$$

where two main sources of savings (the left hand side) are equal to the demand for capital (the right hand side). If saving and investment in the private sector roughly equal to each other, then the current account is mainly driven by the government budget stance. The "twin deficits" hypothesis is based on this framework, which seems to explain the US experience from 1980 to 1986 when its fiscal budget deficit increased from 2.7% to 5% and its current account deficit rose from 0 to 3.5% (measured as a share of GDP). However, the "twin deficits" mechanism soon broke down by the late 1990s when the federal budget stance shifted to surplus and the US current account deficit became worse. Nor is the twin deficit hypothesis able to explain why a number of major countries, including Germany and Japan, continue to run large current account surpluses despite sizable government budget deficits (Bernanke, 2005). For the case of the US Mann (2002) argues that it is the advance in information technology that has changed the economy. It is argued that the investment boom in the 1990s has placed a wedge between private investment and savings. On one hand, the US new technology development suggested attractive investment opportunities and boosted the US stock market to an unprecedented level, leading to a surge in both foreign direct investment and portfolio equity investment. On the other hand, the robust economy and stable low unemployment rate have strengthened households' confidence, which encourages consumption and greatly slashes saving. The US personal saving rate dramatically declined from 7.7% in 1992 to less than 1% in 2007.[1] Though intuitive, such perspective is too restrictive to the domestic side of the story: it suggests that the deterioration in the US current account mainly reflects economic development within the US itself, and it simply assumes that foreigners are responding to the US saving-investment gap passively.

An alternative perspective sees the current account imbalances as the collective outcome of saving and investment decisions in a global framework. Bernanke (2005) argues that the major factor is the emergence of an excess saving, the global saving glut, from some industrial and developing economies. With aging population, slowly growing or declining labor forces, and a dearth of domestic investment opportunities, the mature industrial economies except the US have a strong motive for saving and seek to lend abroad. Yet the excess saving from industrial

[1] Source: Bureau of Economic Analysis

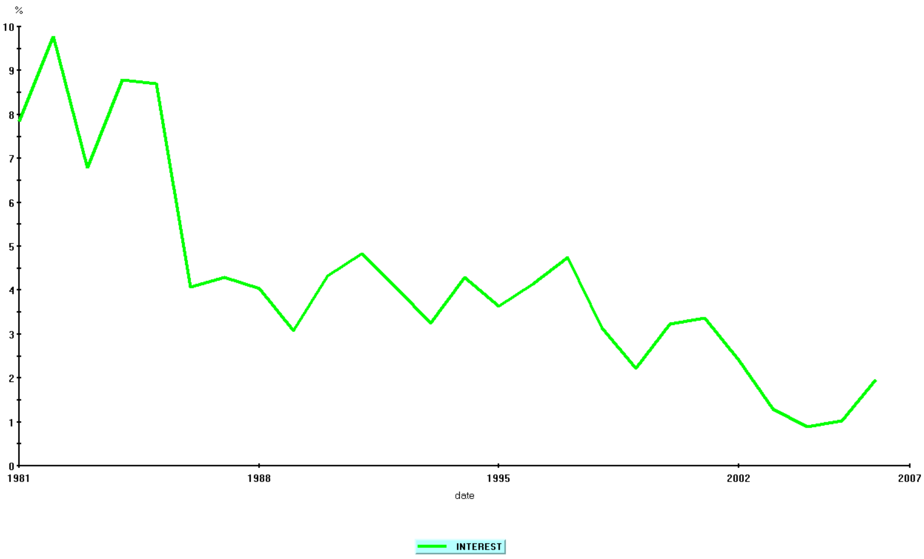

Figure 1: Yield on US 10-year federal government security deflated by inflation. Source: OECD Main Economic Indicators

economies is only a fraction of the global saving glut. The substantial portion comes from the developing economies and the oil producing countries. Before financial crisis in the late 1990s most developing economies were net debtors in their international investment positions. After the painful experience of 1997 financial crisis many emerging economies started to adopt new strategies for managing international capital flows. A typical strategy, adopted in Korea and Thailand for example, is to build up a large stock of foreign reserves as a buffer against potential capital outflows at the time of crisis. Besides, many developing countries are pursuing the export-led-growth strategy which is associated with currency peg and official foreign exchange intervention. Foreign reserves are accumulated in the context of official exchange intervention, in the hope of promoting export by reducing exchange rate fluctuation. Another important factor that contributes to the global saving glut is the sharp rise in oil prices. With higher revenues from oil exports the current account surpluses have increased substantially in the Middle East, Russia, Nigeria and other major oil exporting countries. The global-saving-glut perspective helps to explain why the rest of the world are willing to finance the US deficit, and why is there a relatively low long-term real interest rates in the world today (Figure 1).

Nonetheless current account imbalances are only part of the story. The focus on current account emphasizes the flows of international capital but neglects the stock positions. After all, as commented by Mann (2002), current account is not a fundamental economic force in itself. It is one manifestation of the underlying disequilibria in different markets. A third perspective suggests that global imbalances are better analyzed by taking into account the determination of both the flows and the stocks of capital. Different factors discussed in previous paragraphs could be reasonably viewed as various shocks to supplies or demands of assets in the

Table 1: International Investment Position (as a ratio to GDP) in 2004. Source: Lane and Milesi-Ferretti, 2006

Country	Gross assets	Gross liabilities	Net assets
Canada	99%	112%	-13%
France	212%	206%	6%
Germany	167%	159%	8%
Italy	105%	124%	-19%
Japan	89%	51%	38%
United Kingdom	357%	371%	-14%
United States	84%	107%	-23%
Argentina	88%	136%	-48%
Brazil	28%	78%	-50%
China	55%	47%	8%
India	23%	34%	-11%
United Arab Emirates	252%	206%	46%
Venezuela	89%	73%	16%
Russia	67%	66%	1%

international markets. There are other advantages to follow this perspective. For one reason, it looks at a larger picture. The volume of international financial transactions is far more enormous and is increasing more rapidly than the international trade flows, due to on-going financial liberalization and innovation. Secondly, apart from the gains in trading goods and services, there is potential gain from diversifying a nation's portfolio allocation. A country may obtain better trade-off of risk and return when it has the access to different assets in different currencies and different maturities which could be imperfectly correlated to each other. Finally, the stock positions of assets and liabilities suggest the stance of a nation's external budget, which bears the important question of sustainability of the external imbalances and the associated flows of capital. Note that the international capital markets are highly integrated: a country's positive net foreign assets holding implies net liabilities for others. But the total net holdings should add up to zero for the world. If someone is accumulating assets indefinitely, someone else must be accumulating liabilities indefinitely, which is not sustainable. Therefore the international investment positions represent a more fundamental picture of the external imbalances. For example, the stock of foreign assets held by the US has surged from 25% of GDP in 1976 to 104% of GDP in 2006, while its gross liabilities has surged from 16% of GDP in 1976 to 123% of GDP in 2006. The negative net asset position is simply the accumulated current account deficits over the past decades. In a cross-country dimension, Table 1 shows the diverse development of the international investment positions for several advanced and emerging economies in 2004. Certain countries, such as the United Arab Emirates, become large net creditors, while some countries, including the US, become large net debtors.

In this paper I follow the third perspective which places assets holdings and relative returns at the centre of investigation. Lane and Milesi-Ferretti (2003, 2006) document a tremendous increase in gross assets and liabilities holdings since 1970 in both advanced and developing countries. During 1970-2004, the sum of total assets and liabilities measured as a proportion to GDP have increased from 45% to 302% for the industrial economies, and from 15% to 102% for the emerging countries.[2] The growing cross-border holdings have substantially enhanced global linkages far beyond the international trades. A country with large exposure to foreign assets holdings is now subject to sizable wealth effect from the fluctuations in exchange rate and prices of assets. This so-called valuation effect has attracted wide interests in the study of the external financial adjustment (Gourinchas and Rey, 2007), and in the investigation of international transmission mechanism of monetary policy (Tille, 2005). Moreover the presence of a number of large net debtor and net creditor countries raise the important issue of solvency. With a net liabilities up to 20% of annual GDP it may be necessary to rethink about the sustainability of the US' external position.

However, the question of sustainability or long-run solvency is not very satisfactorily addressed in the literature. Partly this is due to lack of data. More importantly it is a problem of methodology. The concept of sustainability was vaguely defined; *ad-hoc* filtering is often adopted in empirical studies, but leaving the questions about long-run properties unanswered; and no empirical investigation has been done properly in a multi-country framework taking into account the interdependence of cross-border holdings of assets and liabilities. A major contribution in this paper is to propose a formal and testable long-run solvency condition following a general approach developed by Pesaran (2008). The theory is based on a proper definition of steady state and could be easily imbedded into a conventional VECM model. This method is general in nature and can be readily applied to any solvency problem of either an individual or an economy as a whole. As an application we carefully study the external imbalances of the US using the quarterly data from Gourinchas and Rey (2007). The empirical results show that imperfect substitutability between domestic and foreign assets has played a significant role in the long-run solvency problem of the US.

The rest of the paper is planed as follows. Section 2 surveys the existing approaches and documents some resent contributions to the research on external imbalances, from both a short-run and a long-run perspective. Section 3 defines the long-run solvency condition and proposes its testable implications. The relevant econometrics techniques are briefly discussed. Section 4 presents the empirical results and discusses the implications. Section 5 concludes.

2 A Review of the Literature

Much of the theoretical research on external imbalances is based on a neoclassical setup where current account dynamics is captured as the optimal equilibrium outcome of rational behaviour featuring forward looking characteristics. This provides a useful framework for answering questions like how external position may respond to various shocks in the short run. We start the discussion with the literature on the short-run dynamics of external imbalances, with special attention to the valuation effect recently emphasized by Gourinchas and Rey (2007). However, the short-run perspective does not consider the problem of long-run solvency. An implicit but

[2] See Gourinchas (2007) for the list of industrial and emerging economies.

so far unquestioned assumption in many open economy models is that the intertemporal budget constraint is satisfied in every period, in the past and in the future, as an accounting identity. We argue that this assumption implies solvency and thus should be tested formally. We shall briefly review studies on the long-run properties of external imbalances.

2.1 Short-run Dynamics of External Imbalances

A modern synthesis to the analysis of current account imbalances was developed in 1980s in the context of the intertemporal open economy model. The dynamics of current account is typically characterized as the result of forward looking decisions by households and investment decisions by firms. In line with the permanent income hypothesis, the intertemporal approach to current account places consumption smoothing at the centre of the theoretical framework. Domestic consumers are better off with access to international capital markets where they could optimize the intertemporal allocation of consumption through borrowing and lending. For example, a home country technology shock that raises expected future income will induce domestic consumers to borrow from abroad, and thereby resulting in a current account deficit for the home country. This basic idea has inspired many empirical studies in the attempt to explain current account dynamics. Sheffrin and Woo (1990) carried out the first empirical analysis on four countries. They start with a small open economy model based on a representative consumer maximizing expected life-time utility subject to the intertemporal budget constraint and the transversality condition. Utility is assumed to depend on consumption only, and follow a conventional time-separable functional form. In order to have a closed-form solution they further assume quadratic utility (thereby certainty equivalence), a fixed discount factor β , and a fixed interest rate r, which satisfy $\beta(1+r)=1$. Substituting the first order conditions from utility maximization into the budget constraint they obtain the core result that the current account CA_t is equal to the expected present value of the future stream of the changes in net output, where net output NO_t is defined as GDP less investment and government spending. Mathematically this is

$$CA_t = -E_t[\sum_{i=t+1}^{\infty}(\frac{1}{1+r})^{i-t}\Delta NO_i], \tag{1}$$

where $E_t[\cdot]$ stands for the expectation operator conditional on information available at time t; and Δ is the first difference operator. To examine the present value model (1) empirically they followed the testing strategy developed by Campbell (1987) and Campbell and Shiller (1987) who initially used the method to study consumption behaviour. The basic idea is to proxy the expected future values by the past information using an unrestricted Vector Autoregressive (VAR) model. Suppose the underlying data generating process is a VAR in $\mathbf{z}_t = (\Delta NO_i, CA_t)'$, given by

$$\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_{t-1} + \boldsymbol{\epsilon}_t, \tag{2}$$

where $\mathbf{z}_t$ is assumed to be stationary, and $\boldsymbol{\epsilon}_t$ is a vector of stationary stochastic disturbances with zero means.[3] Notice, there is no deterministic components (intercept and trend) in model (2), which implies that the unconditional mean of $\mathbf{z}_t$ is zero. The conditional expected future value can be written as

$$E_t[\mathbf{z}_{t+h}] = \mathbf{\Phi}^h \mathbf{z}_t,\ h = 1, 2, ... \tag{3}$$

Substitute (3) into (1) we have the predicted current account, $\widehat{CA}_t$, given by

$$\widehat{CA}_t = \mathbf{\Psi} \mathbf{z}_t\ , \tag{4}$$

where $\mathbf{\Psi} = -\frac{1}{1+r}(1, 0)\mathbf{\Phi}(\boldsymbol{I} - \frac{1}{1+r}\mathbf{\Phi})^{-1}$. If the intertemporal model is valid $\widehat{CA}_t$ is then equal to CA_t, implying that $\mathbf{\Psi} = (0, 1)$. These constraints can be formally tested by a Wald statistic. Sheffrin and Woo find some supporting evidence from Belgium and Denmark, but the constraints are clearly rejected for Canada and the UK.

The basic present value model fails in the empirical test because the estimated current account derived from the VAR model is too smooth compared to the actual level. This is not surprising since the stylized model is rather simple in nature: utility is risk neutral and time-separable; the gross interest rate is exogenous and fixed at the level equal to the discount factor, which induces excess capital movement; goods are homogenous; and domestic and foreign assets are perfect substitutes. Subsequent researches have considered various extensions to improve the empirical performance of the basic model. Ghosh and Ostry (1997) propose that relaxing risk neutrality and incorporating precautionary saving improve the model's prediction on current account volatility. Gruber (2004) introduces preference with habit by relaxing time-separable utility assumption. He shows that habit formation can improve the model's informal prediction fit significantly. Bergin and Sheffrin (2000) suggest that allowing extra uncertainties from time varying foreign interest rate and relative prices between tradable and non-tradable goods may also improve the fit of the present value model. Other important extensions include country-specific fiscal shock and world interest rate shock. Although these factors are important and they have contributed a closer fit of the basic model to the actual data, the core theoretical restrictions on the coefficient vector $\mathbf{\Psi}$ is too often rejected in empirical studies (Nason and Rogers, 2003). Moreover the assumption that current account CA_t is stationary with zero unconditional mean as specified in model (2) seems not very realistic because we have observed prolonged current account deficits or surpluses in a number of economies.

Despite the unsatisfactory empirical performance of the present value model, it is argued that abandoning the intertemporal approach is unwarranted. The basic idea underlying the intertemporal approach is consumption smoothing which offers useful insight about an open economy's short-run response to various shocks. As remarked by Obstfeld (2001) the intertemporal approach is a good starting point for thinking about the important and interrelated policy issues of external balance and external sustainability. But there must be something missing either in the macro model or in the econometric method for the poor empirical performance. Recent

[3] Further lags can be introduced. By stacking the model properly it will take the same form as (2) eventually. See Sheffrin and Woo (1990) for details.

studies suggest that focusing on current account is somewhat misled. Instead, we should focus on the determination of a country's gross foreign assets and liabilities. Partly, there is serious data problem as current account is usually recorded at historical cost in the National Income and Product Accounts and the Balance of Payments. The book-value accounting measure neglects the capital gains and losses on gross assets and liabilities due to exchange rate and assets prices movements. The discrepancy between book-value and market-value has become much larger after the tremendous increase in cross-border holdings of foreign assets and liabilities[4]. A proper measure should keep track of the market value of assets and liabilities. The data on current account from the National Income and Product Accounts may not be a good measure for the change of a country's net foreign wealth, although in the standard intertemporal approach they are treated as the same.[5] More importantly, current account is not a fundamental economic force in itself, but a temporary equilibrium outcome of the underlying interactions of supply and demand for goods and assets. The portfolio allocation of national wealth and the associated dynamics in the stocks of foreign assets and liabilities bear more important questions, such as the international transmission mechanism of monetary policies and the sustainability of the external imbalances. These considerations have shifted research interests to a country's international investment position.

Tille (2005) documents the detailed composition of the US international investment portfolio. He finds that the US portfolio is highly leveraged in currencies and in types of assets: it is short in assets denoted in the US dollar and long in assets denoted in foreign currencies; it is short in low-return, less risky debts and banking assets, and long in high-return, risky foreign direct investment and equities. Consequently a depreciation in the dollar may lead to a non-negligible capital gain for the US, stabilizing its external imbalances. However, the home currency depreciation may generate just the opposite effect for the emerging economies whose liabilities are mainly denoted in foreign currencies. These wealth effects, if become large, may have important implications for the international transmission mechanism of monetary policy which in turn affects the values of currencies. To formally examine the effect of financial integration on the monetary transmission mechanism, Tille (2005) extends the standard two-country open economy model[6] to include cross-country holdings of various financial assets. The impact of a permanent unexpected monetary shock that depreciates the home currency is studied by calibration, conditional on the composition of a country's investment portfolio (which is called the structure of financial integration). By careful parameterization to mimic the situation in the US, Tille (2005) shows that the welfare gain from an exchange rate depreciation for the home country is sizable; the magnitude of the effect depends on the specific structure of the integration. If cross-border holdings take the form of debt instrument, monetary expansion with valuation effect leads to welfare gain six times of the benchmark case without financial integration. If cross-border holdings are mainly in the form of equity, the total welfare gain with

[4] See Gourinchas and Rey, 2005

[5] By definition, current account equals to the change in a country's net foreign assets position in the intertemporal model (Obstfeld and Rogoff, 1996).

[6] The setup by Obstfeld and Rogoff, 1995.

valuation effect is about twice of the benchmark case. Thus the benefit of financial integration is at least as large as that from nominal rigidities.

Yet a major limitation of Tille's (2005) model is that the structure of financial integration is exogenously given. No portfolio optimization is considered and all assets are perfect substitutes in the steady state yielding identical returns. It is not capable to explain why a country's portfolio may be highly leveraged. On the other hand, using a portfolio-balance model Blanchard, Giavazzi and Sa (2005) suggest that imperfect substitutability between domestic and foreign assets helps to explain the dynamics of the net foreign wealth and the associated movement in real exchange rate of the US. Incorporating home-bias in assets holdings and in consumption their model enables one to consider shifts in assets preference as well as shifts in the demands for foreign goods. They conclude that two driving forces for the substantial increase of the US current account deficit and the large swing in real dollar value in the past decade are a shift towards foreign goods from the US and a shift towards dollar assets from the rest of the world. These factors are absent in the general equilibrium model where domestic and foreign assets are perfect substitutes. Blanchard, Giavazzi and Sa (2005) also suggest that imperfect substitutability between domestic and foreign assets may give rise to the so-called valuation effect proposed by Lane and Milesi-Ferretti (2002) and Gourinchas and Rey (2007).

Alternative to the conventional trade channel the adjustment of a country's net foreign wealth may take place through the channel of capital gains and losses on gross assets and liabilities, the so-called "valuation effect". Gourinchas and Rey (2007) explore this idea extensively in their study of the international financial adjustment for the US. To see the intuition, define NA_t as the stock of net foreign assets at time t, and consider the following accumulation identity:

$$NA_{t+1} = \tilde{R}_t \cdot NA_t + NX_t \ , \tag{5}$$

where NX_t is the net export flow and $\tilde{R}_t$ is the gross portfolio return on the net foreign assets position. Notice, in addition to the investment income, such as interest payments and dividend yields, the gross return $\tilde{R}_t$ comprises of the capital gains and losses due to local asset prices movement and exchange rate fluctuations. Let NI_t denotes the investment income balance. The current account CA_t is equal to the sum of net export plus the investment income. Thus equation (5) can be rearranged as follows:

$$\begin{aligned} NA_{t+1} - NA_t &= (\tilde{R}_t - 1)NA_t + NX_t \qquad (6) \\ &= [(\tilde{R}_t - 1)NA_t - NI_t] + [NI_t + NX_t] \\ &\equiv VA_t + CA_t \ , \end{aligned}$$

in which the valuation effect is defined by $VA_t \equiv (\tilde{R}_t - 1)NA_t - NI_t$. Equation (6) shows that the valuation term VA_t could potentially account for a significant proportion of the total change in net foreign wealth. Thus using data on book-value measure of current account could seriously distort the empirical result even when the intertemporal approach is a valid theoretical model. For the case of the US the valuation term implies that to rebalance its net foreign asset position the adjustment may take place through a change in the returns on US assets held by foreigners relative to the return on foreign assets held by the US. Such wealth transfer may possibly occur via depreciation of the dollar, because the US portfolio is highly leveraged in currency with most of its debts denoted in the dollar (Tille, 2005). Following these intuitions, Gourinchas and Rey (2007) construct a present value model to examine the dynamics of external imbalances. It should be noticed that their model only consider the cyclical movements of the international financial adjustment, taking the long-run equilibrium as given. To be more precise, they view the world economy as a stochastic economy with deterministic trends. The slow-moving trends in exports, imports, foreign assets, and liabilities[7] are attributed to the gradual process of trade and financial integration. But they do not attempt to model these trends of structural changes; instead, they focus on the external imbalances *in deviation* from these trends. The intertemporal budget constraint is assumed to be satisfied in all periods and all states of the world. Under certain assumptions, they log-linearize the intertemporal constraint around the slow moving trends and arrive at the core present value model given below:

$$nxa_t \simeq -\sum_{j=1}^{\infty} \rho^j E_t[\tilde{r}_{t+j} + \Delta\widetilde{nx}_{t+j}] \ , \tag{7}$$

where the cyclical external imbalances nxa_t is defined by

$$nxa_t \equiv |u^a| \ln(\hat{A}_t/\bar{A}_t) - \left|u^l\right| \ln(\hat{L}_t/\bar{L}_t) + |u^x| \ln(\hat{X}_t/\bar{X}_t) - |u^m| \ln(\hat{M}_t/\bar{M}_t) \ .$$

$\hat{A}_t$, $\hat{L}_t$, $\hat{X}_t$ and $\hat{M}_t$ are the ratios of assets, liabilities, exports and imports to domestic wealth respectively; $\bar{A}_t$, $\bar{L}_t$, $\bar{X}_t$ and $\bar{M}_t$ are the corresponding trend components. Parameters u^a, u^l, u^x, u^m and ρ are evaluated at the steady state.[8] Intuitively, nxa_t can be interpreted as the deviation from the trend of the ratio of net exports to net foreign assets ($\frac{NX_t}{NA_t}$). The detrended net export growth, $\Delta\widetilde{nx}_t$, is defined by

$$\Delta\widetilde{nx}_t \equiv |u^x| \, \Delta \ln(\hat{X}_t/\bar{X}_t) - |u^m| \, \Delta \ln(\hat{M}_t/\bar{M}_t) - \epsilon_t^{\Delta w} \ ,$$

in which $\epsilon_t^{\Delta w}$ is the cyclical component of the growth in domestic wealth. The valuation effect is captured by the detrended portfolio return $\tilde{r}_{t+j}$ which is defined as

$$\tilde{r}_t \equiv \frac{u^a}{|u^a|} \ln(\tilde{R}_t/\bar{R}_t) \ ,$$

where $\bar{R}_t$ is the trend component of the gross portfolio return, $\tilde{R}_t$, on the corresponding net foreign asset position. The present value model (7) suggests that a country can improve its net external position either through a net export growth ($\Delta\widetilde{nx}_t > 0$) or through a higher return on its net foreign asset portfolio ($\tilde{r}_t > 0$, which is the valuation channel), or both. In practice the detrended components of each series are obtained separately by using Hodrick-Prescott filter set

[7] These are measured proportional to wealth.

[8] See Gourinchas and Rey (2007) for the construction of these parameters.

to filter out cycles of periods more than 50 years. To test the key implication (7) empirically, one needs a long enough data of the gross assets and liabilities at market value. Gourinchas and Rey (2005) have conducted detailed estimates of the US gross assets and liabilities covering the period of 1952Q1-2004Q1. Using this newly constructed dataset, they follow the same present value model testing strategy discussed before and show that the resulting cross-equation restrictions cannot be rejected at conventional level of significance. Further, they decompose the unconditional variance of nxa_t into two parts: one due to the contribution of trade channel ($\Delta\widetilde{nx}_t$) and the other due to the valuation channel ($\tilde{r}_t$). They find that valuation effect accounts for about 27% of the total variance of nxa_t, from which they conclude that although valuation effect does not replace the need for an ultimate adjustment through higher exports, it profoundly transforms the nature of the external adjustment process.

The perspective which emphasizes the role of valuation effect on external adjustment is very interesting and inspiring. But the *ad-hoc* filtering process makes it somewhat difficult to interpret their empirical results. Looking at the cyclical movements of external imbalances leaves too much unanswered about the economic nature of the long-term trends. A rebalancing process in external constraint is not just about how deviations may adjust towards the steady state. Secondly the present value model (7) is derived following the assumption that there is a solvency constraint on the secular trends, but they do not examine this. They assume common stochastic trends, and thus cointegration, among the trade shares and the foreign assets shares without empirically investigating the maintained positions. They apply HP filter separately to each series instead. Moreover, the *ad-hoc* filtering process is criticized for generating unknown distortion to data. Harvey and Jaeger (1993) show that HP filter can generate arbitrary cycles. Gourinchas and Rey (2007) argue that because they use HP filter to eliminate only very low frequencies components the detrended series still contain most information of the data. But the parameter of HP filter is not unique and often chosen arbitrarily. In their robustness tests Gourinchas and Rey (2007) show that using different filters, such as Christiano and Fitzgerald's (2003) asymmetric filter, does not alter their main conclusion. Overall, the filtering technique is silent about the economic nature of slow moving trends, leaving the question of long-run solvency unanswered. A different approach is needed to study the long-run properties of the external imbalances, which is taken up in Section 3 of this paper. Nonetheless, the perspective on valuation effect suggests a novel and important aspect of a country's external adjustment process. It highlights a potentially critical role played by the increasing cross-border holdings of foreign assets, and the associated sizable wealth transfer induced by assets prices and exchange rate movements.

2.2 Theories on the Long-run Sustainability

Unlike researches on the short-run dynamics of external imbalances, there are relatively few studies on the long-run properties of these imbalances. Faruqee (1995), Gagnon (1996), Alberola, Cervero, Lópex and Ubide (1999) and Lane and Milesi-Ferretti (2004) suggest that there is a positive long-run relation between net foreign assets and the real exchange rate. The intuition behind is that debtor countries may tend to have more depreciated real exchange rate. But their theoretical rationale is rather loose, partly because the statistical properties of the "steady state" are not properly considered. Corsetti and Konstantinou (2004) argue that in addition to the great ratios - consumption to output and investment to output ratios[9] - there is a long-run relation between the log of net foreign assets and the log of GDP. But there is a problem in their argument since net foreign assets may be negative for a long period of time. It is not appropriate to take log on the net foreign assets, and hence their log-linearization procedure in the derivation of the long-run relation is invalid.

A critical but not very well addressed question is the long-run sustainability of external imbalances. The major deficiency in the literature seems to be the lack of a clear notion of sustainability. Mann (2002) suggests that a sustainable external imbalance at a point in time is the one that does not induce significant changes in fundamental variables, such as consumption, investment, interest rate, or exchange rate. Her proposition focuses on the interests and dividends payments arising from the gross stock of liabilities. If financing obligations stemming from the external imbalance become large enough to affect domestic consumption and business investment, or to affect the determination of interest rate and exchange rate, then the adjustment process will get underway towards rebalancing. Intuitive as it sounds, Mann has not suggested how the proposition could be tested empirically. The ratio of current account deficit to GDP or the share of US assets in the global investor's portfolio seem to be informative about the sustainability problem, but it is unknown what will be the benchmark sustainable values for these measures, and how they might be estimated. Though the present value models in section 2.1 provide useful insights about the short-run dynamics of external imbalances, they are not concerned with the long-run properties. Standard open economy models typically assume that the intertemporal budget constraint is satisfied in every period in the past and in the future, and in the every state of the world. This assumption automatically ensures long-run sustainability. Testing this assumption seems to be a hopeful way of assessing long-run solvency.

Naturally, a sustainable path of an economy means it is not violating its budget constraint. To analyze the long-run sustainability problem we should focus on whether or not the intertemporal budget constraint will be satisfied in the future, or *ex ante*. Existing literature has considered cointegration tests on the present value constraints in the solvency problems of the government finance and the external trade balance. For example, Ahmed and Rogers (1995) start from the government budget balance to derive a present value constraint as follow:

$$E_t \sum_{j=0}^{\infty} ({}_t s_{t+j} G_{t+j}) - E_t \sum_{j=0}^{\infty} ({}_t s_{t+j} T_{t+j}) + (1 + r_{t-1}) B^g_{t-1} = \lim_{N \to \infty} E_t ({}_t s_{t+N} B^g_{t+N}) \ , \qquad (8)$$

where G_t is the government spending, T_t is the tax revenue and B^g_t is the stock of government debt, all in real terms; ${}_t s_{t+j}$ is the marginal rate of substitution between consumption in period t and $t + j$; and r_{t-1} is the real interest rate. They argue that if the government satisfies its intertemporal budget constraint for every period in the future then the forward limiting expected

[9] See King, Plosser, Stock and Watson (1991)

term on the right hand side of equation (8) should be zero. Intuitively, solvency requires that asymptotically the government cannot leave a debt that has a positive expected present value. No Ponzi scheme condition requires the expected present value of government debt to be non-negative. These amount to a conventional transversality condition (TVC) defined in the present context as:

$$\lim_{N\to\infty} E_t({}_ts_{t+N}B^g_{t+N}) = 0 \ .$$

After imposing TVC, equation (8) is simplified as

$$\Delta E_t \sum_{j=0}^{\infty}({}_ts_{t+j}G_{t+j}) - \Delta E_t \sum_{j=0}^{\infty}({}_ts_{t+j}T_{t+j}) = -(G_t + r_{t-1}B^g_{t-1} - T_t) \ . \tag{9}$$

Therefore if G_t, B^g_{t-1} and T_t are $I(1)$ processes, and if the left hand side of equation (9) is stationary, then the present value constraint implies a cointegrating relation among G_t, B^g_{t-1} and T_t as specified on the right hand side of (9). This is equivalent to saying that the government budget deficit is stationary.[10] Ahmed and Rogers use residuals based cointegration tests to show that empirically the implied cointegration holds for the US over 1792-1992 and for the UK over 1830-1992, from which they conclude that government budget balance is solvent in the long run. Then they apply exactly the same technique to study the international trade balance and propose that sustainability requires current account deficit to be stationary. For simplicity they assume an equal rates of returns for domestic and foreign assets in every period, and focus on net foreign debt only. Such simplification neglects one of the important consequences of financial globalization. With growing stocks of cross-border holdings of financial assets an open economy is exposed to larger capital gains or losses due to fluctuation in exchange rates and relative rates of return, even if the net balance is zero. Nonetheless, they arrive at the following conclusion. Let X_t and M_t denote the real export and import respectively, and B^f_t denotes the real net foreign debt. They conclude that a cointegrating relation as follow

$$X_t - M_t - r_{t-1}B^f_{t-1} \sim I(0) \tag{10}$$

is a necessary and sufficient condition for a sustainable trade balance. However their empirical findings are not consistent. At first they suggest that the net foreign debt B^f_t is nonstationary by Phillips-Perron unit-root test. Then they find there are two cointegrations for the US. Besides the cointegrating relation in equation (10), their estimation suggests an additional cointegrating relation between export and import:

$$X_t - M_t \sim I(0) \ , \tag{11}$$

which is inconsistent with cointegration (10) if net foreign debt B^f_t is nonstationary.

A potential problem of their approach is that the government's net debt is different from a country's net foreign debt. Notice that the government's revenue, expenditure and interests payment on the government bonds are all denoted in domestic currency. However, a coun-

[10] Notice $(T_t - G_t - r_{t-1}B^g_{t-1})$ is essentially the budget deficit at time t.

try's international investment portfolio could be highly leveraged in currencies and in assets categories. Therefore, a country's external imbalance could be subject to the valuation effects suggested by Tille (2005) and Gourinchas and Rey (2007). Moreover, the return on foreign assets may not be equal to the interest payment on liabilities due to imperfect substitutability between domestic and foreign assets (Blanchard et al. 2005). These suggest that focusing on net foreign assets position alone may not be appropriate. Instead a general approach should consider the gross assets and liabilities positions.

3 Long-Run Solvency Condition

To properly answer the question on long-run solvency we follow the general approach developed by Pesaran (2008). A distinct feature is that we consider a general intertemporal budget constraint which explicitly differentiates between assets that are valued in different currencies and yielding different returns. In this way we are able to incorporate the valuation effects as an additional channel for external financial adjustments. We are interested in whether the valuation effect have contributed any to the growing imbalances from a long-run perspective, which has not been addressed yet. By adopting a proper definition of steady state we avoid the use of filters and focus on the long-run trend components of each series.

Let's start from the home country's intertemporal budget constraint in nominal term:

$$\tilde{Y}_t + \frac{\tilde{S}_t}{\tilde{S}_{t-1}}(1 + R_t^*)\tilde{A}_{t-1} - (1 + R_t)\tilde{L}_{t-1} = \tilde{C}_t + \tilde{G}_t + \tilde{I}_t + \tilde{A}_t - \tilde{L}_t \ , \tag{12}$$

where $\tilde{Y}_t$ denotes the GDP; $\tilde{S}_t$ is the nominal exchange rate measured as the domestic price of foreign currency (therefore an increase in $\tilde{S}_t$ represents depreciation); $\tilde{A}_t$ denotes the stock of foreign assets held by the home country and $\tilde{L}_t$ is the stock of liabilities, both written in terms of domestic currency; R_t denotes the nominal rate of return on domestic assets held by foreigners, valued in home currency; R_t^* denotes the nominal rate of return on foreign assets held by the home country, valued in foreign currency. $\tilde{C}_t$, $\tilde{G}_t$ and $\tilde{I}_t$ denote nominal volumes of consumption, government spending and investment respectively. Writing the intertemporal budget constraint in this way we implicitly assume that all the foreign assets held by the home country are valued in foreign currency while the entire stock of liabilities is valued in domestic currency. This is relevant to the case of the US and a number of industrial economies. But it seems inappropriate for the emerging economies whose liabilities are mostly valued in foreign currencies. One can extend equation (12) to allow for the case where both assets and liabilities comprise of different currencies components. Such a setup is useful for the analysis of short-run international transmission mechanism of monetary policy (see Tille, 2005). But it turns out to be not much different from the simple setup (12) in terms of long-run relations. For the ease of illustration I use the simple setup throughout the paper.

We have a more informative insight about the evolution of external imbalances by rearranging identity (12) as follows. Let $\widetilde{NX}_t = \tilde{Y}_t - (\tilde{C}_t + \tilde{G}_t + \tilde{I}_t)$ denotes the nominal net export flow, we have

$$\widetilde{NX}_t + [\frac{\tilde{S}_t}{\tilde{S}_{t-1}}(1+R_t^*) - 1]\tilde{A}_{t-1} - R_t\tilde{L}_{t-1} = \Delta(\tilde{A}_t - \tilde{L}_t) \ . \tag{13}$$

Equation (13) is a general stock-flow accounting identify which highlights that changes in net assets position are determined by the flow of trade balance, the income flows from the stocks of foreign assets and liabilities, and the capital gains or losses. It is essentially the same as equation (6), only that we are now explicitly modelling the asset and liabilities sides of the valuation term.

The absolute values of $\widetilde{NX}_t$, $\tilde{A}_t$ and $\tilde{L}_t$ are growing exponentially. To stabilize the system we divide each variable by the stock of wealth ($\tilde{W}_t$) measured by household's net worth[11]. The intuition comes from Merton's portfolio allocation model (1971) which suggests that the portfolio shares $\frac{\tilde{A}_t}{\tilde{W}_t}$ and $\frac{\tilde{L}_t}{\tilde{W}_t}$ are constant in the steady state if assets and liabilities are not perfect substitutes. One may also consider using GDP as an alternative denominator[12]. Nonetheless, the denominator should capture the common factors, i.e. economic growth and increasing international economic integration. Deflating equation (13) by wealth also gets rid of nominal prices which are unlikely to affect the long-run rebalancing process significantly.

To be precise, we have:

$$\frac{\widetilde{NX}_t}{\tilde{W}_t} + \frac{\tilde{S}_t}{\tilde{S}_{t-1}}(1+R_t^*)\frac{\tilde{W}_{t-1}}{\tilde{W}_t}\frac{\tilde{A}_{t-1}}{\tilde{W}_{t-1}} - (1+R_t)\frac{\tilde{W}_{t-1}}{\tilde{W}_t}\frac{\tilde{L}_{t-1}}{\tilde{W}_{t-1}} = \frac{\tilde{A}_t}{\tilde{W}_t} - \frac{\tilde{L}_t}{\tilde{W}_t} \ . \tag{14}$$

So equation (14) is defined in terms of real magnitudes. Denote variables measured in real term without a '~'. So the real rates of return on domestic assets and liabilities are given by

$$1 + r_t = \frac{1+R_t}{1+\pi_t} \text{ and } 1 + r_t^* = \frac{1+R_t^*}{1+\pi_t^*} \ ,$$

where π_t and π_t^* stand for domestic and foreign inflation respectively. The real exchange rate is given by

$$s_t = \frac{\tilde{S}_t P_t^*}{P_t} \ ,$$

where P_t (P_t^*) is the domestic (foreign) aggregate price level. Let $nx_t = \frac{NX_t}{W_t}, a_t = \frac{A_t}{W_t}$, $l_t = \frac{L_t}{W_t}$ and $g_t = \frac{W_t}{W_t} - 1$. Equation (14) can be neatly written as

$$nx_t + \rho_{a,t} \cdot a_{t-1} - \rho_{l,t} \cdot l_{t-1} = \Delta(a_t - l_t) \ , \tag{15}$$

where

$$\rho_{a,t} = \frac{1+r_t^*}{1+g_t}\frac{s_t}{s_{t-1}} - 1, \text{ and}$$

$$\rho_{l,t} = \frac{r_t - g_t}{1+g_t} \ .$$

[11] See Flow of Funds Accounts, Table B100.42

[12] In our empirical study of the US using wealth to deflate assets and liabilities produces more sensible results. See the appendix for details.

To compare with early studies we can derive the standard present value model from equation (15). Rearrange equation (15) as follow:

$$na_{t-1} = \frac{1}{1+\rho_{l,t}}[na_t - \delta_t \cdot a_{t-1} - nx_t] \ , \tag{16}$$

where $na_t = a_t - l_t$ is the net foreign assets position, and $\delta_t = \rho_{a,t} - \rho_{l,t}$ measures the real return differential between domestic and foreign assets. If the intertemporal budget constraint (16) is expected to be satisfied in one period ahead, we have

$$na_t = E_t[\frac{1}{1+\rho_{l,t+1}} na_{t+1}] - E_t[\frac{1}{1+\rho_{l,t+1}}(\delta_{t+1} \cdot a_t + nx_{t+1})] \ . \tag{17}$$

Suppose $\rho_{l,t} > 0$ for all t, and that the external budget constraint is expected to be satisfied in every period in the future, we can iterate equation (17) forward. And if we further assume that the transversality condition is also satisfied

$$\lim_{h\to\infty} E_t[(\Pi_{j=1}^{h} \frac{1}{1+\rho_{l,t+j}}) na_{t+h}] = 0 \ ,$$

then we have the standard present value model as below:

$$na_t = -E_t[\sum_{j=1}^{\infty}(\Pi_{i=1}^{j} \frac{1}{1+\rho_{l,t+i}})\delta_{t+j} \cdot a_{t-1+j}] - E_t[\sum_{j=1}^{\infty}(\Pi_{i=1}^{j} \frac{1}{1+\rho_{l,t+i}}) nx_{t+j}] \ , \tag{18}$$

where the first term on the right hand side is the valuation effect and the second term is the present value of all the future net trade flows. Equation (18) suggests that a current deficit in external position can be financed either through an increase in the future net exports ($nx_{t+j} > 0$), or through higher relative return from assets over the interest payment on liabilities ($\delta_{t+j} > 0$), or both. This is exactly what model (7) implies in Gourinchas and Rey (2007). But our approach differs from model (7) in the perspective. GR (2007) looks at the short-run dynamics and their model is about deviations from the unknown trends; but we are interested in the long run and our model is about the levels of variables.

Equation (18) shows that the valuation term is the expected present value of the return differentials multiplied the stock of foreign assets. If domestic and foreign assets are perfect substitutes and that arbitrage-free condition always holds, there will be zero return differential ($\delta_t = 0$ for all t), then the valuation effect will disappear. Our model will degenerate to the case similar to Ahmed and Rogers (1995). But if there is imperfect substitutability between different assets such that $\delta_t \neq 0$ we have valuation effect as an additional channel in country's external adjustment process. This is what Blanchard, Giavazzi and Sa (2005) suggest in their portfolio balance model. It should be noticed that model (18) (and all present value models) is derived under the assumption that the intertemporal budget constraint is satisfied in expectation for every period in the future. This is a critical but so far unquestioned assumption. We should clarify shortly that such assumption requires that the open economy is solvent in the long run.

Solvency or sustainability means the budget constraint is not violated in the past and in the future. For any finite initial value the intertemporal budget constraint (15) holds trivially when looking backward. A more interesting question is whether it is expected to hold looking forward. If we believe the economy is solvent h periods ahead we expect the intertemporal budget constraint (15) to be satisfied at that time. Mathematically this is expressed as

$$E_t[nx_{t+h} + \rho_{a,t+h}a_{t-1+h} - \rho_{l,t+h}l_{t-1+h} - \Delta(a_{t+h} - l_{t+h})] = 0 \ , \tag{19}$$

where E_t denotes expectation taken with respect to information available at time t. We call condition (19) the general solvency condition in the sense that information set at time t contains all information available to the domestic investors as well as to the foreigners. Equation (19) takes into account not only how much the home economy wants to borrow (lend) but also how much the rest of the world is willing to lend (borrow). However, it is important to note that sustainability is defined relative to the specific information set, and what appears to be violation of sustainability may reflect the limited information set used by the econometrician. For example, if there is a discovery of a large natural resource, a country will run prolonged balance of payments deficits while the investment in exploiting the resource is made. This may appear unsustainable if the information set does not contain the resource discovery. So we need to be careful in constructing and interpreting an empirical assessment of long-run sustainability. Nonetheless, a general long-run solvency condition is that the intertemporal budget constraint should be satisfied in the steady state.

As a little digression we need to clarify what is meant by steady state. Steady state is an equilibrium concept, and most macroeconomic models implicitly or explicitly assume that everything is either constant or growing at fixed rate in the steady state. From an econometrics point of view such perspective is rather restricted since many economic time series appear to be nonstationary. Some arbitrary transformations, such as filtering or differencing, are usually proposed to deal with the non-stationarity problem. However the transformation procedure may introduce unknown distortion to data or may erase key information. For example, Harvey and Jaeger (1993) show that HP filter can generate arbitrary cycles. The choice of parameter in HP filter is arbitrary. On the other hand if we are interested in the relation among the levels, first differencing will not provide any useful results. Instead we prefer to avoid filtering and here we adopt a more general definition of steady state following Garratt, Lee, Pesaran and Shin (2006) and Dees, Pesaran, Smith and Smith (2008). This is appropriate for both stationary and non-stationary stochastic processes. To be precise, we have the following:

Definition 1 *For a vector process $\mathbf{z}_t$ of dimension $m \times 1$ its "steady state" or "permanent" value, $\mathbf{z}_t^p$, is defined by*

$$\mathbf{z}_t^p = \lim_{h\to\infty} \{E_t[\mathbf{z}_{t+h} - \boldsymbol{g}_z h]\} \ , \tag{20}$$

where $\boldsymbol{g}_z$ is the $m \times 1$ vector of deterministic growth component of $\mathbf{z}_t$. (All or some of the elements of $\boldsymbol{g}_z$ could be zero.)

Notice that we essentially define the steady state value by the Beveridge-Nelson stochastic trend component, which is also interpreted as the permanent component (see Garratt, Robertson and Wright, 2006). Using the Wold decomposition theorem, which states that any stationary stochastic process can be expressed as the sum of a deterministic and a stochastic moving-average component, we can easily interpret definition (20). Suppose $\mathbf{z}_t$ is stationary. It can be written in moving average form:

$$\mathbf{z}_t = \boldsymbol{\alpha}_z + \boldsymbol{C}_z(L)\boldsymbol{\varepsilon}_{z,t} \ ,$$

where $\boldsymbol{C}_z(L) = \boldsymbol{C}_{z,0} + \boldsymbol{C}_{z,1}L + \boldsymbol{C}_{z,2}L^2 + ...,$[13] and the matrices $\{\boldsymbol{C}_{z,i},\ i = 0, 1, 2, ...\}$ are absolute summable.[14] $\boldsymbol{\varepsilon}_{z,t}$ is a vector of white noise process with zero mean, and $\boldsymbol{\alpha}_z$ is the unconditional mean of $\mathbf{z}_t$. So the permanent component of $\mathbf{z}_t$ is given by

$$\mathbf{z}^p = \lim_{h\to\infty} E_t[\mathbf{z}_{t+h}] = E[\mathbf{z}_t] = \boldsymbol{\alpha}_z \ .$$

The time subscript for the steady state value of stationary $\mathbf{z}_t$ can be omitted because the permanent component in this case is fixed. This is consistent with the conventional interpretation. On the other hand, if $\mathbf{z}_t$ is a nonstationary $I(1)$ process with a deterministic trend, we may start from the first difference of $\mathbf{z}_t$. Suppose $\mathbf{z}_t$ is first difference stationary, then $\Delta\mathbf{z}_t$ can be written as:

$$\Delta\mathbf{z}_t = \boldsymbol{g}_z + \boldsymbol{C}_z(L)\boldsymbol{\varepsilon}_{z,t} \ .$$

Notice $\boldsymbol{C}_z(L) = \boldsymbol{C}_z(1) + \boldsymbol{C}_z^*(L)(1-L)$, so

$$\mathbf{z}_t = \mathbf{z}_0 + \boldsymbol{g}_z t + \boldsymbol{C}_z(1)\sum_{j=0}^{t}\boldsymbol{\varepsilon}_{z,t-j} + \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,t} - \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,0} \ ,$$

where $\mathbf{z}_0$ is the initial value of $\mathbf{z}_t$. By definition (20) the steady state value of $\mathbf{z}_t$ is given by

$$\mathbf{z}_t^p = \lim_{h\to\infty} E_t[\mathbf{z}_{t+h} - \boldsymbol{g}_z h] = \mathbf{z}_0 + \boldsymbol{g}_z t + \boldsymbol{C}_z(1)\sum_{j=0}^{t}\boldsymbol{\varepsilon}_{z,t-j} - \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,0} \ .$$

We should include subscript t for the steady state value in this case because $\boldsymbol{Z}_t^p$ contains the sum of all the past innovations. The steady state value could be changing over time even when the deterministic growth is zero. Notice that $\mathbf{z}_t^p$ satisfies the following equation

$$\mathbf{z}_t^p = \mathbf{z}_{t-1}^p + \boldsymbol{g}_z + \boldsymbol{C}_z(1)\boldsymbol{\varepsilon}_{z,t} \ .$$

So $\mathbf{z}_t^p$ is a random walk with drift. $\mathbf{z}_t^p$ is the so called Beveridge-Nelson stochastic trend component. An advantage of the Beveridge-Nelson transitory and permanent decomposition is its uniqueness: all transitory components will vanish when the forecasting horizon goes to infinity.

Now we define the long-run solvency condition in general.

Definition 2 *An open economy is solvent in the long run if its intertemporal budget constraint*

[13] With a little abuse of notation I use L here to denote for the lag operator.

[14] The matrices $\{\boldsymbol{C}_{z,i},\ i = 0, 1, 2, ...\}$ are absolute summable if $\{[\mathrm{Trace}(\boldsymbol{C}_{z,i}\boldsymbol{C}'_{z,i})]^{1/2}, i = 0, 1, 2, ...\}$ are absolute summable.

(15) is satisfied in the steady state. Mathematically this is expressed as

$$\lim_{h\to\infty} E_t[nx_{t+h} + \rho_{a,t+h}a_{t-1+h} - \rho_{l,t+h}l_{t-1+h} - \Delta(a_{t+h} - l_{t+h})] = 0 \qquad (21)$$

where $\rho_{a,t}$ and $\rho_{l,t}$ are defined in equation (15).

A country is expected to be able to finance its external imbalance in the infinite future if it is believed to remain solvent. The conditional expectation suggests again sustainability is defined relative to the information set available. But the idea underlying equation (21) is general in nature, and could be readily applied to any solvency problem for either an individual or an economy as a whole. A test of equation (21) is therefore a formal and important assessment for the sustainability of external imbalances.

3.1 Simplifying the Long-Run Solvency Condition

It is difficult to test condition (21) directly because expectations are not observable. Moreover the intertemporal budget constraint (15) is not linear because of the products of two variables, i.e. $\rho_{a,t} \cdot a_{t-1}$ and $\rho_{l,t} \cdot l_{t-1}$. To conduct an empirical study we need to simplify the model. In particular we need to consider the stochastic properties of various variables that enter equation (21) and the nonlinear structure.

First, it is very likely that exports (x_t),imports (m_t),assets (a_t), and liabilities (l_t) follow nonstationary processes. There could be deterministic trends, but more important is the presence of unit-root components. If not external imbalances should be mean reverting, so that the open economy can adjust to new balance quickly. However we have not observed such adjustments in the past decades in the US and a number of emerging economies. In fact the trade volume, the stocks of assets and liabilities are growing much faster than output;[15] and the imbalances in trades and in net foreign assets positions have been worsening. Such development is largely due to the structural changes stemming from increasing global integration in trade and in financial markets. Trade flows are spurred by reduced tariffs and trade barriers, declining transport costs, increasing use of telecommunication, and the development of multinational companies. The gross holdings of assets and liabilities have more than tripled as a share of GDP[16] because of progressing financial innovation and liberalization on capital control. Overall international interdependence has grown significantly among industrial economies and among developing economies. We regard $I(1)$ process to be a convenient way to capture these structural changes. Thus it is not unreasonable to propose the following assumption.

Assumption 1 *$x_t, m_t, a_t,$ and l_t are $I(1)$ processes which could possibly contain linear deterministic trends.*

[15] See IMF World Economic Outlook, and Lane and Melisi-Ferriti (2006).
[16] see Lane and Melisi-Ferriti 2006

For simplicity and practical consideration we rule out the possibility of $I(2)$ processes in the study. So the first difference of net foreign assets position is stationary, that is

$$\lim_{h\to\infty} E_t[\Delta(a_{t+h} - l_{t+h})] = \alpha_{na} \ ,$$

where α_{na} is a constant. Two remaining terms $\rho_{a,t+h}$ and $\rho_{l,t+h}$ are determined by real growth rate in domestic wealth, real rates of returns on assets and liabilities, and the percentage change in real exchange rate. There is a consensus, at least to a certain extent, about the properties of these variables both in economic and statistical terms.

Follow the literature in economic growth we assume that the real growth rate of wealth follows a stationary process. Secondly we assume the marginal return to capital is fixed along the balanced growth path following the neoclassical growth model (Solow, 1956). This is consistent with the stylized findings in Kaldor (1961). Binder and Pesaran (1999) consider a general setup of stochastic growth, and they show that even when output in level contains unit-root, capital per effective labour will converge to a steady-state probability distribution where the limit of capital-per-effective-labour is a time-invariant random variable with non-degenerate probability distribution function. Note that the aggregate real returns on assets is largely driven by the marginal return to capital[17] therefore it is not unrealistic to assume that r_t and r_t^* are stationary. Finally we assume relative Purchasing Power Parity (PPP) to hold in the steady state. The rationale is the Law of One Price. But we do not need a strong assumption of absolute PPP being satisfied in the long run. There could be persistent departure from absolute PPP due to transportation cost, trade barriers and tariffs, non-tradable goods, and productivity differentials (the Harrod-Balassa-Samuelson effect). Instead we assume a much weaker form of PPP which states that the growth rate of real exchange $g_{s,t}$ is stationary. This implies that inflation differential between domestic and foreign economies is reflected in the percentage change in nominal exchange rate. Empirical studies on relative PPP provide less controversial conclusion in the long run (Taylor and Taylor 2004). In sum we propose the following assumptions:

Assumption 2 *The real growth rate of wealth, the real rates of portfolio return and the percentage change in real exchange rate are stationary:*

$$g^p = \lim_{h\to\infty} E_t[g_{t+h}] \ , \tag{22}$$

$$r^p = \lim_{h\to\infty} E_t[r_{t+h}] \ , \tag{23}$$

$$r^{*p} = \lim_{h\to\infty} E_t[r^*_{t+h}] \ , \tag{24}$$

$$g^p_s = \lim_{h\to\infty} E_t[g_{s,t+h}] \ , \tag{25}$$

where $g_{s,t+h} = \triangle s_{t+h}/s_{t+h-1}$.

Assumption 2 effectively ensures $\rho_{a,t+h}$ and $\rho_{l,t+h}$ are stationary. So in the steady state they

[17] Under standard profit maximization the real rate of return will be equal to the marginal return to physical capital.

are constant.

$$\rho_a^p = \lim_{h\to\infty} E_t[\rho_{a,t+h}] \tag{26}$$

$$\rho_l^p = \lim_{h\to\infty} E_t[\rho_{l,t+h}] \ , \tag{27}$$

This is important because now we can simplify the nonlinear products involving an $I(0)$ and an $I(1)$ processes in condition (21). But it should be emphasized that $\rho_{a,t}$ and $\rho_{l,t}$ will not necessarily be equal to each other even in the steady state. Although returns to physical capital in different countries may converge, it does not require ρ_a^p to be equal to ρ_l^p in the long run at least for two reasons. The composition of international investment portfolio may vary considerably across different countries in terms of currencies and riskiness; the returns on different government bonds are not necessarily identical because of differences in credit worthness. We will see later that only when domestic and foreign assets are perfect substitute in the steady state will ρ_a^p be equal to ρ_l^p.

Finally consider the steady state values of the products involving an $I(0)$ and an $I(1)$ variables, i.e. $\rho_{a,t+h} \cdot a_{t-1+h}$ and $\rho_{l,t+h} \cdot l_{t-1+h}$. Following the elementary property that $E(wv) = E(w)E(v) + cov(w, v)$ for any stochastic variables w and v, and notice we have already specified the infinite-horizon forecasts for the first moments, the remaining question is what would be values of

$$\lim_{h\to\infty} cov_t[\rho_{a,t+h}, a_{t-1+h}] \text{ and } \lim_{h\to\infty} cov_t[\rho_{l,t+h}, l_{t-1+h}] \ .$$

It turns out that under some mild conditions the infinite-horizon forecast of the second moment involving an $I(0)$ and an $I(1)$ variables is a constant. Formally:

Proposition 3 *Suppose w_t is a scalar $I(0)$ process, and v_t is a scalar $I(1)$ process such that*

$$w_t = \alpha_w + c_w(L)\epsilon_t$$

$$\Delta v_t = \alpha_v + c_v(L)\xi_t \ ,$$

where α_w and α_v are the unconditional means of w_t and Δv_t respectively; ϵ_t and ξ_t are white noise processes with zero means; $c_i(L) = c_{i,0} + c_{i,1}L + c_{i,2}L^2 + ...$ and $\sum_{j=0}^{\infty} |c_{i,j}| < \infty$ for $i = w, v$.

If $E_{t-1}[\epsilon_t\xi_t] = \sigma_{\epsilon\xi}$ is a finite constant, and $E_{j-1}[\epsilon_t\xi_s] = 0$ for $t \neq s$ and $j = \min\{s, t\}$ then

$$\lim_{h\to\infty} cov_t[w_{t+h}, v_{t+h}] = \sigma_{wv} \ ,$$

where $|\sigma_{wv}| < \infty$, and $cov_t[\cdot, \cdot]$ denotes the covariance conditional on information at time t.

Proof. See appendix A. ■

Following proposition 3 we have the following simplification:

Assumption 3

$$\lim_{h\to\infty} \{cov_t[\rho_{a,t+h}, a_{t-1+h}] - cov_t[\rho_{l,t+h}, l_{t-1+h}]\} = \theta, \tag{28}$$

where θ is a finite constant.

We argue equation (28) is not too strong as an assumption since we have allowed for contemporaneous correlation between the innovations to an $I(0)$ and $I(1)$ processes. Now we can propose the key result of the paper - the simplified long-run solvency condition.

Proposition 4 *Under assumptions 1-3, if a co-trending condition is satisfied such that*

$$g_{nx} + \rho_a^p \cdot g_a - \rho_l^p \cdot g_l = 0, \tag{29}$$

where g_z is the constant deterministic growth rate of z_t for $z = nx$, a, and l; and ρ_a^p and ρ_l^p are the steady state values of $\rho_{a,t+h}$ and $\rho_{l,t+h}$ respectively, then a sustainable economy satisfies a long-run solvency condition given by

$$nx_t^p + \rho_a^p \cdot a_t^p - \rho_l^p \cdot l_t^p = \alpha \ , \tag{30}$$

where the permanent components are defined by the Beveridge-Nelson trend (definition 1), and α is a constant.

Specially if net export ($nx_t = x_t - m_t$) is stationary, then the long-run solvency condition (30) can be simplified as two long-run relations:

$$x_t^p - m_t^p = \alpha_{nx}, \ \text{and} \tag{31}$$

$$a_t^p - \lambda l_t^p = \alpha_{al} \ , \tag{32}$$

where $\lambda = \frac{\rho_l^p}{\rho_a^p}, 0 < |\lambda| < \infty$, α_{nx} and α_{al} are constants.

Proof. see appendix B. ■

We label equations (31) and (32) as the key stock-flow equilibrium. It is derived from the general long-run solvency condition (21) under certain mild assumptions. Intuitively, condition (29) is the equilibrium among the deterministic trend component of the variables in the budget constraint; and condition (30) is the equilibrium among the stochastic trend components. Solvency requires that the deterministic trends and the stochastic trends should match respectively.

Because ρ_a^p and ρ_l^p are constant so equation (30) is now linear. A more informative insight could be obtained if we rearrange equation (30) as follow:

$$na_t^p \approx -[nx_t^p + (\rho_a^p - \rho_l^p)a_t^p]/\rho_l^p \ , \tag{33}$$

where $na_t^p = a_t^p - l_t^p$, and the constant α is omitted. It suggests that in the steady state the imbalance in net foreign assets can be sustained by the flow of net export nx_t^p and the valuation effect which is the relative return differential $(\rho_a^p - \rho_l^p)$ times the gross assets position a_t^p.

We emphasize that ρ_a^p may not be equal to ρ_l^p, which is one of the major findings distinguish our research from the early literature. By definition

$$\rho_a^p - \rho_l^p = \lim_{h\to\infty} E_t\{\frac{(1+r^*_{t+h})(1+g_{s,t+h}) - (1+r_{t+h})}{1+g_{t+h}}\} .$$

Thus the gap between ρ_a^p and ρ_l^p is essentially the return differential between domestic and foreign assets, taking into account the fluctuation in real exchange rate. Early studies usually impose arbitrage-free condition, at least in the long-run, resulting in equal real returns from different investments. In the present context, arbitrage in the international capital market where there is perfect substitutability between domestic and foreign assets will ensure $\rho_a^p = \rho_l^p$. The rationale follows the Uncovered Interest Parity (UIP) hypothesis with a flexibility in recognizing various short-run departures, such as trading noise, asymmetric information and transaction costs.

However it is questionable whether UIP may actually hold. Studies have shown that the US economy has long enjoyed a favourable earning advantage on its foreign assets relative to its payment on liabilities (Gourinchas and Rey, 2005). Although it has transformed from net creditor into net debtor in the mid 1980s and its net assets position has been worsened ever since, the US is able to maintain a generally positive net international investment income (Obstfeld and Rogoff, 2005). This might be largely due to two factors. First, many central banks are holding a huge amount of US treasury bills and dollar reserves, especially in the emerging markets and the oil-producing economies. These assets generate very low or even negative real returns. In 2003, for example, only 38% of the U.S. liabilities are in the form of equity (both portfolio equity and foreign direct investment). But in contrast Americans hold 60% of their gross foreign assets in the form of equities and foreign direct investment. Second, the US dollar remains as the world's main foreign reserve and vehicle currency in international trade, even when the inflationary and opportunities cost attached to dollar reserves are considerably large. These stylized facts suggest there is more than what arbitrage equilibrium could capture and it is important to examine the hypothesis carefully. We therefore do not impose arbitrage-free condition in the model. Instead we allow for a gab between ρ_a^p and ρ_l^p to capture the current status of the international monetary order. This gap is reflected in the value of λ in the stock-flow equilibrium (32). Recall $\lambda = \frac{\rho_l^p}{\rho_a^p}$, so a value of λ different from one is against the arbitrage-free condition in the steady state.

Allowing for non-zero return differential we can incorporate the valuation effect in the long-run external adjustments. The second term on the right hand side of equation (33) captures the valuation channel emphasized by Blanchard, Giavazzi and Sa (2005) and Gourinchas and Rey (2007). These papers introduce valuation effect in a short-run framework. But our model shows that the valuation term may also be important in the long run. Once if the domestic and foreign assets are not perfect substitutes the solvency problem becomes asymmetric. Consequently, it is not the net position but the gross positions that matter. A country can run net debt for ever as long as it has favourable excess return from assets over liabilities, which is essentially what equation (32) is saying. We interpret the persistent return differential as some sort of seigniorage power analogous to the power of a government in the public finance who can hold net debt forever because its ability to print money. Gourinchas and Rey (2005) use the term 'exorbitant privilege' to describe the special status of the US in the international monetary order.

But how far would such special status of the US continue in the future? Consider central banks in Asia and Middle-East who have huge dollar reserves that are subject to sizable capital loss due to depreciating dollar value. It is reasonable to diversify the foreign reserves towards other currencies, such as the Euro. Sensitive investors may trigger or aggravate this shift by massive withdrawal of their US assets when they perceive the potential risks have been accumulated to certain level. This could generate a great concern about the potential collapse of the dollar, which in turn aggravates the incentives to shift from dollar denoted assets. It seems that the relative earning advantage of the US investors could not last forever. On the other hand, a benign perspective proposed by Dooley, Landau and Garber (2004) argues that the developing economies following export-led growth strategy will not easily shift from dollar reserves until they mature into the central economy. This so-called revised Bretton Woods System hypothesis suggests that the unique status of the US economy as the world financial centre and the supplier of international liquidity will last for a much longer time than some pessimistic predictions. Given the unresolved debates over the special status of the US economy, an empirical study that is able to provide some evidence to distinguish between different theories is useful. We argue that our general model method a coherent framework for testing long-run solvency, and the key parameter λ have important implications for the international monetary order.

3.2 Cointegrating Relations

Following reasonable simplifications we derive the stock-flow equilibrium (31)-(32) as the long-run solvency conditions. But these two relations are not directly testable because we do not observe expectation and the permanent components. To estimate the stock-flow equilibrium we need the following proposition which states that the linear relation of the permanent components is equivalent to the cointegrating relation of the levels of the time series.

Proposition 5 *Suppose each element in a $m \times 1$ stochastic vector $\mathbf{z}_t$ is $I(1)$. A linear relation of the permanent component of $\mathbf{z}_t$ captured by*

$$\boldsymbol{\beta}' \mathbf{z}_t^p = 0 \ , \tag{34}$$

where vector $\boldsymbol{\beta} \neq \mathbf{0}$ is equivalent to the cointegrating relation given by

$$\boldsymbol{\beta}' \mathbf{z}_t \sim I(0) \ , \tag{35}$$

and $\boldsymbol{\beta}$ is the cointegrating vector. Here the permanent component is defined by the Beveridge-Nelson stochastic trend.

Proof. See appendix. ■

Follow proposition 5 the long-run solvency conditions (32) for an open economy are given by

$$x_t - m_t = \alpha_{nx} + \zeta_{1,t} \sim I(0), \text{ and} \tag{36}$$

$$a_t - \lambda \cdot l_t = \alpha_{al} + \zeta_{2,t} \sim I(0) \tag{37}$$

where the stationary zero mean disturbances ($\zeta_{1,t}$ and $\zeta_{2,t}$) represents short-run departure from the long-run equilibrium. Parameter $\lambda = \frac{\rho_l^p}{\rho_a^p}$ can be estimated and will provide important implication on the relative return in the steady state. A value significantly different from one is evidence against perfect substitutability between domestic and foreign assets.

We want to emphasize that the cointegrating relations (36)-(37) would imply long-run solvency only under the condition that the co-trending restriction (29) is satisfied. A careful empirical study should examine all the hypotheses under the theory and we think these questions are better analyzed within a cointegrating-VAR framework. The likelihood-based cointegrating analysis is able to test the number of long-run relations. The co-trending restriction can be tested before it can be confidently imposed into the model. The identification problem and the economic theory could be investigated by the over-identifying restrictions test. A brief summary below may suffice to illustrate the econometrics technique.

Suppose the data generating process for a $m \times 1$ vector $\mathbf{z}_t$ is given by a VAR(p) model as below:

$$\mathbf{z}_t = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \boldsymbol{t} + \boldsymbol{\Phi}_1 \mathbf{z}_{t-1} + ... + \boldsymbol{\Phi}_p \mathbf{z}_{t-p} + \boldsymbol{u}_t \ , \tag{38}$$

where $\boldsymbol{u}_t$ is a vector of serial uncorrelated disturbances with zero mean. It can be reparameterised as a Vector Error Correction Model (VECM):

$$\boldsymbol{\Delta}\mathbf{z}_t = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 t - \boldsymbol{\Pi}\mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \boldsymbol{\Delta}\mathbf{z}_{t-i} + \boldsymbol{u}_t \ , \tag{39}$$

where

$$\boldsymbol{\Pi} = \boldsymbol{I}_m - \sum_{i=1}^{p} \boldsymbol{\Phi}_i, \ \boldsymbol{\Gamma}_i = - \sum_{j=i+1}^{p} \boldsymbol{\Phi}_j, i = 1, ..., p-1.$$

Usually the deterministic components $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ are assumed to satisfy certain restrictions to ensure desired properties in the VAR model (38), such as a linear deterministic trend.[18] If the elements of $\mathbf{z}_t$ are $I(0)$, $\boldsymbol{\Pi}$ will be full rank; if the elements of $\mathbf{z}_t$ are $I(1)$ but not cointegrated then it must be that $\boldsymbol{\Pi} = \mathbf{0}$. The interesting situation is when the elements of $\mathbf{z}_t$ are $I(1)$ and cointegrated in r independent cointegrating relations, then we have deficient rank, i. e. rank($\boldsymbol{\Pi}$)$= r < m$. The number of cointegrations can be tested by likelihood-based method developed by Johansen (1988,1991), i.e. the 'trace' statistic and the 'maximum eigenvalue' statistic. The distributions of these two statistics are not standard because of the presence of unit-root; and they depend crucially on the structure of the deterministic components - the intercept and the trend. For example, if there is a deterministic trend in the VECM, we may restrict the trend to be within the cointegrating relation so as to avoid quadratic trend in the

[18]see Juselius (2006) Ch6 for detailed discussion.

level of $\mathbf{z}_t$. Critical values of the 'trace' and 'maximum eigenvalue' statistics corresponding to different specifications of VECM are simulated by Monte Carlo method, and reported in Johansen (1991) and Pesaran, Shin and Smith (2000).

Under rank deficiency $\mathbf{\Pi}$ can be decomposed as $\mathbf{\Pi} = -\boldsymbol{\alpha}_z\boldsymbol{\beta}'_z$ in which $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ are $(m \times r)$ full rank matrices. So the cointegrating relations can be written as $\boldsymbol{\beta}'_z\mathbf{z}_t = \boldsymbol{\xi}_{zt}$ where stationary disturbances $\boldsymbol{\xi}_{zt}$ are the short-run deviations from the long-run equilibrium. However $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ are not separately identifiable because for any non-singular matrix $\boldsymbol{Q}$, we have $\mathbf{\Pi} = -\boldsymbol{\alpha}_z\mathbf{Q}\mathbf{Q}^{-1}\boldsymbol{\beta}'_z$. The coefficient matrices $\boldsymbol{\alpha}^*_z = \boldsymbol{\alpha}_z\boldsymbol{Q}$ and $\boldsymbol{\beta}'^*_z = \boldsymbol{Q}^{-1}\boldsymbol{\beta}'_z$ would be observationally equivalent to $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ respectively. To identify the long-run relations we need to impose a total of $r \times r$ exactly identifying restrictions, r of which are normalizations. Economics theory can then be tested by the over-identifying restrictions. In the present context, let $\mathbf{z}_t = (x_t, m_t, a_t, l_t)'$ and we may model the long-run solvency conditions (36)-(37) in a general VECM as follow:

$$\Delta\mathbf{z}_t = \boldsymbol{\alpha}_0 - \boldsymbol{\alpha}_z\boldsymbol{\beta}'(\mathbf{z}_{t-1} - \mathbf{g}_z(t-1)) + \sum_{i=1}^{p-1}\mathbf{\Gamma}_i\Delta\mathbf{z}_{t-i} + \boldsymbol{u}_t \ ,$$

where $\boldsymbol{u}_t$ is the vector of reduced form white noise errors with zero mean. The co-trending restriction is captured by $\boldsymbol{\beta}'\mathbf{g}_z = 0$ which could be tested by an over-identifying restriction.[19]

4 Empirical Tests on the Long-Run Solvency Conditions

As an application we study the US long-run solvency problem. There is a subtle issue on data. By definition, current account measures the change in a country's net foreign wealth. However, in practice this is not the case because national statistics, such as the National Income and Product Accounts and the Balance of Payments, usually reports current account in book value, which neglects capital gains stemming from assets prices and exchange rate movements[20]. A better estimate of change in net foreign wealth should keep track of the market value of gross assets and liabilities. Only until the 1980s did a number of national statistical agencies, such as the US Bureau of Economics Analysis, start to collect information on the gross assets and liabilities holdings at market value. Lane and Milesi-Ferretti (2006) provide annual estimates of gross assets and liabilities at market value for over 140 countries covering the period 1970-2004. Though the data on developing countries are less comprehensive, their study provides scholars a set of very useful annual estimates for understanding the global external imbalances. For the case of the US Gourinchas and Rey (2005, 2007) have constructed quarterly data on the gross foreign assets and liabilities positions at market value covering a much longer period of 1952Q1-2004Q1. This dataset is perfect for examining the long-run solvency conditions.

Table 2: the US data

Data	Description and Source	Notation in MFit
x_t	Export/Wealth; 1952Q1-2004Q1 Source: Gourinchas and Rey (2007)	XW
m_t	Import/Wealth; 1952Q1-2004Q1 Source: Gourinchas and Rey (2007)	MW
a_t	Gross Assets/Wealth; 1952Q1-2004Q1 Source: Gourinchas and Rey (2007)	AW
l_t	Gross Liabilities/Wealth; 1952Q1-2004Q1 Source: Gourinchas and Rey (2007)	LW
$\tilde{W}_t$	Household net worth (wealth); 1952Q1-2004Q1 Source: Flow of Funds Accounts	
$r_{a,t}$	Real total returns on foreign assets Source: Gourinchas and Rey (2005)	
$r_{l,t}$	Real total returns on domestic assets (liabilities) Source: Gourinchas and Rey (2005)	
$\rho_{a,t}$	$\rho_{a,t} = \frac{1+r_{a,t}}{1+g_{w,t}} - 1$	RHOA
$\rho_{l,t}$	$\rho_{l,t} = \frac{1+r_{l,t}}{1+g_{w,t}} - 1$	RHOL

4.1 The US Data

The time series used in the study are briefly summarized in Table 2. Gourinchas and Rey (2005, 2007) also provide estimates of the aggregate real returns $r_{a,t}$ on assets and $r_{l,t}$ on liabilities.[21] The aggregate real return is the sum of investment income plus the revaluation of the gross assets at the end of period. These are probably the best available estimates of the portfolio returns in gross assets and liabilities. We can thus construct a fairly good proxy for the values of $\rho_{a,t}$ and $\rho_{l,t}$.

A plot of the series may suffice to reveal the salient feature of the data. Figure 2 shows that the assets a_t and liabilities l_t are growing significantly, but with a higher growth rate in the liabilities roughly since 1978. Meanwhile the net export flow is fluctuating around zero before 1978 but soon deteriorates to deficit and remains negative since then (See Figure 4). Figure 2 also suggests unit-root properties in the assets and liabilities stocks, which are confirmed by the Augmented Dickey-Fuller (ADF) test reported in the appendix. It seems the net export flow may also follow an $I(1)$ process as it has deviated from the value of zero significantly. But there are strong arguments supporting comovement between exports and imports (See Figure 3). One important observation is that a country tends to import similar goods that they export. Moreover, demands shocks can be transmitted across countries through trades and financial interdependence. Here we do not take any *a priori* perspective and leave the long-run relation between exports and imports to be estimated by the VECM.

[19] See Pesaran, Shin and Smith (2000).
[20] see Gourinchas and Rey (2005)
[21] A detailed description of the data construction can be found in Gourinchas and Rey (2005).

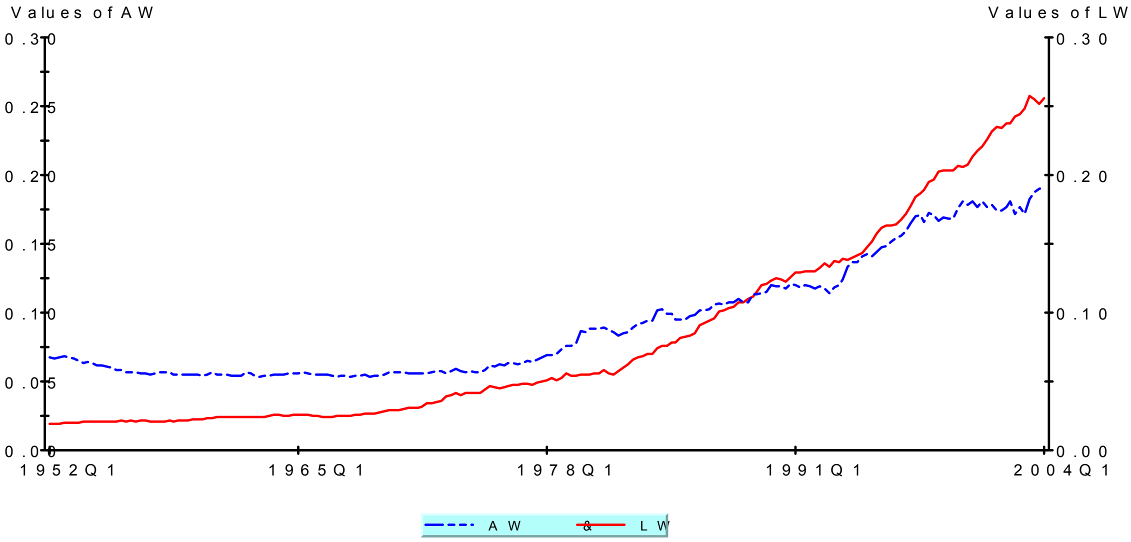

Figure 2: Ratios of Assets (AW) and Liabilities (LW) to Wealth

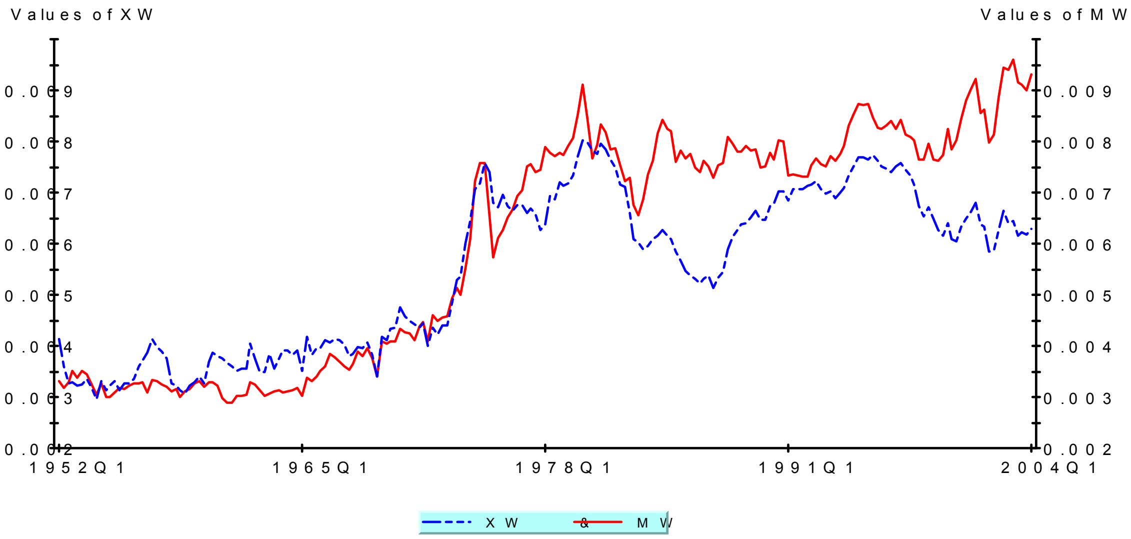

Figure 3: Ratios of Exports (XW) and Import (MW) to Wealth

Table 3: Unit-root tests on variables RHOA and RHOL, 1952Q3-2004Q1

Variable	DF	ADF(1)	ADF(2)	ADF(3)	ADF(4)
$\rho_{a,t}$	-14.4707	-9.3830	-7.1348	-5.5359	-5.5820
$\rho_{l,t}$	-13.4442	-9.3464	-7.9086	-4.9633	-4.6104

The Dickey-Fuller regressions include an intercept but not a trend. 95% critical value=-2.8759

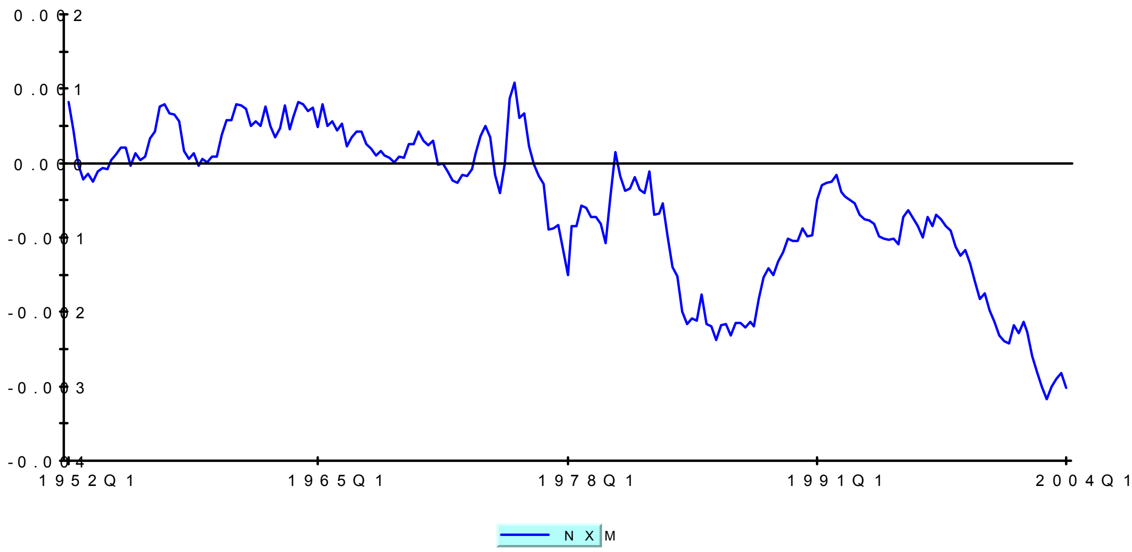

Figure 4: Ratio of net export to Wealth

Prior to estimation it is particularly important to examine Assumption 2. If $\rho_{a,t}$ and $\rho_{l,t}$ are not stationary their steady state values will not be constant, subsequently the long-run solvency conditions will not have a nice linear structure. Here we examine the constructed proxies of $\rho_{a,t}$ and $\rho_{l,t}$. Table 3 reports the ADF tests on the two series. Both are unambiguously stationary and the plots in the appendix confirm the stationarity. This simple exercise offers certain confidence on the linear long-run relation (30) and our modelling strategy which does not include $\rho_{a,t}$ and $\rho_{l,t}$ explicitly in the VECM.

4.2 Cointegration Analysis

The long-run solvency conditions include the co-trending restriction and two cointegrations below:

$$x_t - m_t = \alpha_{nx} + \zeta_{1,t} \sim I(0), \text{ and} \tag{40}$$

$$a_t - \lambda \cdot l_t = \alpha_{al} + \zeta_{2,t} \sim I(0), \tag{41}$$

where $\zeta_{1,t}$ and $\zeta_{2,t}$ are stationary stochastic disturbances with zero mean. These conditions could be neatly written as

$$\zeta_t = \boldsymbol{\beta}'\mathbf{z}_t - \alpha \ , \tag{42}$$

where $\mathbf{z}_t = (x_t, m_t, a_t, l_t)'$ and

$$\boldsymbol{\beta} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -\lambda \end{bmatrix}.$$

The co-trending restriction implies that there should be no deterministic trend component within the cointegration (42). But Figure 2 and 3 show that x_t, m_t, a_t, and l_t are all trending upwards over the sample period. One possible modelling strategy is to use VECM without deterministic trend. This is essentially imposing co-trending restriction on the model without testing it. However it is not easy to distinguish between a trend stationary process from a unit-root process, so the better way is to start from a general specification that allows for both deterministic and stochastic trends. We also want to test the co-trending restriction before we can comfortably impose it. Therefore we start from a VECM model with a restricted deterministic trend and unrestricted intercept. This ensures the deterministic trend component in the underlying VAR process is linear. Then the co-trending restriction can be tested by an over-identifying restriction on the cointegration. Specifically we embed the long-run relations into a general VECM given by

$$\Delta \mathbf{z}_t = \boldsymbol{\alpha}_0 - \boldsymbol{\alpha}_z \boldsymbol{\beta}_1' (\mathbf{z}_{t-1} - \mathbf{g}_z (t-1)) + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{z}_{t-i} + \boldsymbol{u}_t \ , \qquad (43)$$

where $\boldsymbol{u}_t$ is the vector of reduced form serially uncorrelated errors with zero means.[22] The co-trending restriction implies that $\boldsymbol{\beta}' \mathbf{g}_z = 0$. We follow the standard maximum likelihood estimation method developed by Johansen (1988, 1995) to estimate cointegrating-VAR model (43) and test the related hypotheses. These are done in MicroFit 5.0.[23]

Note that for the purpose of testing the number of cointegrating relations it is unnecessary to check whether all elements in vector $\mathbf{z}_t$ are $I(1)$ process before carrying out Johansen's tests on the cointegrating rank order, so long as there is no element in $\mathbf{z}_t$ of integrated order higher than one. The estimated rank order will tell how many stationary relations are there in the model. A scalar stationary process, if exists in $\mathbf{z}_t$, is interpreted as a reduced form cointegrating relation, which will be counted in the cointegrating rank order.

The first step is to specify the number of lag in the underlying unrestricted VAR. The number of lag should be selected properly so that the likelihood based tests of cointegration rank is valid. Usually this requires fitting an unrestricted VAR to the data where a large number of lag is selected to eliminate miss-specifications, especially serial correlation in the residuals. Garratt, Lee, Pesaran and Shin (2006) suggest that one may follow Akaike and Schwarz information criteria. Here we follow Garratt et al. (2006). Akaike information criterion (AIC) selects the lag order to be 6 or 7, while Schwarz information criterion selects only one lag. The conflicting results from different information criteria are not surprising because they impose

[22] The seasonal effects are not significant in our data. The likelihood-ratio tests suggest it's not necessary to include seasonal dummies in the VECM.

[23] MicroFit 5.0, developed by Pesaran, M.H. and Pesaran, B., 2008, forthcoming

Table 4: Cointegration Rank Tests, 1952Q1-2004Q1

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r = 0$	$r = 1$	33.2908	31.7900	29.1300
$r \leq 1$	$r = 2$	25.2668	25.4200	23.1000
$r \leq 2$	$r = 3$	18.5551	19.2200	17.1800
$r \leq 3$	$r = 4$	4.6613	12.3900	10.5500
(b) Trace Statistics				
$r = 0$	$r = 1$	81.7741	63.0000	59.1600
$r \leq 1$	$r = 2$	48.4832	42.3400	39.3400
$r \leq 2$	$r = 3$	23.2164	25.7700	23.0800
$r \leq 3$	$r = 4$	4.6613	12.3900	10.5500

Specification: VECM(7) with unrestricted intercept and restricted trend

different penalties methods. Lütkepohl and Saikkonen (1999) suggest that selecting a higher order lag length than the truth may result in an increase in the mean square error of forecasts, while under fitting the lag order may generate serial correlation. Given that the problem of underestimating the lag order is more serious than overfitting it, we proceed with 7 lags in the VECM.

We specify unrestricted intercept and restricted trend in the VECM(7) for the cointegrating analysis. MicroFit 5.0 reports Johansen's "trace" and "maximal eigenvalue" statistics with the associated 90% and 95% critical values. They are summarized in Table 4.

Both statistics suggest that there are two cointegrating relations among the elements in vector $\mathbf{z}_t$ so we proceed with $r = 2$. The exactly identified long-run relations are given below

$$1 \cdot x_t + \underset{(0.26101)}{-0.47149} m_t + 0 \cdot a_t + \underset{(0.0090916)}{-0.0019810} l_t + \underset{(0.2023E-4)}{-0.3295E-4} t = \hat{\alpha}_{xm} + \hat{\zeta}_{1,t} \ , \tag{44}$$

$$0 \cdot x_t + \underset{(1.8384)}{-0.46751} m_t + 1 \cdot a_t + \underset{(0.064272)}{-0.54551} l_t + \underset{(0.1444E-3)}{-0.6111E-4} t = \hat{\alpha}_{al} + \hat{\zeta}_{2,t} \ , \tag{45}$$

where figures in the parenthesis are the asymptotic standard errors.

The more economically meaningful long-run relations are obtained by imposing the over-identifying restrictions, i.e. the co-trending restriction $\boldsymbol{\beta}'\mathbf{g}_z = 0$ and the other restrictions on $\boldsymbol{\beta}$ given by equation (42). But these restrictions should be tested before they could be confidently accepted. We first leave the coefficients for import and liabilities as unspecified but impose the rest of the restrictions on the VECM. These restrictions are accepted by the log-likelihood ratio (LR) test using bootstrapping technique to generate the critical value in order to adjust for small sample size. So we obtain two long-run relations as follow.

$$x_t - \underset{(0.17467)}{0.67006} m_t = \hat{\alpha}_{xm} + \hat{\zeta}_{1,t} \ , \tag{46}$$

$$a_t - \underset{(0.079843)}{0.49255} \; l_t = \hat{\alpha}_{al} + \hat{\zeta}_{2,t} \; . \tag{47}$$

The coefficient of m_t in equation (46) is very closed to one. Actually, after imposing all the restrictions in (42) as the over-identifying restrictions we obtain two long-run relations given as:

$$x_t - m_t = \hat{\alpha}_{xm} + \hat{\zeta}_{1,t} \; , \tag{48}$$

$$a_t - \underset{(0.052612)}{0.55578} \; l_t = \hat{\alpha}_{al} + \hat{\zeta}_{2,t} \; . \tag{49}$$

The LR statistic for all the over-identifying restrictions[24] is 13.0149 while the bootstrap critical value at 95% significance level is 21.0133. Thus the co-trending restriction is satisfied and the estimated long-run relations are consistent with our theory. These suggest the long-run solvency conditions are satisfied in the US over the period of 1952-2004.

The estimate for λ is 0.556. A further restriction that $\lambda = 1$ is rejected. Recall that $\lambda = \frac{\rho_l^p}{\rho_a^p}$, so $\lambda \neq 1$ implies that there is imperfect substitutability between domestic and foreign assets. Moreover, the magnitude of the estimate is meaningful. Notice $\hat{\lambda} = 0.556 < 1$ implies $\rho_a^p > \rho_l^p$, thus the US is enjoying a greater returns on its foreign assets compared to the payment on its liabilities. This is exactly what Gourinchas and Rey (2005) suggested in their empirical study where they constructed the aggregate real rates of return for the US assets and liabilities.

A robustness test to our analysis is to use the constructed proxies of $\rho_{a,t}$ and $\rho_{l,t}$ to calculate an estimate of λ. The sample means are $\bar{\rho}_a = 0.007849$ and $\bar{\rho}_l = 0.002605$, implying that $\bar{\lambda} = 0.3319$. Notice $\bar{\lambda}$ is of the similar magnitude as the estimates from VECM models above. Especially it is positive and significantly less than one.

How may we interpret the results above? The key message is that the US external budget constraint is satisfied in the long-run and we have evidence against the hypothesis of perfect substitutability or arbitrage-free condition in the international capital market. The systematic premium on the US earning may reflect the higher risk it bears in its investment. But more importantly, it highlights the unique status of the US which is the financial centre of the world economy. Its ability to supply worldwide accepted currency for trades and foreign reserves is pretty much similar to the seigniorage power of a government. The central role in the international monetary order has entitled the US the 'exorbitant privilege' so that it can borrow cheaply to finance its mounting deficits. But there is one difficulty in this explanation looking at the period before the collapse of Bretton Woods system. The US actually had surplus during the early days and had only developed mild deficits before the 1980s. Given these facts it is difficult to imagine the US would need 'easy' borrowing to sustain its external position in the early period. Possibly there may be a structural break and thus we carry out careful structural stability tests to examine this idea.

[24] including the co-trending restriction

4.3 Structural Stability Tests on Long-run Relations

A careful review on the plots of data also suggests that there may be a structural break around 1978-1984. It is clear that a_t and l_t start to increase more rapidly since 1978 roughly. Before 1978 the net export is fluctuating around or slightly above zero, but it turns to and remains negative afterwards. It is attempting to conceive that the international economy has entered a different regime from 1980s onwards where trades and financial interdependence have accelerated. Our main interest is therefore to examine the structural stability of the long-run relations, namely equations (48) and (49). Section 4.2 concludes that the US is enjoying easy borrowing from the rest of the world to finance its debt. But it is unclear when did the US start to run this 'strategy'. It is also questionable whether it is necessary for the US to rely on this strategy in the early periods of 1950s-1970s when its deficit was little.

We first consider a recursive test of fixed cointegrating vectors[25]. The null hypothesis is that the long-run relations captured by matrix $\boldsymbol{\beta}$ are constant over a reference period which in our case is the full sample period. This reflects the underlying assumption in the previous section that there is a unique parameter regime over the full sample. We use the recursive test to detect whether there is a structural break and when.

In the first step we obtain the full-sample estimate of the cointegrating matrix given by

$$\hat{\boldsymbol{\beta}}_0 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -0.55578 \\ 0 & 0 \end{bmatrix}$$

in which the first column corresponds to equation (48) and the second corresponds to equation (49). The last row of $\hat{\boldsymbol{\beta}}_0$ represents the co-trending restriction. We use $\hat{\boldsymbol{\beta}}_0$ as the reference value to test the alternative hypothesis that there is a structural break in long-run relations around 1977-1985. We focus on structural break in this specific period because of hints from the plots of data, and because of the consideration that we should have subsamples covering a long-enough period for testing long-run relations. The second step is to test whether the reference value $\hat{\boldsymbol{\beta}}_0$ is in the space spanned by $\hat{\boldsymbol{\beta}}_{t_i}$, where $\hat{\boldsymbol{\beta}}_{t_i}$ is the estimate of $\boldsymbol{\beta}$ based on the sample $1, 2, ..., t_i$; for $t_i = 1977Q1, 1977Q2, ..., 1985Q4$.For each subsample we specify VECM(7) with unrestricted intercept and restricted trend. In short, the recursive test can be summarized as testing

$$H_0 : \hat{\boldsymbol{\beta}}_0 \in sp(\hat{\boldsymbol{\beta}}_{t_i}),\ t_i = 1977Q1, 1977Q2, ..., 1985Q4$$

against

$$H_1 : \hat{\boldsymbol{\beta}}_0 \notin sp(\hat{\boldsymbol{\beta}}_{t_i}),\ t_i = 1977Q1, 1977Q2, ..., 1985Q4$$

This can be carried out by the likelihood ratio test of over-identifying restrictions for each subsample, where the test statistic is asymptotically distributed as χ^2 with 6 degrees of freedom.

[25] See Juselius, 2006, Ch9.2

We also use the bootstrap critical value at the 5% significance level to take into account small sample size. The result is given in the following figure where the test values have been scaled by the 95% quantiles of the $\chi^2(6)$ and the bootstrap simulation respectively. A value larger than 1.0 means rejection of the null hypothesis.

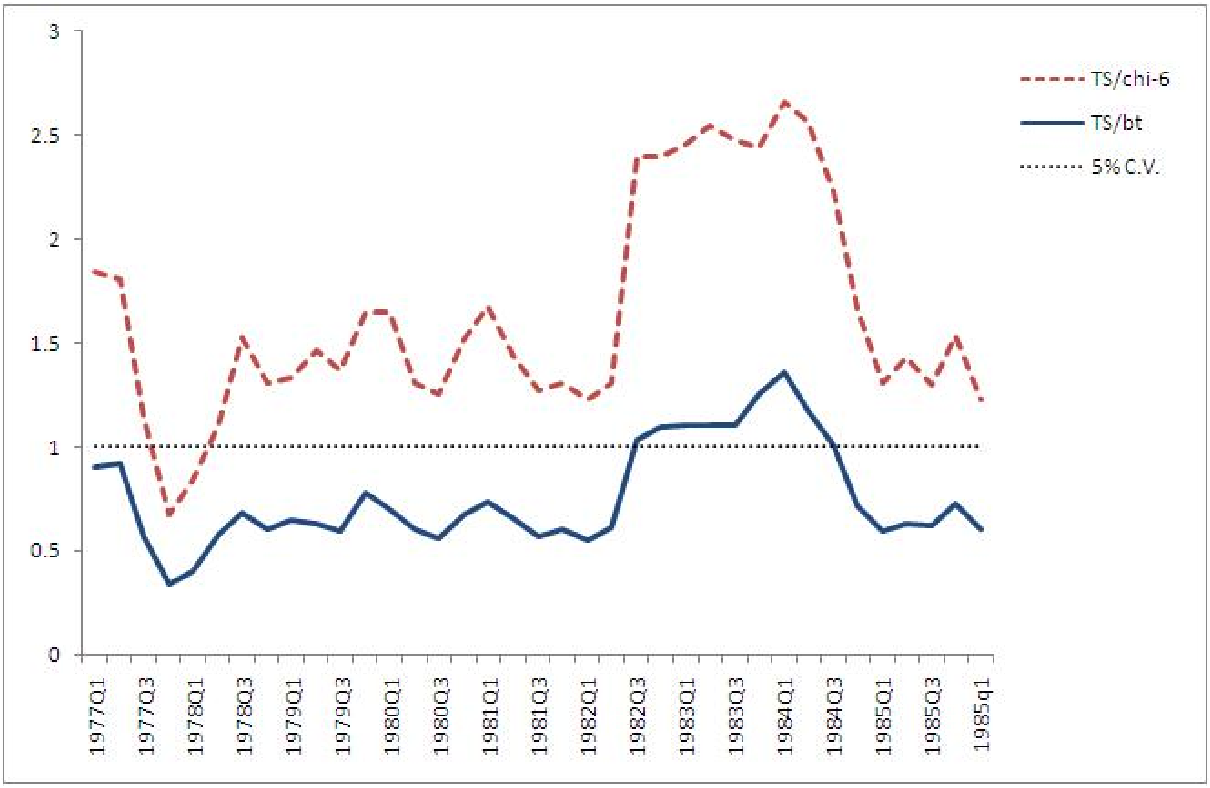

Figure 5: Recursive Test on Structural Stability. (A value greater than one indicates structural break.)

The dash-line, which is the test statistics scaled by the 95% quantile of $\chi^2(6)$, rejects constant cointegrating matrix nearly at each point of our recursive test. But after adjusting for the small sample size by bootstrap technique, the solid line suggests there is more likely a structural break around 1982Q3-1984Q3. We conclude that there is a structural break in long-run relations around 1982Q3-1984Q3.

Notice that the rejection of the null hypothesis of constant cointegrating matrix could be due to two potential reasons. It could be due to a violation of the long-run solvency conditions; or it could be due to a parameter shift in λ. Clearly the implications are different for each case, and we need a careful study of what actually cause the structural break. To examine formally we split the whole sample into two subsamples: $\{1952Q1-1982Q3\}$ and $\{1982Q4-2004Q4\}$[26]. Then we repeat the same cointegrating analysis using VECM model to verify whether the long-run solvency conditions are met in each subsamples, and to examine the estimates of λ. We should point out that for each subsample we specify different number of lags in the VECM

[26] In the robustness tests we also consider other different break points around 1977-1985 and we obtain similar results.

Table 5: Cointegration Rank Tests for Subsample 1: 1952Q1-1982Q3

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r = 0$	$r = 1$	39.7442	31.7900	29.1300
$r \leq 1$	$r = 2$	28.4707	25.4200	23.1000
$r \leq 2$	$r = 3$	17.3769	19.2200	17.1800
$r \leq 3$	$r = 4$	5.3597	12.3900	10.5500
(b) Trace Statistics				
$r = 0$	$r = 1$	90.9516	63.0000	59.1600
$r \leq 1$	$r = 2$	51.2073	42.3400	39.3400
$r \leq 2$	$r = 3$	22.7366	25.7700	23.0800
$r \leq 3$	$r = 4$	5.3597	12.3900	10.5500

Specification: VECM(7) with unrestricted intercept and restricted trend

based on Akaike Information Criterion (AIC). This allows the flexibility of changing short-run dynamics which should not affect the estimation of cointegration.

4.3.1 Subsample 1: 1952Q1-1982Q3

Following AIC we fit a VECM(7) of the form (43) to the data. The maximum eigenvalue test and trace statistics in Table 5 show there are two cointegrating relations.

The long-run solvency conditions are satisfied, as confirmed by the over-identifying restrictions tests. The resulting long-run relations are given by

$$x_t - m_t = \hat{\alpha}^1_{xm} + \hat{\zeta}^1_{1,t} \ , \tag{50}$$

$$a_t - \underset{(0.58866)}{1.3642} \ l_t = \hat{\alpha}^1_{al} + \hat{\zeta}^1_{2,t} \ , \tag{51}$$

in which $\hat{\lambda}_1 = 1.3642$ is close to one. Actually an over-identifying restriction that $\hat{\lambda}_1 = 1$ cannot be rejected using the bootstrap critical value. However, $\hat{\lambda}_1$ is significantly different from the full-sample estimate (equation (49)). This is confirmed by the rejection of imposing $\hat{\lambda}_1 = \hat{\lambda} = 0.556$. Therefore we conclude that in the first subsample the long-run solvency conditions are met with $\hat{\lambda}_1 = 1$.

The implications are interesting. While the long-run solvency is met in this subsample, we find a significantly different estimate of λ. The estimate $\hat{\lambda}_1 = 1$ implies equal real returns on domestic and foreign assets during this period. It confirms our intuition that it is not necessary for the US to obtain systematic earning advantage in the early period especially before the collapse of the Bretton Woods system. There seems to have perfect substitutability between domestic and foreign assets which can be interpreted as the result of arbitrage. A subsequent question is what happens from 1983 onwards. Is it possible that there might be a shift in λ?

Table 6: Cointegration Rank Tests for Subsample 2: 1982Q4-2004Q1

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r = 0$	$r = 1$	47.5993	31.7900	29.1300
$r \leq 1$	$r = 2$	24.3647	25.4200	23.1000
$r \leq 2$	$r = 3$	12.5250	19.2200	17.1800
$r \leq 3$	$r = 4$	9.2972	12.3900	10.5500
(b) Trace Statistics				
$r = 0$	$r = 1$	90.4195	63.0000	59.1600
$r \leq 1$	$r = 2$	42.8202	42.3400	39.3400
$r \leq 2$	$r = 3$	18.4555	25.7700	23.0800
$r \leq 3$	$r = 4$	9.2972	12.3900	10.5500

Specification: VECM(7) with unrestricted intercept and restricted trend

4.3.2 Subsample 2: 1982Q4-2004Q1

Following AIC we fit VECM(7) to the data. Table 6 shows that the maximum eigenvalue test and trace statistics confirm there are two cointegrations. The over-identifying restrictions test confirms the long-run solvency is satisfied, and the estimated cointegrations are given by:

$$x_t - m_t = \hat{\alpha}^2_{xm} + \hat{\zeta}^2_{1,t} \ , \tag{52}$$

$$a_t - \underset{(0.030704)}{0.50603} \ l_t = \hat{\alpha}^2_{al} + \hat{\zeta}^2_{2,t} \ , \tag{53}$$

Interestingly, $\hat{\lambda}_2 = 0.50603$ is significantly different from $\hat{\lambda}_1 = 1$, implying imperfect substitutability between domestic and foreign assets in this subsample. $\hat{\lambda}_2 < 1$ reveals that the US is enjoying an excess return of gross assets over gross liabilities.

From the analysis above we conclude that it is a shift in λ that causes a structural break in the cointegrating matrix. Long-run solvency are satisfied in both subsamples, but the relative returns of domestic assets versus foreign assets are different. Our analysis shows from 1983 onwards the favorable return premium has become systematic and increasingly important in financing the US' debt. Gourinchas and Rey (2005) show that the US is enjoying a sizeable excess return of gross assets over gross liabilities, and the excess return has increased after the collapse of the Bretton Woods system. Our analysis supports this view, and is consistent with the stylized fact that the US is investing more in high returns and risky assets compared to the rest of the world.

By questioning the hypothesis of perfect substitutability between domestic and foreign assets we seem to have uncovered the "true" long-run relations for the US external imbalance. Despite its tremendous liabilities accumulated the US is able to maintain solvent because the favourable return premium it enjoys. It may be safe to say that it is the willingness on the rest of the world to hold dollar denoted assets even at the cost of low returns that keeps the US external imbalance sustainable. We interpret the unique status of the US as a kind of seigniorage power. This also suggests strategical accumulation of the US treasury bills and the dollar, as the result of the export-led growth policy for example, may have a bigger role than what conventional perspective would perceive. Another critical implication is that the so-called valuation effect is not only important in the short run but also in the long run, as long as there is systematic return differential. The early studies which focus on the net asset position alone are misled.

Our finding is more in line with the Revised Bretton Woods System hypothesis[27] which states that the unique status of the US being the financial centre of the world will last for a very long time. The key is that there are sufficiently large number of countries who are currently following or about to follow the export-led growth strategy supported by undervalued currency, capital control and official outflows in the form of accumulation of reserve asset claims on the centre country- the US.

However it certainly does not mean that the US can expand its deficit forever. The willingness to hold dollar denoted assets, no matter for what strategy motives, may eventually conflict with the interest on economic returns from international investment. When the dollar reserves have to be accumulated at the expense of sizable national wealth due to declining dollar value, central banks may need to consider diversifying their foreign reserves towards other currencies, say the Euro. This seems to be a relevant issue especially when the real values of dollar reserves held by the central banks in China and in oil-producing countries have become more and more vulnerable to substantial capital loss. We do not think the US can further exploit its "exorbitant privilege" or seigniorage power in the future, and the finding that the US is increasingly dependent on the rest of the world to finance its deficit is actually alarming. But given the depth and liquidity of the market for the US dollar the transitional process can only take place slowly.

5 Conclusion

In this paper we propose a formal definition of long-run solvency and show that it can be empirically tested by cointegrating analysis. This method can be applied to long-run solvency problem of either an individual or an economy as a whole. A particular interesting application is to examine the sustainability of external imbalances. Our results suggest that early studies which only focus on the net foreign assets position at book value is misled. A country with huge cross-border holdings of financial assets is exposed to significant capital gains or losses due to currency and asset prices movements. Even in the long run imperfect substitutability between domestic and foreign assets has played a significant role in the external financial adjustment process. The systematic return premium enjoyed by the US is nothing new. But it is novel to show empirically that this effect has contributed to the long-run sustainability of the external imbalances of the US.

Generally speaking, the US has not run into potential trouble of insolvency yet. But the finding that the US has increasingly relied on the willingness of the rest of the world to finance its external imbalances is alarming indeed. No guarantee can be assumed that the US can

[27] see Dooley, Landau and Garber (2004).

maintain its exorbitant privilege forever. Investors have long been concerned about the trend of dollar depreciation in recent years which results in huge capital losses on dollar denoted assets. A potential collapse of the US dollar is not entirely impossible in the future (Krugman 2007). The tremendous official accumulation of the dollar reserves, especially in the emerging Asian economies and the oil-producing countries, cannot be sustainable.

Nonetheless we agree that further adjustment through depreciation in the dollar is inevitable for the US to rebalance its external position. This may have important implication for the international monetary transmission mechanism, which will be an interesting research topic. Moreover, return premium enjoyed by the US implies adverse effect on the rest of the world. It is thus important to examine the external imbalances for other countries. A global perspective is required to fully capture the interdependence of the financial linkages between different economies. This deserves serious future research.

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A Proof of Proposition 3

Lemma 6 *Suppose x_t and y_t are scalar $I(0)$ processes which can be written in the infinite moving average form following the Wold's decomposition theorem:*

$$x_t = \alpha_x + c_x(L)\varepsilon_t \ ,$$

$$y_t = \alpha_y + c_y(L)\eta_t \ ,$$

where α_x and α_y are the unconditional means of x_t and y_t respectively; ε_t and η_t are white noise with zero means; $c_i(L) = c_{i,0} + c_{i,1}L + c_{i,2}L^2...$ and $\sum_{j=0}^{\infty} |c_{i,j}| < \infty$ for $i = x, y$.

If $E_{t-1}[\varepsilon_t\eta_t] = \sigma_{\varepsilon\eta}$ where $|\sigma_{\varepsilon\eta}| < \infty$; and $E_{j-1}[\varepsilon_s\eta_t] = 0$ for $s \neq t$ and $j = \min\{s, t\}$ then

$$\lim_{h\to\infty} cov_t[x_{t+h}, y_{t+h}] = \sigma_{xy}$$

where $|\sigma_{xy}| < \infty$.

Proof.

$$cov_t[x_{t+h}, y_{t+h}] = E_t[(x_{t+h} - E_t(x_{t+h}))(y_{t+h} - E_t(y_{t+h}))].$$

Note that $E_t(x_{t+h}) = \alpha_x + c_{x,h}\varepsilon_t + c_{x,h+1}\varepsilon_{t-1} + ...$, therefore

$$x_{t+h} - E_t(x_{t+h}) = c_{x,0}\varepsilon_{t+h} + c_{x,1}\varepsilon_{t+h-1} + ... + c_{x,h-1}\varepsilon_{t+1} \ .$$

Similarly

$$y_{t+h} - E_t(y_{t+h}) = c_{y,0}\eta_{t+h} + c_{y,1}\eta_{t+h-1} + ... + c_{y,h-1}\eta_{t+1} \ .$$

Thus

$$\begin{aligned} cov_t[x_{t+h}, y_{t+h}] &= E_t[(\sum_{s=0}^{h-1} c_{x,s}\varepsilon_{t+h-s})(\sum_{j=0}^{h-1} c_{y,j}\eta_{t+h-j})] \\ &= E_t[(\sum_{s=0}^{h-1}\sum_{j=0}^{h-1} c_{x,s}c_{y,j}\varepsilon_{t+h-s}\eta_{t+h-j}] \\ &= \sigma_{\varepsilon\nu}\sum_{s=0}^{h-1} c_{x,s}c_{y,s} \ . \end{aligned}$$

where the last step uses the properties that $E_{t-1}[\varepsilon_t\eta_t] = \sigma_{\varepsilon\nu}$, $E_{j-1}[\varepsilon_s\eta_t] = 0$ for $s \neq t$ and $j = \min\{s, t\}$, and the law of iterated expectation. So

$$\lim_{h\to\infty} cov_t[x_{t+h}, y_{t+h}] = \sigma_{\varepsilon\nu} \cdot \lim_{h\to\infty}\{\sum_{s=0}^{h-1} c_{x,s}c_{y,s}\}$$

Note that $|c_{x,s}c_{y,s}| \leq \frac{1}{2}(c_{x,s}^2 + c_{y,s}^2)$, so

$$\begin{aligned}\left|\lim_{h\to\infty}\{\sum_{s=0}^{h-1} c_{x,s}c_{y,s}\}\right| &\leq \lim_{h\to\infty}\{\sum_{s=0}^{h-1} |c_{x,s}c_{y,s}|\} \\ &\leq \frac{1}{2}\lim_{h\to\infty}\{\sum_{s=0}^{h-1} c_{x,s}^2 + c_{y,s}^2\} < \infty,\end{aligned}$$

because $\sum_{j=0}^{\infty}|c_{i,j}| < \infty$ for $i = x, y$. Therefore

$$\left|\lim_{h\to\infty} cov_t[x_{t+h}, y_{t+h}]\right| < \infty$$

■

Using the result of Lemma 6 we can now prove proposition 3:

Proof. Given that $w_t = \alpha_w + c_w(L)\epsilon_t$, so

$$w_{t+h} - E_t(w_{t+h}) = \sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s} \ .$$

Note that $\Delta v_t = \alpha_v + c_v(L)\xi_t$, thus

$$v_t = v_0 + \alpha_v t + c_v(1)\sum_{j=0}^{t}\xi_{t-j} + c_v^*(L)\xi_t - c_v^*(L)\xi_0$$

where $c_v(L) = c_v(1) + c_v^*(L)(1-L)$. Therefore

$$v_{t+h} - E_t(v_{t+h}) = c_v(1)\sum_{j=0}^{h-1}\xi_{t+h-j} + \sum_{s=0}^{h-1} c_{v,s}^*\xi_{t+h-s}$$

where $\sum_{s=0}^{h-1} c_{v,s}^*\xi_{t+h-s}$ is a stationary stochastic process.

$$\begin{aligned}\lim_{h\to\infty} cov_t[w_{t+h}, v_{t+h}] &= \lim_{h\to\infty} E_t[(w_{t+h} - E_t(w_{t+h}))(v_{t+h} - E_t(v_{t+h}))] \\ &= \lim_{h\to\infty} E_t[(\sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s})(c_v(1)\sum_{j=0}^{h-1}\xi_{t+h-j} + \sum_{s=0}^{h-1} c_{v,s}^*\xi_{t+h-s})] \\ &= c_v(1)\lim_{h\to\infty} E_t[(\sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s})(\sum_{j=0}^{h-1}\xi_{t+h-j})] \\ &\quad + \lim_{h\to\infty} E_t[(\sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s})(\sum_{s=0}^{h-1} c_{v,s}^*\xi_{t+h-s})] \ .\end{aligned}$$

By Lemma 7 the second component, $\lim_{h\to\infty} E_t[(\sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s})(\sum_{s=0}^{h-1} c_{v,s}^*\xi_{t+h-s})]$, is a finite constant. Consider the first term:

$$c_v(1)\cdot\lim_{h\to\infty} E_t[(\sum_{s=0}^{h-1} c_{w,s}\epsilon_{t+h-s})(\sum_{j=0}^{h-1}\xi_{t+h-j})] = c_v(1)\sigma_{\epsilon\xi}\cdot\lim_{h\to\infty}[\sum_{s=0}^{h-1} c_{w,s}],$$

which is also a finite constant. Thus $\lim_{h\to\infty} cov_t[w_{t+h}, v_{t+h}]$ is a finite constant. ■

B Proof of Proposition 4

Lemma 7 *Let $f(t,h) = E_t[(g_{nx} + \rho_{a,t+h}g_a - \rho_{l,t+h}g_l)h]$. Under the co-trending restriction (29) and the conditions that $(\rho_{a,t+h} - \rho_a^p)$ and $(\rho_{l,t+h} - \rho_a^p)$ are stationary processes with zero mean, then for each t we have*

$$f(t,h) = E_t[(g_{nx} + \rho_{a,t+h}g_a - \rho_{l,t+h}g_l)h] \xrightarrow{p} 0, \ \text{as } h \to \infty.$$

Proof. [28]

$$\begin{aligned} f(t,h) &= E_t[(g_{nx} + \rho_{a,t+h}g_a - \rho_{l,t+h}g_l)h] \\ &= [(g_{nx} + \rho_a^p g_a - \rho_l^p g_l)h] + g_a E_t[(\rho_{a,t+h} - \rho_a^p)h] - g_l E_t[(\rho_{l,t+h} - \rho_l^p)h]\ . \end{aligned}$$

Under the co-trending restriction that $g_{nx} + \rho_a^p g_a - \rho_l^p g_l = 0$, we have

$$f(t,h) = g_a E_t[(\rho_{a,t+h} - \rho_a^p)h] - g_l E_t[(\rho_{l,t+h} - \rho_l^p)h]\ .$$

Now consider $E_t[(\rho_{a,t+h} - \rho_a^p)h]$ in which $(\rho_{a,t+h} - \rho_a^p)$ is assumed to be stationary with zero mean. Following the Wold decomposition theorem, $(\rho_{a,t+h} - \rho_a^p)$ can be written as

$$(\rho_{a,t+h} - \rho_a^p) = \sum_{j=0}^{\infty} c_{a,j}\varepsilon_{t+h-j}\ ,$$

where ε_t is white noise with zero mean satisfying $E[|\varepsilon_t|] < K_1 < \infty$ for all t, $|c_{a,j}| < K_2 |\lambda|^j$ with $|\lambda| < 1$, and K_1 and K_2 are positive constants which do not depend on j or t. Let $v_{t,h} = E_t[(\rho_{a,t+h} - \rho_a^p)h]$, so

$$v_{t,h} = h\sum_{j=0}^{\infty} c_{a,j+h}\varepsilon_{t-j}, \ \text{with } E(v_{t,h}) = 0.$$

Therefore

$$E[|v_{t,h}|] < h\sum_{j=0}^{\infty} |c_{a,j+h}| \cdot E[|\varepsilon_{t-j}|] < K_1K_2 h|\lambda|^h \sum_{j=0}^{\infty} |\lambda|^j = \frac{K_1K_2}{1-|\lambda|} h|\lambda|^h\ .$$

Note that $h|\lambda|^h < K_3 < \infty$, and by l'Hôital's rule we have

$$\lim_{h\to\infty}(h|\lambda|^h) = 0.$$

Therefore

$$\sup_t E[|v_{t,h}|] < \infty, \ \text{for all } h;$$

[28] This proof follows Pesaran (2008).

and

$$\lim_{h\to\infty}\{\sup_t E[|v_{t,h}|]\} = 0.$$

Following the generalized Chebyshev's inequality, for any constant $\delta > 0$ we have

$$\Pr\{|v_{t,h}| > \delta\} \leq \frac{E[|v_{t,h}|]}{\delta},$$

So

$$\lim_{h\to\infty} \Pr\{|v_{t,h}| > \delta\} = 0.$$

Thus

$$g_a E_t[(\rho_{a,t+h} - \rho_a^p)h] \xrightarrow{p} 0, \text{ as } h \to \infty,$$

where $\xrightarrow{p}$ denotes convergence in probability. Similarly $g_l E_t[(\rho_{l,t+h} - \rho_l^p)h] \xrightarrow{p} 0$, as $h \to \infty$. Therefore

$$f(t,h) \xrightarrow{p} 0, \text{ as } h \to \infty.$$

■

Now we prove proposition 4:

Proof. We detrend nx_t, a_t, and l_t by their own deterministic growth components, and group the deterministic trends together. Condition (21) can be written as:

$$\begin{aligned} &\lim_{h\to\infty} E_t[(nx_{t+h} - g_{nx}h) + \rho_{a,t+h}(a_{t-1+h} - g_a h) - \rho_{l,t+h}(l_{t-1+h} - g_l h) - \Delta(a_{t+h} - l_{t+h})] \\ &\quad = -\lim_{h\to\infty} E_t[(g_{nx} + \rho_{a,t+h} g_a - \rho_{l,t+h} g_l)h] \ . \end{aligned} \tag{54}$$

The conditional forecasts for deterministically detrended series should go to limiting values when h goes to infinity, namely $|\lim_{h\to\infty} E_t[z_{t+h} - g_z h]| < \infty$,for $z = nx, a$ and l. Following Proposition 3 we have $\lim_{h\to\infty} cov_t[\rho_{a,t+h}, a_{t-1+h}]$ and $\lim_{h\to\infty} cov_t[\rho_{l,t+h}, l_{t-1+h}]$ as finite constants. Notice that the product of two limiting values is also finite, so

$$\left|\lim_{h\to\infty} E_t[\rho_{a,t+h}] \cdot \lim_{h\to\infty} E_t[a_{t-1+h} - g_a h]\right| < \infty \ .$$

Therefore

$$\begin{aligned} \left|\lim_{h\to\infty} E_t[\rho_{a,t+h}(a_{t-1+h} - g_a h)]\right| &= \left|\lim_{h\to\infty} \{E_t[\rho_{a,t+h}]E_t[a_{t-1+h} - g_a h] + cov_t[\rho_{a,t+h}, a_{t-1+h}]\}\right| \\ &\leq \left|\lim_{h\to\infty} E_t[\rho_{a,t+h}] \cdot \lim_{h\to\infty} E_t[a_{t-1+h} - g_a h]\right| + \\ &\quad \left|\lim_{h\to\infty} cov_t[\rho_{a,t+h}, a_{t-1+h}]\right| \\ &< \infty \ , \end{aligned}$$

where the first step uses $cov_t[\rho_{a,t+h}, a_{t-1+h}] = cov_t[\rho_{a,t+h}, a_{t-1+h} - g_a h]$. Similarly

$$\left| \lim_{h\to\infty} E_t[\rho_{l,t+h}(l_{t-1+h} - g_l h)] \right| < \infty \ .$$

So the left hand side of equation (54) is finite. For equation (54) to hold consistently the sum of the deterministic trends cannot explode:

$$\left| \lim_{h\to\infty} E_t[(g_{nx} + \rho_{a,t+h} g_a - \rho_{l,t+h} g_l)h] \right| < \infty \ .$$

Using lemma 8 we conclude that under the co-trending restriction (29) the right hand side of equation (54) converge in probability to zero.

Therefore equation (54) can be approximated by:

$$\begin{array}{c} \lim_{h\to\infty}\{E_t[nx_{t+h}] + E_t[\rho_{a,t+h}]E_t[a_{t-1+h} - g_a h] + cov_t[\rho_{a,t+h}, a_{t-1+h}] + \\ -E_t[\rho_{l,t+h}]E_t[l_{t-1+h} - g_l h] - cov_t[\rho_{l,t+h}, l_{t-1+h}] - E_t[\Delta(a_{t+h} - l_{t+h})]\} = 0. \end{array} \tag{55}$$

Notice, we have

$$\begin{aligned} \lim_{h\to\infty} E_t[\rho_{a,t+h}]E_t[a_{t-1+h} - g_a h] &= \lim_{h\to\infty} E_t[\rho_{a,t+h}] \cdot \lim_{h\to\infty} E_t[a_{t-1+h} - g_a h] \\ &= \rho_a^p \cdot a_t^p \ . \end{aligned}$$

Similarly,

$$\lim_{h\to\infty} E_t[\rho_{l,t+h}]E_t[l_{t-1+h} - g_l h] = \rho_l^p \cdot l_t^p \ .$$

Substitute equation (28) into (55) we have

$$nx_t^p + \rho_a^p \cdot a_t^p - \rho_l^p \cdot l_t^p = \alpha_{na} - \theta \ . \tag{56}$$

Specially if net export nx_t is stationary, we have

$$\lim_{h\to\infty} E_t[nx_{t+h}] = nx^p, |nx^p| < \infty.$$

Substitute into equation (56) we have

$$\rho_a^p \cdot a_t^p - \rho_l^p \cdot l_t^p = \alpha_{na} - \theta - nx^p,$$

where $|\alpha_{na} - \theta - nx^p| < \infty$. ■

C Proof of Proposition 5

Proof. It's trivial to show that cointegration (35) implies equation (34). Here we show that equation (34) also implies cointegration (35). Recall the Beveridge-Nelson decomposition which states that any $I(1)$ process can be written as the sum of a stochastic trend and a stationary component. Given that $\mathbf{z}_t$ is a vector of $I(1)$ process by assumption, we write $\Delta\mathbf{z}_t = \boldsymbol{g}_z + \boldsymbol{C}_z(L)\boldsymbol{\varepsilon}_{z,t}$.[29] $\mathbf{z}_t$ can be decomposed as:[30]

$$\mathbf{z}_t = \mathbf{z}_0 + \boldsymbol{g}_z t + \boldsymbol{C}_z(1)\sum_{j=0}^{t}\boldsymbol{\varepsilon}_{z,t-j} + \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,t} - \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,0} \ , \tag{57}$$

where $\mathbf{z}_0$ is the initial value, $\boldsymbol{g}_z$ is the deterministic growth component of $\mathbf{z}_t$, and $\boldsymbol{C}_z(1) + \boldsymbol{C}_z^*(L)(1-L) = \boldsymbol{C}_z(L)$. Thus the permanent component of $\mathbf{z}_t$ is given by

$$\mathbf{z}_t^p = \lim_{h\to\infty} E_t[\mathbf{z}_{t+h} - \boldsymbol{g}_z h] = \mathbf{z}_0 + \boldsymbol{g}_z t + \boldsymbol{C}_z(1)\sum_{j=0}^{t}\boldsymbol{\varepsilon}_{z,t-j} - \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,0} \ . \tag{58}$$

Substitute (57) and (58) into (34) we have

$$\boldsymbol{\beta}'(\mathbf{z}_t - \boldsymbol{C}_z^*(L)\boldsymbol{\varepsilon}_{z,t}) = 0 \ ,$$

which gives (35) immediately. ■

D Data Source

The stocks of assets and liabilities at market value, and the total returns on the US international investment portfolio are provided by Gourinchas and Rey (2007). They use data on the net and gross foreign asset positions from the US bureau of Economic Analysis (BEA) and the Federal Reserve Flows of Funds Accounts (FFA). Following official classifications they split the US net foreign portfolio into four categories: debt (corporate and government bonds), equity, foreign direct investment, and other. The "other" category includes mostly bank loans and trade credits. It also contains gold reserves. Their strategy is to reconstruct market value estimates of the gross external assets and liabilities of the US that conform to the BEA definitions by using FFA flow and position data and valuation adjustments. The total returns for each class of financial assets are constructed as follows. For equity and FDI, they use quarterly total returns on the broadest stock market indices available in each country. The total return on debt is a weighted average of the total quarterly return on 10-year government bonds and the three-month interest rate on government bills, with weights reflecting the maturity structure of debt assets and liabilities. The total return on other assets and liabilities is computed using three-month interest rates. All returns are adjusted for US inflation by subtracting the quarterly change in the personal consumer expenditure deflator. In all cases, they use end-of-period exchange rates to convert local currency capital gains and total returns into dollars. A complete description of the data can be founded in Gourinchas and Rey (2005).

We use the aggregate returns from Gourinchas and Rey (2005) to construct proxies for $\rho_{a,t}$ and $\rho_{l,t}$. Below are plots of the two series.

[29] $\boldsymbol{C}_z(L) = \boldsymbol{C}_{z,0} + \boldsymbol{C}_{z,1}L + \boldsymbol{C}_{z,2}L^2 + ...$, and the matrices $\{\boldsymbol{C}_{z,i},\ i = 0, 1, 2, ...\}$ are absolute summable.

[30] See Katarina Juselius, 2006, ch5 pp 84-85

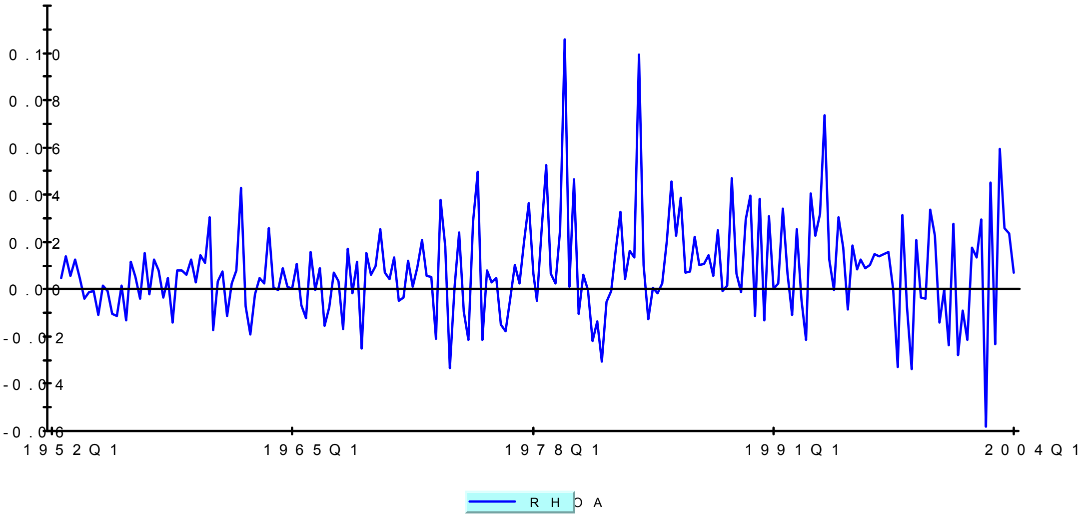

Figure 5: RHOA

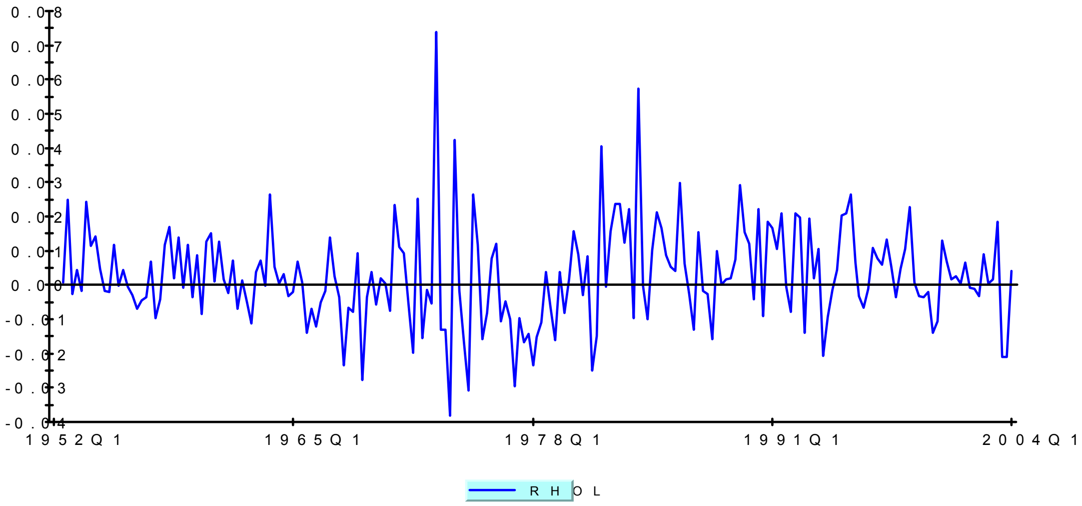

Figure 6: RHOL

E Unit-root Tests on the US Data

For the level, the ADF regressions include an intercept and a trend. The 95% critical value is -3.4328. For the first differences, the ADF regressions include an intercept but not a trend. The 95% critical value is -2.8758.

Table 7: ADF Test on US Data

Unit-root tests on level					
Variable	DF	ADF(1)	ADF(2)	ADF(3)	ADF(4)
x_t	-1.4667	-1.6024	-2.0213	-2.1925	-2.2686
m_t	-2.3800	-2.7565	-2.5563	-2.2908	-2.0872
a_t	-1.7150	-1.6897	-1.6951	-1.6934	-1.6827
l_t	1.0213	.92767	1.0753	1.0126	.93897
Unit-root tests on first difference					
Variable	DF	ADF(1)	ADF(2)	ADF(3)	ADF(4)
Δx_t	-13.3936	-8.2485	-6.7432	-5.9672	-6.0983
Δm_t	-12.2597	-10.1552	-9.3341	-8.6481	-8.2701
Δa_t	-14.9067	-9.1419	-7.0578	-5.6900	-5.7405
Δl_t	-10.3950	-7.1561	-5.0980	-4.0717	-3.7855

F Robustness Tests

F.1 Why Use Wealth as the Denominator in Equation (14)

In Section 3 we use wealth to deflate assets and liabilities. The intuition follows Merton's portfolio allocation model (1971) which suggests that the portfolio shares $\frac{A_t}{W_t}$ and $\frac{L_t}{W_t}$ are fixed in the steady state as long as assets and liabilities are not perfect substitute. Gourinchas and Rey (2007) follow this argument in constructing their present value model and in their empirical study. From a practical perspective we can also see that using wealth instead of output as the denominator ensures linearity in the data (see Figure 2), therefore a linear VECM is appropriate.

Alternatively, we have considered using GDP to deflate assets and liabilities. The drawback is that the ratios of assets and liabilities to GDP seem to grow in a non-linear pattern. (see Figure 7 below) The corresponding empirical results are not very sensible because the linear VECM cannot handle non-linear pattern properly.

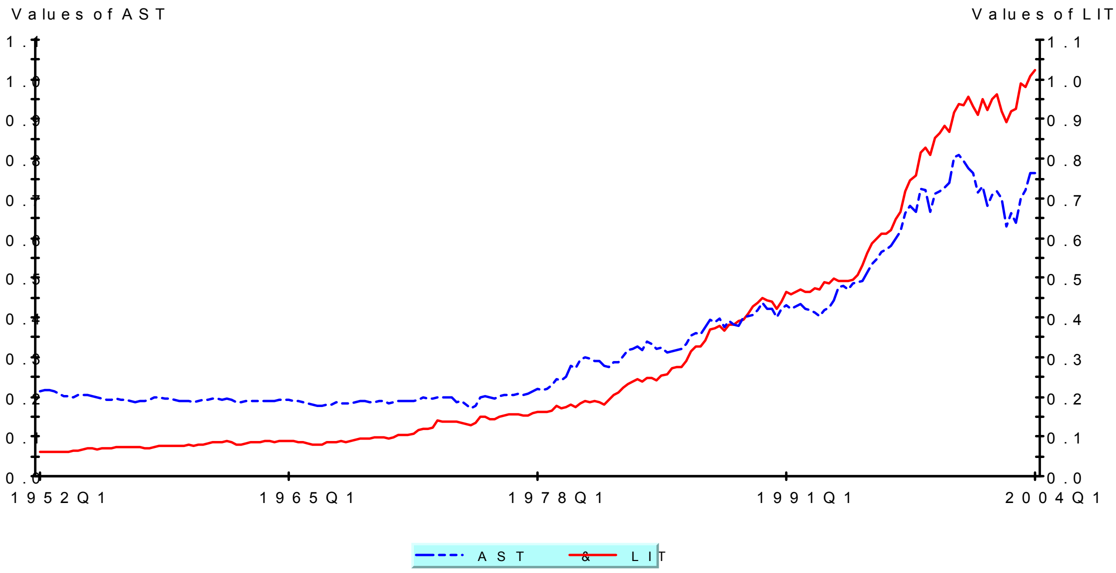

Figure 7: Ratios of Assets (AST) and Liabilities (LIT) to GDP. USA. 1952-2004

F.2 Robustness to the Choice of Break Point

In Section 4.3 we select 1982Q2 as the break point to split the full sample into two subsamples. The long-run relations in the first period are quite robust to different choices of the break point. However, the cointegrating analysis in the second period is not very stable if the starting date is changed. This is mainly due to the limited sample size we have. Notice that when we follow the system approach where 4 variables ($x_t, m_t,$ a_t and l_t) are modeled in one VECM there are many coefficients to be estimated. Many of the estimated parameters are statistically insignificant. Alternatively we explore the idea of parsimonious approach which excludes export and import but looks at assets and liabilities only. The intuition is that if we agree net export is stationary over a longer horizon, the long-run solvency condition boils down to a cointegration between assets and liabilities. The parsimonious approach is a robustness test to our theory. But the trade-off for estimating the subsystem is that a large number of lags might be needed to approximate the true data generating process. Therefore we also investigate the sensitivity of the cointegrating analysis to the choice of lag order in the VECM.

Let $\mathbf{z}_t = (a_t, l_t)'$. We start with a VECM featured with unrestricted intercept and restricted trend. We first fit the VECM to the full sample (1952Q1-2004Q1). Then we repeat the recursive structural stability test on the long-run relation. If there is a structural break then we carry out the cointegrating analysis for each subsample separately.

F.2.1 2-Variable Model on Full Sample

Following AIC we fit VECM(9) to the full sample. Table 8 reports Johansen's likelihood tests on the cointegrating rank order. The maximum eigenvalue statistics and the trace statistics

Table 8: Cointegrating rank test. 2-Variable VECM, 1952Q1-2004Q1

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r=0$	$r=1$	20.4268	19.2200	17.1800
$r\leq 1$	$r=2$	9.1621	12.3900	10.5500
(b) Trace Statistics				
$r=0$	$r=1$	29.5889	25.7700	23.0800
$r\leq 1$	$r=2$	9.1621	12.3900	10.5500

confirm that a_t and l_t are cointegrated. The number of cointegration is not sensitive to the number of lag orders.

Proceed with $r=1$. The co-trending restriction is the only over-identifying restriction and is confirmed by bootstrap technique at 95% significance level. The estimated relation is given by:

$$a_t - \underset{(0.097284)}{0.41861} \; l_t = \hat{\alpha}_{al} + \hat{\zeta}_{2,t} \; , \tag{59}$$

Notice the key parameter $\hat{\lambda} = 0.41861$ is significantly different from zero and less than one, implying imperfect substitutability between domestic and foreign assets. This is confirmed by the rejection of a further restriction imposing $\hat{\lambda} = 1$. Moreover, the sign and magnitude of the estimates are sensible. Notice $\hat{\lambda} < 1$ implies $\rho_a^p > \rho_l^p$, thus the US is enjoying excess return on foreign assets compared to the payment on liabilities. This is similar to the finding in the system approach where all four variables are modelled together.

F.2.2 Structural Stability on Long-run Relation

Again the critique to the full sample result is that there might be a structural break in the long-run relation around 1977-1985. To formally examine the structural stability we repeat the recursive test of fixed cointegrating vector. The null hypothesis is that the long-run relation captured by cointegrating vector $\boldsymbol{\beta}' = [1, -\lambda]$ is fixed over the full sample period.

First we obtain a reference value for $\boldsymbol{\beta}$ from the whole-sample estimate. We use $\hat{\boldsymbol{\beta}}_0' = [1, -0.41861, 0]$, where the last element corresponds to the co-trending restriction. The second step is to test whether the reference value $\hat{\boldsymbol{\beta}}_0$ is in the space spanned by $\hat{\boldsymbol{\beta}}_{t_i}$, where $\hat{\boldsymbol{\beta}}_{t_i}$ is the estimate of $\boldsymbol{\beta}$ based on the sample $1, 2, ..., t_i$; for $t_i = 1977Q1, 1977Q2, ..., 1985Q4$. This can be carried out by the likelihood ratio test of over identifying restrictions for each subsample, where the test statistic is asymptotically distributed as $\chi^2(2)$. We also use the bootstrap critical value at the 5% significance level to take into account small sample size. The result is given in the following figure where the test values have been scaled by the 95% quantile of the $\chi^2(2)$ and the bootstrap simulation respectively. A value larger than 1.0 means rejection of the null hypothesis.

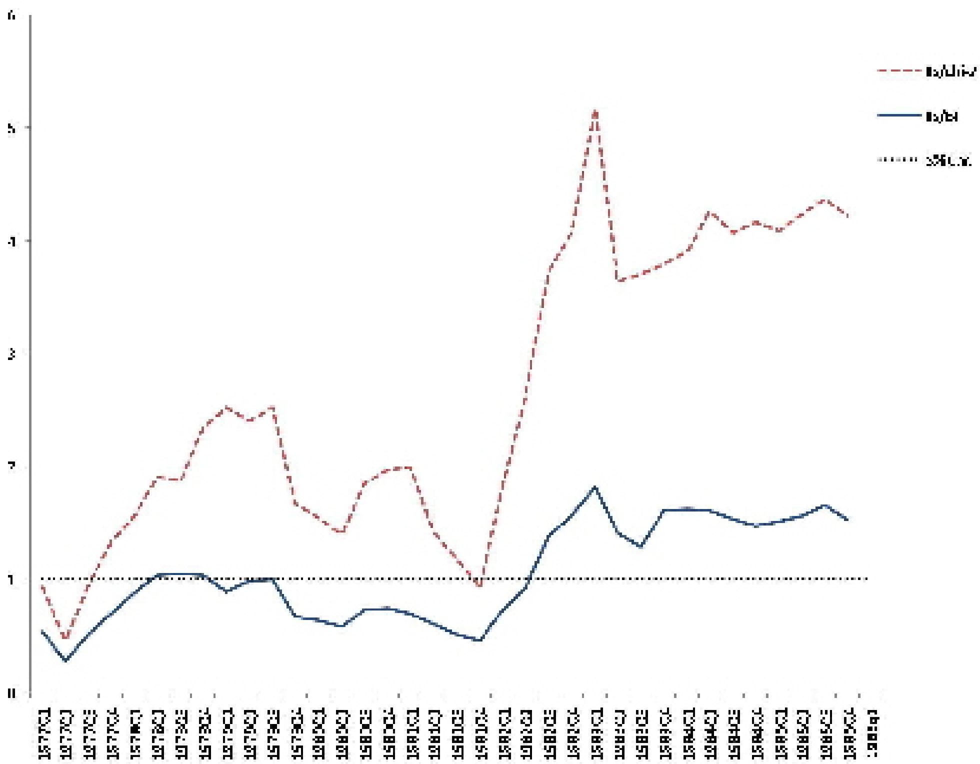

Figure 8: Recursive Test on Constant Cointegrating Vector (a value greater than one indicates structural break)

In Figure 8 the dash-line, which is test statistics scaled by the 95% quantile of $\chi^2(2)$, rejects constant cointegrating vector nearly at each point of the recursive test. But after adjusting for the small sample size using bootstrap, the solid line suggests there is a structural break first around 1978-1979. We conclude from the recursive test above that the null hypothesis of constant cointegrating vector $\boldsymbol{\beta}$ cannot be accepted in the full sample.

To carefully study what have actually caused the structural break we split the full sample into two subsamples: $\{1952q1 - 1978q1\}$ and $\{1978q2 - 2004Q1\}$. We repeat the same cointegrating analysis for each subsample and examine the stability of λ.

F.2.3 Subsample 1: 1952Q1-1978Q1 (105 obs)

We fit VECM(5) to the data. Table 9 shows that the maximum eigenvalue test and the trace statistics confirm that there is one cointegrating relation over 1952Q1-1978Q1.

The long-run solvency condition is satisfied, as confirmed by the over-identifying restrictions test. The resulting long-run relation is given by

$$a_t - \underset{(0.25724)}{0.84486}\; l_t = \hat{\alpha}_{al}^1 + \hat{\zeta}_{2,t}^1 \;, \tag{60}$$

in which $\hat{\lambda}_1 = 0.84486$ is very close to one. Actually an over-identifying restriction that $\hat{\lambda}_1 = 1$ cannot be rejected using the bootstrap critical value. However, $\hat{\lambda}_1$ is significantly different

Table 9: Cointegrating rank test, 1952Q1-1978Q1

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r=0$	$r=1$	22.1704	19.2200	17.1800
$r \leq 1$	$r=2$	6.7537	12.3900	10.5500
(b) Trace Statistics				
$r=0$	$r=1$	28.9241	25.7700	23.0800
$r \leq 1$	$r=2$	6.7537	12.3900	10.5500

Table 10: Cointegrating rank test, 1978Q2-2004Q1

H_0	H_1	Test Statistics	95% Critical Value	90% Critical Value
(a) Maximum eigenvalue statistics				
$r=0$	$r=1$	19.7765	19.2200	17.1800
$r \leq 1$	$r=2$	4.282	12.3900	10.5500
(b) Trace Statistics				
$r=0$	$r=1$	24.0591	25.7700	23.0800
$r \leq 1$	$r=2$	4.2826	12.3900	10.5500

from the estimate using the whole sample (equation (59)). Therefore we conclude that in this subsample the long-run solvency conditions are met with $\hat{\lambda}_1 = 1$.

F.2.4 Subsample 2: 1978Q2-2004Q1 (104 obs)

We fit VECM(4) to the data. Table 10 shows that the maximum eigenvalue test and trace statistics confirm there is one cointegration.

The over-identifying restriction test confirms the long-run solvency is satisfied, and the estimated cointegration is given by:

$$a_t - \underset{(0.037078)}{0.47994} \; l_t = \hat{\alpha}_{al}^2 + \hat{\zeta}_{2,t}^2 \,, \tag{61}$$

Interestingly, $\hat{\lambda}_2 = 0.47994$ is significantly different from $\hat{\lambda}_1 = 1$, implying imperfect substitutability between domestic and foreign assets in this subsample. $\hat{\lambda}_2 < 1$ reveals that the US is enjoying an excess return of gross assets over gross liabilities.

It is important to emphasize that selecting different break point does not alter the main results in the parsimonious approach. We have investigated other break points, such as 1977q4, 1978q2, or 1978q3, and we obtain similar results as above. In addition, the cointegrating analysis for each of the sub-sample is not sensitive to the lag order in VECM.

Over all, the parsimonious approach which excludes export and import provides consistent results as the system approach. The robustness of the subsystem model with respect to the choice of break point is very appealing. This exercise confirms the robustness of the findings we have in the system approach.