Explaining External Asset Allocation: A Multi-Country Model with Preference Heterogeneity*

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Abstract

One of the most defining features of economic development in the past twenty years has been the growth in cross-border financial asset holdings. This paper proposes a micro-founded, multi-country model with endogenous external asset allocation and preference heterogeneity in consumption tastes. The model is solved by generalising the method for determining country asset portfolios proposed by Devereux and Sutherland (2008) to assets denominated in different currencies and more than two countries. This paper deals with three stylised facts of external asset allocation: heterogeneity across asset classes and countries as well as no short selling in aggregate. The proposed model yields a rich set of theoretical results relating country portfolios to macroeconomic fundamentals and consumption preferences and is rich enough to replicate the stylised facts in theoretical calibrations. In an empirical application the model successfully replicates heterogeneity across asset classes and, to an extent, countries as well as no aggregate short selling of external assets.

Keywords: External assets, portfolio choice, preference heterogeneity

JEL Codes: F40, F41, G11

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1 Introduction

One of the most defining features of economic development in the past twenty years has been the growth in cross-border financial asset holdings. This fact has been well documented by Lane and Milesi-Ferretti (2001, 2007). This paper proposes a multi-country dynamic stochastic general equilibrium model with preference heterogeneity in consumption tastes and endogenous portfolio choice. The proposed model is then evaluated empirically by verifying its ability to match three key stylised facts on external asset allocation.

First, according to Lane and Milesi-Ferretti (2001, 2007), stocks of external assets and liabilities are generally positive, that is no short-selling of assets is observed on an aggregate basis. In more than 60 percent of cases on average, depending on the data considered, the proposed model replicates this fact even though no short selling constraints are imposed. Second, there is heterogeneity among countries in average external asset positions across two broad asset classes this paper considers - bonds and equity. For 18 countries analysed in this paper, which account for the majority of world GDP and external assets, the proposed model correctly replicates the sign of 29 to 35 external asset positions in different asset classes from a total of 36. Third, assets issued by some countries account for a larger share of external asset portfolios of other regions than would be suggested by their share of aggregate output. A notable example of this phenomenon is the UK. This fact is also, to an extent, replicated by the model.

There are several reasons it is important to consider external asset allocation both across asset classes and across countries. First, external asset portfolios have now become large enough for fluctuations in exchange rates and asset prices to cause very significant reallocations of wealth across countries (Lane and Milesi-Ferretti, 2007).[1] Second, observed external asset portfolios reflect global imbalances, whose danger to the world financial stability for several years has been the subject of vigorous debate in the literature (Mendoza et al., 2007).[2]

This paper is the first to consider bilateral asset allocation, that is allocation of external assets not only between bonds and equity, but also between different countries within the same asset class. The data set on bilateral stocks of external assets used in this paper is constructed by Kubelec and Sá (2009) and covers 60 to 80 percent of the total of world's external assets. For most specifications, the root mean square error of model prediction of asset allocations in bonds or equity is less than the standard deviation of allocations suggesting that the model explains some of the variance in external asset allocations.

In terms of methodology, this paper generalises the solution method for optimum asset allocation proposed by Devereux and Sutherland (2007, 2008) to a framework with multiple countries, potentially different consumption tastes and two kinds of assets - bonds and equity. Assets are denominated in the currencies of the issuing regions. The key idea, due to Devereux and Sutherland (2008), is that time variation in portfolio allocation is irrelevant for determining first-order behaviour of macroeconomic variables like consumption or price level. Therefore one

[1] See also, for example, Cavallo and Tille (2006) as well as Tille (2008).

[2] See also, for example, Blanchard et al. (2005), Obstfeld and Rogoff (2005, 2007), Ghironi et al. (2007), Hausmann and Sturzenegger (2007).

can solve for the optimal asset portfolio allocation by combining a second-order approximation of the portfolio selection condition with a first-order approximation to the remaining parts of the model.

This paper shows how such a combination needs to be adjusted when assets are denominated in different currencies. It also shows that the key equilibrium condition of the model, when more than two countries are present, still has an exact solution when expressed in the matrix form and, in addition, provides some insights on how the existence of multiple solutions can be analysed. In a contribution similar to this paper, Dedola and Straub (2008) solve for optimum portfolio choice with a different set of available assets and homogeneous preferences, but restrict themselves to considering just three countries.

This paper also provides several theoretical results regarding optimum asset allocation and its relationship to consumption tastes, terms of trade, presence of government spending and persistence of stochastic shocks. Some of the results replicate earlier contributions in a richer, stochastic setting with a more realistic structure of asset markets. Thus, this paper obtains the result originally due to Cole and Obstfeld (1991), who argue that if countries produce specialised traded goods, real asset returns are equalised across countries due to commodity trade even if there is no trade in financial assets. This paper also replicates the result of Baxter, Jermann, and King (1998), who analyse equity of traded and nontraded industries separately and show that the optimal holdings of equity of nontraded goods industries will depend on the assumed elasticity of substitution between traded and nontraded goods.

The proposed model of external asset allocation features a world with an arbitrary number of economies, each populated by a continuum of identical, infinitely lived consumers, who receive stochastic traded and nontraded endowments. Consumers derive utility from consumption of a variety of goods and holding real money balances and can trade their traded endowments and financial assets. The model allows for a non-zero stock of net external assets in the initial steady state. Different from previous literature, the model also allows for heterogeneity in consumption preferences across different countries.

The influence of heterogeneity of countries on external asset allocation has been previously considered by Mendoza et al. (2007), who focused on the heterogeneity in the level of financial development of countries and they show that countries with more advanced financial markets will accumulate foreign liabilities. This paper shows that key features of external asset allocation can be replicated even if financial markets are assumed to have the same level of development.

The optimal steady state foreign asset portfolio allocation is found by considering the first and second-order approximations of the model around the explicitly derived steady state. A similar method has been first proposed by Judd and Guu (2001), who use Taylor approximations for asset demand around the equilibrium that would prevail if there were no uncertainty and hence no risk. Other contributions to this literature are by Evans and Hnatkovska (2005), who develop a solution method, which relies on perturbation methods with continuous-time approximations, and Tille and Van Wincoop (2007), whose approach is essentially similar to the one by Devereux and Sutherland (2008).

Section 2 of this paper describes the measurement of external assets and the key stylised facts on external asset allocation across asset classes as well as countries. Section 3 describes the model and the solution method for the portfolio allocation problem. Section 4 outlines some theoretical results obtained from the solution and provides potential theoretical explanations for the results obtained in Section 5. Section 5 discusses data collection, estimation of certain parameters used in the empirical application of the model as well as the method and the results of the empirical application of the model. Finally, Section 6 concludes.

2 Stylised Facts on External Asset Allocation

The main source of data on aggregate stocks of external assets and liabilities is the updated and extended version of the data set constructed by Lane and Milesi-Ferretti (2007). To calculate the composition of external asset portfolios, this paper uses the data set containing portfolio allocations for 18 countries compiled by Kubelec and Sá (2009). The 18 countries included in the data set are Argentina, Australia, Brazil, Canada, China, France, Germany, Hong Kong, India, Italy, Japan, Korea, Mexico, Portugal, Singapore, Spain, United Kingdom and the United States. In total these countries account for between 60 and 80 percent of the world's total external assets.

Kubelec and Sá (2009) obtain the estimates of portfolio composition of individual countries by combining existing data sources[3] on asset portfolio composition, where they are available, with estimates of portfolio weights of various assets generated by gravity models.

The focus of this paper is on external asset holdings by the private sector in the time period from 1990 to 2005.[4] There are two reasons for focusing on this period. First, for many of the countries in the data set it coincides with the removal of capital controls and is therefore most consistent with the assumption of free capital mobility. Second, there is more of actual data available for this time period in the aggregate data set by Lane and Milesi-Ferretti (2007). To obtain private sector asset holdings for individual countries portfolio composition weights from Kubelec and Sá (2009) are multiplied by the aggregate stocks of external assets from Lane and Milesi-Ferretti (2007) after removing, to the maximum possible extent, government assets and liabilities. Note that the portfolio composition weights do not necessarily sum up to one, since countries may have external assets in other countries not covered in the data set. After obtaining external assets, external liabilities are inferred from the accounting identity, that is, for example, the liabilities of the UK in bonds, for example, are assumed to equal the total assets of other countries in UK bonds. The reason the focus is also only on the private sector external asset holdings is because governments face incentives in their external asset allocation that are different from those of private agents. In most cases, for example, governments do

[3]These sources include OECD International Direct Investment database and United Nations Conference on Trade and Development for foreign direct investment, the IMF coordinated portfolio survey for equity and debt securities as well as the Locational Banking Statistics by the Bank for International Settlements for debt securities.

[4]The data set by Kubelec and Sá (2009) covers the time period from 1980 to 2005.

not maximise their own consumption and instead may be concerned with the distribution of wealth, etc.

In the data external assets and liabilities are split into the following broad categories: portfolio equity, foreign direct investment, debt (including portfolio debt and other investment), financial derivatives and foreign exchange reserves. In this paper we abstract from financial derivatives, because data on those, even in aggregate, is not available for the majority of the time period considered. We also abstract from foreign exchange reserves. Even though foreign exchange reserves form a sizeable component of external assets for many Asian countries, they are unlikely to be explained well by a model of endogenous portfolio choice with utility maximisation as a central goal of economic agents.

The focus of this paper is therefore on the remaining asset categories - portfolio equity, foreign direct investment and debt. Portfolio equity holdings denote ownership of shares of companies and mutual funds below the 10 percent threshold (Lane and Milesi-Ferretti, 2007). FDI can include equity holdings that are above 10 percent, greenfield investments as well as foreign property investment. Thus, FDI, on the one hand, can be lumped with equity because there is little conceptual difference between, for example, a 9 percent and a 16 percent stake in a foreign company, but, on the other hand, greenfield investments and foreign property typically entail much higher transaction costs than a simple purchase of equity. This paper considers both cases with FDI modelled as equity as well as with FDI excluded from the data. Subsequently portfolio equity and FDI together will be referred to as total equity. Debt securities consist of portfolio debt securities and other debt instruments, which include loans, deposits and trade credits. These asset classes will be collectively referred to as bonds.

The model developed in Section 3 allows solving for the steady state portfolio allocation.[5] The empirical objective of the model is to match long-run average external holdings and composition of external assets. Thus, it is necessary to establish whether external asset holdings are stable over some time period.

To investigate stability, this paper employs Quandt-Andrews test for one or more unknown structural breakpoints in the ratio of net external assets (including as well as excluding FDI) to GDP. The advantage of the Quandt-Andrews test is that the Chow breakpoint test is performed for all dates within a certain time period and the test statistics are summarised into one statistic for the test of null hypothesis of no breakpoints in the time period considered (Andrews, 1993; Andrews and Ploberger, 1994). The null hypothesis of no structural break between years 1993 and 2002 is unambiguously not rejected only for six series out of the total of 36, therefore it is appropriate to split the sample into several parts. The likeliest location for breakpoints in 19 out of 36 series are years 2001 and 2002. This reflects the nature of the dataset, which, for years prior to 2001 is comprised mostly of estimated values. A further six series have breakpoints in the years 1995 to 1997, whereas two series have likeliest breakpoints before 1995.[6] This paper

[5]It is possible to extend the method to solve for portfolio allocation dynamics as well, however, given the scarcity of data on asset allocation, its low frequency and short sample period available this is not pursued in this paper.

[6]Detailed results are given in Table A.1 in Appendix A.

therefore splits the sample period in three parts: from 2001 to 2005, since data availability is best for that period, and two other periods of comparable length - 1990 to 1995 and 1995 to 2000.

The sample period from 2001 has another distinctive characteristic in the fact that five countries in the sample - France, Germany, Italy, Portugal and Spain have adopted a common currency in use for all transactions - the euro. Given that the model assumes that every country has its own currency and the fact that the EMU countries can be expected to have similar macroeconomic conditions[7], the five countries adopting the euro have been grouped into a single EMU region, thus reducing the number of countries to 14 for the last sample period.

Table A.2 in Appendix A shows the average aggregate private sector net external asset holdings calculated using the aggregate external positions from the External Wealth of Nations dataset. Table A.3 shows the average private sector net external asset holdings as a share of GDP from 1990 to 2005, which are calculated using bilateral holdings for only the 18 countries in the dataset of Kubelec and Sá (2009). [8] A negative entry in Table A.3 means that liabilities in this category exceed assets. For example, the US has net liabilities in bonds and net assets in total equity and portfolio equity in all three time periods considered.

Table A.3 shows that there is considerable heterogeneity in aggregate holdings of external assets. Some countries have net assets in total or portfolio equity (e.g. the United States for all time periods, Germany in total, but not portfolio equity for all time periods), whereas the majority of others have net liabilities (e.g. Brazil and Mexico for all time periods in total as well as portfolio equity). Germany, France and the UK have net assets in bonds for all time periods, whereas others (e.g. Australia, Canada) have net liabilities. After removing foreign exchange reserves China emerges with a small negative (and approaching positive) position in bonds, and Japan has a relatively large positive position in bond

The reader may argue that removing foreign exchange reserves from consideration removes an important phenomenon that has contributed to the extent of the global imbalances. Some literature on the determination of currency reserves includes Papaioannou et al. (2006) and, in particular, Devereux (2009), who constructs a model of the interaction between an emerging market and an advanced economy in which an optimal general equilibrium portfolio structure implies that emerging market economies simultaneously build up a stock of foreign exchange rate reserves while receiving FDI flows from the advanced economy. That model, however, postulates differences between capital markets of emerging and developed markets. The point of this paper is to show that heterogeneity in asset allocations can be replicated to some extent even without postulating such differences and in a purely utility optimising model. Therefore, one can argue that foreign exchange reserve accumulation is not an essential phenomenon for global imbalances.

Tables A.4 through to A.12 in Appendix A illustrate the composition of portfolio equity

[7]Note that this is only assumed for a time period from 2001 to 2005.

[8]There is a subtle point here that average next external asset holdings as a share of GDP are defined as the ratio of average external assets to the average GDP, rather then the average of the ratios of external assets to GDP. This definition is adopted to ensure that when average net external asset holdings in percent of GDP are multiplied by GDP, they sum up to zero.

assets, total equity and debt securities for three separate periods considered in this paper 1990 to 1995, 1996 to 2000 and 2001 to 2005. A notable feature of the data is that asset allocation in all three asset categories is generally concentrated in the securities issued by just a few countries, of which the US and the UK are the most prominent examples. This concentration seems to have remained fairly stable in equity holdings, but increased in debt holdings. Thus, from 1990 to 1995 other countries have on average been holding 47 percent of their total equity portfolio in the United States (52.3 percent for portfolio equity), followed by 45.2 percent from 1996 to 2000 and 47.5 percent from 2001 to 2005 (48.9 percent and 53.5 percent for portfolio equity respectively). At the same time, the share of debt security portfolio that other countries have on average allocated to the United States has grown from 29.7 percent during the years from 1990 to 1995, to 49.3 percent during the years from 2001 to 2005.

The share of total equity in external assets allocated to the UK region has varied from 14 percent between 1996 and 2000 to 12.6 percent between 2001 and 2005 (13.4 and 14.9 percent respectively for portfolio equity). The average share of debt securities concentrated in the UK is even higher ranging from 23 percent between 1990 and 1995 to 21.4 percent between 2001 and 2005. The UK is particularly notable given that its share of external asset portfolios of other countries is much higher than its share in the total output of the 18 countries considered. This is in part explained by the prominent role of the UK financial industry and the fact that obtaining accurate data for such countries is complicated by the fact that balance of payments statistics are constructed on the basis of the residence principle, that is without accounting for reinvestments of assets Kubelec and Sá (2009) and the fact that UK may be an intermediary of many transactions rather than their ultimate destination. However, as we shall see such a feature can be replicated by a model without some countries acting as financial intermediaries.

Thus, a model that explains external asset allocation must account for three important stylised facts. First, *stocks of external assets are positive*, that is countries do not in aggregate 'go short' in each other's assets. Second, *there is heterogeneity in aggregate average external asset positions*, for example, the US and the UK take long positions in equity and short positions in debt securities. Third, *assets issued by some regions have a larger share of external asset portfolios of other regions than would be suggested by their share of aggregate output.* Such regions are termed financial centres in this paper.

3 A Model of External Asset Allocation

There are X countries in the world indexed by $z = 1, \ldots, X$. Every country z is populated by a continuum of infinitely lived, identical consumers with total mass L_z, whose preferences are described in Section 3.1. The total world population is assumed to be constant and normalised to one, since population growth is unlikely to influence asset allocation over the relatively short time horizons that the model aims to explain. Every period t, each individual in each country z receives an endowment of a traded good unique to that country with a quantity $Y_{z,t}^T$ and an additional endowment of a nontraded good with aggregate quantity $Y_{z,t}^N$.

In every country z, there is a government that levies a lump-sum tax $T_{z,t} \geq 0$ on every individual, issues money $M_{z,t}$ per capita and consumes a bundle of goods $G_{z,t}$ per capita with the same composition of traded and nontraded goods as consumers. Endowments, government spending and money supply are assumed to be exogenous and governed by the processes described in Section 3.2. There is a set of assets to invest in, which consists of riskless bonds and equity for an aggregate mutual fund of the country. All categories of assets are issued by every country z. Assets and their returns are described in Section 3.3.

In each period t, first, new shocks to the endowments of traded and nontraded goods as well as government spending and money supply become known, then returns on assets held from time $t-1$ to time t are determined and paid out. The consumers then decide on assets and money balances to hold from time t to time $t+1$, purchases new assets and consume the rest of the available income. Section 3.4 describes how the steady state external asset allocation is obtained.

3.1 Preferences

The lifetime utility function of every individual in country z is:

$$U_{z,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_{z,s}^{1-\theta} - 1}{1-\theta} + \frac{\chi}{1-\nu} \left[\left(\frac{M_{z,s}}{P_{z,s}} \right)^{1-\nu} - 1 \right] \right). \tag{3.1}$$

In (3.1) $\frac{M_z}{P_z}$ are the holdings of real money balances from country z, $\chi > 0$ is the relative preference for real money balances over consumption and $\nu > 1$ is the coefficient of relative risk aversion for real money balances.[9] The coefficient of relative risk aversion for consumption is $\theta > 0$, which corresponds to the elasticity of intertemporal substitution of $\frac{1}{\theta}$[10] and $0 < \beta < 1$ is the subjective discount factor. Thus, the parameters that regulate attitudes towards risk and discounting are assumed to be the same across all countries.

$C_{z,s}$ in (3.1) is the consumption of a basket of traded and nontraded goods aggregated by a constant elasticity of substitution (CES) index:

$$C_{z,t} = \left[(h_z)^{\frac{1}{\lambda_z}} \left(C_{z,t}^N\right)^{\frac{\lambda_z - 1}{\lambda_z}} + (1-h_z)^{\frac{1}{\lambda_z}} \left(C_{z\Omega,t}^T\right)^{\frac{\lambda_z - 1}{\lambda_z}} \right]^{\frac{\lambda_z}{\lambda_z - 1}}, \tag{3.2}$$

where $C_{z,t}^N$ is the consumption of nontraded goods from country z, $C_{z\Omega,t}^T$ in (3.2) is the consumption of the basket of composite traded goods from X countries of the world (including domestic traded goods) and $0 < h_z < 1$ is a heterogeneous parameter that increases the weight of nontraded goods of region z in the consumption basket. The elasticity of substitution between the

[9]The assumption of a separable utility function in real money balances is made for tractability. Woodford (2003) discusses the reasons that one can ignore real balance effects on the marginal utility of income even without assuming additive separability. The idea involves assuming that money is used in a small amount of transactions, but is essential for those transactions.

[10]The latter parameter is more relevant to the model. It is an unfortunate limitation of the widely used isoelastic utility function that coefficient of relative risk aversion and elasticity of intertemporal substitution are restricted to be reciprocals of each other.

nontraded good and the basket of traded goods is $\lambda_z > 0$. Solving the expenditure minimisation problem yields the aggregate price index corresponding to the consumption basket in (3.2):

$$P_{z,t} = \left[h_z(P^N_{z,t})^{1-\lambda_z} + (1-h_z)\,(P^T_{z\Omega,t})^{1-\lambda_z}\right]^{\frac{1}{1-\lambda_z}}, \tag{3.3}$$

where $P^N_{z,t}$ is the price of nontraded good and $P^T_{z\Omega}$ is the aggregate price index of the consumption basket for traded goods.

The consumption basket for traded goods is given by another CES index:

$$C^T_{z\Omega,t} = \left[\sum_{j\neq z}^{X}\left((\omega_{zj})^{\frac{1}{\gamma_z}}\,(C^T_{zj,t})^{\frac{\gamma_z-1}{\gamma_z}}\right) + (\omega_{zz})^{\frac{1}{\gamma_z}}\,(C^T_{z,t})^{\frac{\gamma_z-1}{\gamma_z}}\right]^{\frac{\gamma_z}{\gamma_z-1}}, \tag{3.4}$$

where $C^T_{zj,t}$ is the consumption of traded goods from region j in region z and ω_{zj} are preference weights in the traded goods basket such that $\sum_{j=1}^{X}\omega_{zj} = 1$ for all z. In (3.4), $\gamma_z > 0$ is the elasticity of substitution between traded goods from different countries. The weight of own traded goods consumption $C^T_{z,t}$ in the overall basket of traded goods is ω_{zz}. Thus ω_{zz} is a heterogeneous parameter that increases the amount of home bias in consumption for country z in the steady state. If $\omega_{zz} > \frac{1}{X}$, households prefer traded goods from their own region more than their average preference for traded goods from other regions and home bias in consumption is present. Standard models with no home bias are nested by the case when $\omega_{zz} = \frac{1}{X}$. If $\omega_{zz} < \frac{1}{X}$, there is a foreign bias in consumption. The aggregate price index for the basket of traded goods is given by:

$$P^T_{z\Omega,t} = \left[\sum_{j\neq z}^{X}\left(\omega_{zj}\left(P^T_{zj,t}\right)^{1-\gamma_z}\right) + \omega_{zz}\left(P^T_{z,t}\right)^{1-\gamma_z}\right]^{\frac{1}{1-\gamma_z}}, \tag{3.5}$$

where $P^T_{zj,t}$ is the price of the traded good from country j in the currency of country z and $P^T_{z,t}$ is the domestic price of the traded good of country z.

Consumption baskets in (3.2) and (3.4) allow for heterogeneity in parameters that regulate tastes for goods from different regions. Preference heterogeneity is introduced in consumption tastes, but not attitudes to risk or discounting (the parameters σ, ν and β) for two reasons. First, the assumption that consumption tastes could be more different than attitudes to risk is intuitive in a sense that agents may more readily disagree over, for example, what brands of goods they prefer than their valuation of the consumption stream or liquidity services from real money balances as such. Second, there is empirical evidence establishing heterogeneity in import demand elasticities for a broad group of countries (Kee et al., 2008).

3.2 Exogenous Driving Forces

The endowments of country z at time t are given by:

$$Y^T_{z,t} = A^T_{z,t}\bar{Y}^T_z, \qquad Y^N_{z,t} = A^N_{z,t}\bar{Y}^N_z, \tag{3.6}$$

where $\bar{Y}_z^T$ is the steady state traded endowment and $\bar{Y}_z^N$ is the steady state nontraded endowment. $A_{z,t}^T$ and $A_{z,t}^N$ are multiplicative stochastic endowment shocks for traded and nontraded endowment respectively. In a nonstochastic steady state $\bar{A}_z^T = 1$ and $\bar{A}_z^N = 1$. The laws of motion for the shocks are:

$$\begin{aligned} \ln A_{z,t}^T &= \rho_z^T \ln A_{z,t-1}^T + \hat{\varepsilon}_{z,t}^T, & \hat{\varepsilon}_{z,t}^T &\sim N\Big(0(\sigma_z^T)^2\Big), & 0 < \rho_z^T < 1, \\ \ln A_{z,t}^N &= \rho_z^N \ln A_{z,t-1}^N + \hat{\varepsilon}_{z,t}^N, & \hat{\varepsilon}_{z,t}^N &\sim N\Big(0, (\sigma_z^N)^2\Big), & 0 < \rho_z^N < 1. \end{aligned} \tag{3.7}$$

Consumption by the government yields no utility to consumers and evolves according to

$$G_{z,t} = \Gamma_{z,t}\bar{G}_z, \tag{3.8}$$

where $\bar{G}_z$ is the steady state government expenditure and $\Gamma_{z,t}$ is the stochastic component, which evolves according to:

$$\ln \Gamma_{z,t} = \rho_z^G \ln \Gamma_{z,t-1} + \hat{\varepsilon}_{z,t}^G, \qquad \hat{\varepsilon}_{z,t}^G \sim N\Big(0, (\sigma_z^G)^2\Big), \quad 0 < \rho_z^G < 1. \tag{3.9}$$

It is again assumed that in the nonstochastic steady state $\bar{\Gamma}_z = 1$. Government spending shocks are introduced to the model in order to ensure that endowment fluctuations are not the only sources of real shocks to the economy. One can define a constant $\bar{g}_z$ as the steady state share of nominal[11] government expenditure in the nominal GDP:

$$\bar{g}_z = \frac{\bar{P}_z\bar{G}_z}{\bar{P}_z^T\bar{Y}_z^T + \bar{P}_z^N\bar{Y}_z^N}. \tag{3.10}$$

In (3.10), $\bar{P}_z$ is the steady state consumption price index, $\bar{P}_z^T$ is the steady state price of traded endowment and $\bar{P}_z^N$ is the steady state price of nontraded endowment.

The budget constraint for the government of country z in real, per capita terms is given by

$$G_{z,t} = T_{z,t} + \frac{M_{z,t} - M_{z,t-1}}{P_{z,t}}. \tag{3.11}$$

Ricardian equivalence holds in this model and one can assume without loss of generality that the government balances its budget every period. In (3.11) $P_{z,t}$ is the consumption price index in period t and $M_{z,t}$ is the period t per capita money supply in country z, which is also assumed to evolve stochastically as

$$M_{z,t} = \Lambda_{z,t}\bar{M}_z, \tag{3.12}$$

where $\bar{M}_z$ is the steady state money supply and $\Lambda_{z,t}$ is the stochastic component, which evolves according to

$$\ln \Lambda_{z,t} = \rho_z^M \ln \Lambda_{z,t-1} + \hat{\varepsilon}_{z,t}^M, \qquad \hat{\varepsilon}_{z,t}^M \sim N\Big(0, (\sigma_z^M)^2\Big), \quad 0 < \rho_z^M < 1. \tag{3.13}$$

[11] Note that throughout this paper the word 'nominal' is used to denote 'measured in monetary terms', rather than measured in constant prices since the model does not feature inflation.

It is assumed that in the nonstochastic steady state $\bar{\Lambda}_z = 1$.

To define exchange rates, let the currency of country 1 be the numeraire and let S_j be the price of country 1 currency in terms of country j currency. The price of country j currency in terms of country z currency is

$$S_{zj,t} = \frac{S_{z,t}}{S_{j,t}}. \tag{3.14}$$

For country z, $S_{zj,t}$ is the domestic price of country j currency and a rise in $S_{zj,t}$ represents a depreciation of the currency of country z against the currency of country j. Naturally, $S_{zz,t} = 1$. It is assumed that the law of one price holds for the prices of traded goods, thus, one can apply (3.14) to write:

$$P^T_{zj,t} = S_{zj} P^T_{j,t} = \frac{S_{z,t}}{S_{j,t}} P^T_{j,t}. \tag{3.15}$$

3.3 Assets and Budget Constraint

External assets are modelled as contracts signed between agents in two different countries. All contracts are assumed to last for one period. Each contract involves one party promising to pay the other party some return in period t in exchange for payment of some price in period $t-1$. The assumption of one period contracts is made for tractability, however, it is not necessary for the solution method adopted. Some asset categories, such as portfolio debt, which is included in debt securities, or portfolio equity, are perhaps better approximated as one period contracts, since they are likely to be motivated by more short-term expectations of macroeconomic fundamentals.

There are two types of contracts that can be signed, which correspond to bonds and equity. To illustrate how asset contracts work consider bond contracts as an example. A country j agent can sign a contract with a country z agent, where country z agent promises to pay country j agent the return on bond in period t in exchange for payment of the price of the bond of country z in period $t-1$. Then, an agent in country j owns bonds issued by agents in country z, which is denoted by a positive position of country j in country z bonds, $B_{jz,t} > 0$. A short position of country j in the country z bonds ($B_{jz,t} < 0$) means that region j agent promises to pay returns to region z agent in period t in exchange for receiving payment of the price of the bond of region z in period $t-1$. Thus, it is a liability of country j to country z. New asset contracts are written every period.

All assets are traded on the globally integrated financial market. Asset prices and returns are denominated in the currency of the issuing country and each asset is assumed to be in zero net supply. The market clearing conditions for all assets issued by country z are given by:

$$\sum_{j=1}^{X} L_j B_{jz,t} = 0, \qquad \sum_{j=1}^{X} L_j K_{jz,t} = 0, \tag{3.16}$$

where B_{jz}, K_{jz}, where $j \neq z$ are the values of country j per capita holdings of respectively bonds and equity issued by country z and measured in the currency of country z. The focus of this paper is on the allocation of assets between $2X$ bonds and equities. The zero net supply assumption and (3.16) merely state that total assets in particular asset category (e.g. country z bonds), which would be denoted by positive $B_{jz,t}$ variables must equal total liabilities in that asset category, which would be denoted by negative $B_{jz,t}$.

To interpret the variable $B_{zz,t}$, consider that the market clearing condition (3.16) for bonds, for example, can be written as: $\sum_{j\neq z}^{X} L_j B_{jz,t} = -L_z B_{zz,t}$, where the left hand side denotes aggregate net holdings of bonds from region z by the rest of the world and the right hand side denotes aggregate net bond liabilities of region z to the rest of the world. According to Lane and Milesi-Ferretti (2007) stocks of external assets and liabilities are generally positive, therefore asset portfolios that match the real data would generally have $B_{jz,t} > 0$ for all $z \neq j$ and $B_{zz,t} < 0$. Note that $B_{zz,t} < 0$ means only that country z has sold some of its issued bonds to other countries, it does not mean that country z aggregate holdings of its issued bonds are negative.

It is a well known result that open economy models with incomplete markets, such as the one proposed in this paper, feature nonstationarity in their equilibrium dynamics. The common methods of addressing the problem are outlined in Schmitt-Grohe and Uribe (2003), who consider a small open economy. These methods include introducing an endogenous discount factor, portfolio adjustment costs, complete asset markets or a debt-elastic interest rate.[12] Given that the focus of this paper is on external asset allocation, the mechanism most appropriate to induce stationarity is the debt-elastic interest rate. Hence in this paper, the interest rate payable on riskless bonds varies with the country's net external asset position in bonds and equities. The per capita net external asset position of country j in bonds and equity is defined in the numeraire currency as:

$$W_{j,t} = \sum_{z=1}^{X} \left[\frac{1}{S_{z,t}} \left(B_{jz,t} + K_{jz,t} \right) \right]. \tag{3.17}$$

If $W_{j,t} < 0$, the country's aggregate external liabilities excess aggregate external assets, that is a country is an international borrower, whereas if $W_{j,t} > 0$ the country is an international lender.

To establish how the interest rate payable on bonds issued by a particular country varies with its net external position in bonds and equity, first, suppose that every country z has a hypothetical riskless real bond, which is a claim on one unit of the consumption bundle consumed by residents of that region. In equilibrium, the expected returns on real bonds would be equal when measured in the same units. Therefore expressing returns on real bonds of country j and country 1 in terms of the consumption bundle of country 1, would yield the real interest parity condition, relating gross returns on real bonds $r_{z,t+1}$ and $r_{1,t+1}$ and real exchange rates $\tilde{S}_{z,t}$ and $\tilde{S}_{z,t+1}$. Real exchange rate of country z to country 1, $\tilde{S}_{z,t} = \frac{P_{1,t} S_{z,t}}{P_{z,t}}$ is the price of the consumption bundle of country 1 in terms of the consumption bundle of country z in

[12] Another method, which does not only eliminate nonstationarity, but makes external asset positions determinate is proposed in Cavallo and Ghironi (2002) and involves overlapping generations. However, since this paper is concerned with medium term (5 years) asset allocation, it is doubtful that the assumption of overlapping generations is appropriate.

period t. The real interest parity condition is:

$$E_t[r_{z,t+1}] = E_t\left[\frac{r_{1,t+1} \times \tilde{S}_{z,t+1}}{\tilde{S}_{z,t}}\right], \tag{3.18}$$

where $r_{z,t+1} = \frac{1}{Q^*_{zB,t}}$, $r_{1,t+1} = \frac{1}{Q^*_{1B,t}}$ and $Q^*_{zB,t}$ is the real price of the real bond issued by region z.

In the model it is assumed that only nominal bonds exist and are traded. Nevertheless, their prices are affected by the risk premium on hypothetical real bonds, since the risk premium, which is decreasing in the ratio of net external assets to GDP, is added to the real returns related by (3.18). The reasons for the risk premium are not modelled explicitly, but could include fears about sustainability of external debt, possible devaluations, etc. Such a risk premium is introduced by, for example, Nason and Rogers (2006), where it varies with the debt to output ratio. Boileau and Normandin (2008) introduce a risk premium in a two country dynamic general equilibrium model, where it is derived from the existence of a financial intermediary facing operating costs. All of these authors impose risk premium on bonds, whose returns are denominated in real variables. The reason for that is that it is necessary to disentangle the risk premium from inflationary expectations and the (second order) effect of inflation risk, in order for it to affect the real consumption decisions of households.

Nominal bonds issued by country z in period $t-1$, similar to Devereux and Sutherland (2008), are modelled as claims on a unit of currency of country z in period t. Gross nominal returns on bonds from country z consist of two parts: $i_{z,t}$ defined such that $i_{z,t}-1$ is the riskless yield to maturity rate and $f(\hat{w}_{z,t-1})$, which is the risk premium. Since bonds are claims on units of currency, the gross nominal return is:

$$i_{z,t} + f(\hat{w}_{z,t-1}) = \frac{1}{Q_{zB,t-1}}, \tag{3.19}$$

where $Q_{zB,t-1}$ is the period $t-1$ price of the nominal bonds issued by country z in the currency of country z and $\hat{w}_{z,t}$ is the deviation of the ratio of per capita net external position in bonds and equity to steady state per capita nominal GDP from its steady state level $\bar{w}_z$ that is:

$$\hat{w}_{z,t} = \frac{\bar{S}_z W_{z,t}}{\bar{P}^T_z \bar{Y}^T_z + \bar{P}^N_z \bar{Y}^N_z} \qquad \bar{w}_{z,t} = \frac{\bar{S}_z \bar{W}_{z,t}}{\bar{P}^T_z \bar{Y}^T_z + \bar{P}^N_z \bar{Y}^N_z}$$

The risk premium $f(\hat{w}_{z,t-1})$ reflects the dependence of the price of the hypothetical real bond on $\hat{w}_{z,t-1}$. The functional form for the risk premium, is given by:[13]

$$f(\hat{w}_{z,t}) = \varsigma\left(e^{-(\hat{w}_{z,t}-\bar{w}_z)} - 1\right), \qquad \frac{\partial f(\hat{w}_{z,t})}{\partial \hat{w}_{z,t}} < 0, \tag{3.20}$$

[13]Lubik (2007) points out that the determinacy and stability properties of models with debt elastic interest rates are sensitive to the choice of the functional form of the risk premium as well as whether the impact of the risk premium is internalised by households. For the functional form considered here, a unique equilibrium always exists.

The assumption that countries are populated by a continuum of atomistic households means that every household perceives aggregate net external asset position in bonds and equities as exogenous, that is it does not internalise the impact of its borrowing decisions on the risk premium.

The fact that $f(\hat{w}_{z,t})$ is decreasing in $\hat{w}_{z,t}$ means that, keeping riskless yield to maturity equal, gross returns on bonds issued by countries whose net external position in bonds and equity is below its steady state level is higher than on bonds issued by countries, whose net external position in bonds and equity is above its steady state level. The parameter $\varsigma > 0$ determines how harshly countries are penalised for changes in net external position in bonds and equities and therefore how quickly the model returns to the steady state. To eliminate nonstationarity in equilibrium dynamics, it is sufficient to impose the risk premium only on bonds.

Gross returns on equity from country z, $d_{z,t}$ are determined by nominal revenues from the sale of traded and nontraded endowments:

$$d_{z,t} = \frac{P^T_{z,t} Y^T_{z,t} + P^N_{z,t} Y^N_{z,t}}{Q_{zK,t-1}}, \tag{3.21}$$

where $Q_{zK,t-1}$ in (3.21) is the period $t-1$ price of equity issued by region z, in the currency of region z, respectively.

Having defined the assets in the model, one can write the nominal budget constraint for in per capita terms for region z as:

$$\begin{aligned}\sum_{j=1}^{X} \Big[S_{zj,t}(B_{zj,t} + K_{zj,t}) \Big] + M_{z,t} &= P^T_{z,t} Y^T_{z,t} + P^N_{z,t} Y^N_{z,t} + M_{z,t-1} - P_{z,t}\Big(C_{z,t} + T_{z,t}\Big) \\ &+ \sum_{j=1}^{X} \left[S_{zj,t} \left(\Big(i_{j,t} + f(\hat{w}_{j,t-1}) \Big) B_{zj,t-1} + d_{j,t} K_{zj,t-1} \right) \right].\end{aligned} \tag{3.22}$$

The budget constraint in (3.22) equates the new stock of bonds and equity as well as money to the difference between the revenues from the sale of traded and nontraded endowments, past stock of money, received or paid returns on assets and expenditure on consumption and taxes.

Combining (3.11) and (3.22) yields an economy-wide budget constraint for region z in per capita, nominal terms and without money balances:

$$\begin{aligned}\sum_{j=1}^{X} \left(S_{zj,t} \left(B_{zj,t} + K_{zj,t} \right) \right) &= \sum_{j=1}^{X} \Big(\Big(i_{j,t} + f(\hat{w}_{j,t-1}) \Big) B_{zj,t-1} S_{zj,t} \Big) \\ &+ \sum_{j=1}^{X} \left(d_{j,t} K_{zj,t-1} S_{zj,t} \right) + P^T_{z,t} Y^T_{z,t} + P^N_{z,t} Y^N_{z,t} - P_{z,t}(C_{z,t} + G_{z,t}).\end{aligned} \tag{3.23}$$

The definition of net external assets in (3.17) allows writing the budget constraint (3.23) for region z measured in numeraire currency:[14]

$$W_{z,t} = \Big(\mathbf{a}'_{z,t}\mathbf{r}_{x,t} + W_{z,t-1}(i_{1,t} + f(\hat{w}_{1,t-1})\Big) + \frac{P^T_{z,t}Y^T_{z,t}}{S_{z,t}} + \frac{P^N_{z,t}Y^N_{z,t}}{S_{z,t}} - \frac{P_{z,t}(C_{z,t} + G_{z,t})}{S_{z,t}}, \qquad (3.24)$$

where $\mathbf{a}'_{z,t}$ is a $2X - 1 \times 1$ vector of all per capita asset holdings in the numeraire currency, apart from the numeraire asset:

$$\mathbf{a}'_{z,t} = \begin{bmatrix} \frac{K_{z1,t-1}}{S_{1,t}} & \frac{B_{z2,t-1}}{S_{2,t}} & \frac{K_{z2,t-1}}{S_{2,t}} & \dots & \frac{B_{zX,t-1}}{S_{X,t}} & \frac{K_{zX,t-1}}{S_{X,t}} \end{bmatrix}$$

and $\mathbf{r}_{x,t}$ is a vector of excess returns adjusted for exchange rate appreciation defined as:

$$\mathbf{r}_{x,t} = \begin{bmatrix} d_{2,t} - (i_{1,t} + f(\hat{w}_{1,t}))\frac{S_{2,t}}{S_{2,t-1}} & \dots & i_{X,t} - (i_{1,t} + f(\hat{w}_{1,t}))\frac{S_{X,t}}{S_{X,t-1}} & d_{X,t} - (i_{1,t} + f(\hat{w}_{1,t}))\frac{S_{X,t}}{S_{X,t-1}} \end{bmatrix}$$

3.4 Model Solution

The solution strategy generalises the method of Devereux and Sutherland (2008) to the choice among assets which are denominated in different currencies and the presence of more than two countries in the model. The method is based on the fact that time variation in portfolio allocation is irrelevant for determining first-order behaviour of macroeconomic variables like consumption or price level. Therefore the optimal asset portfolio can be found by combining a second-order approximation of the portfolio selection condition with a first-order approximation to the remaining parts of the model.

3.4.1 Steady State

The nonstochastic, zero growth and zero inflation steady state of the model described above is a set of X consumptions, $2X$ prices of traded and nontraded goods and $X - 1$ exchange rates $\{\bar{C}_z, \bar{P}^T_z, \bar{P}^N_z, \bar{S}_z\}$ for $z = 1 \dots X$ given the steady state traded and nontraded endowments, government spending, money supply and net external asset position $\{\bar{Y}^T_z, \bar{Y}^N_z, \bar{G}_z, \bar{M}_z, \bar{W}_z\}$ for $z = 1 \dots X$ such that the $2X$ goods market clearing conditions for traded and nontraded goods are satisfied:

$$\begin{aligned} L_z\bar{Y}^T_z &= \sum_{j=1}^{X} L_j \left((1 - h_j)\omega_{jz} \left[\frac{\bar{S}_{jz}\bar{P}^T_z}{\bar{P}^T_{j\Omega}}\right]^{-\gamma_j} \left[\frac{\bar{P}^T_{j\Omega}}{\bar{P}_j}\right]^{-\lambda_j} (\bar{C}_j + \bar{G}_j) \right) \\ \bar{Y}^N_z &= h_z \left[\frac{\bar{P}^N_z}{\bar{P}_z}\right]^{-\lambda_z} (\bar{C}_z + \bar{G}_z), \end{aligned} \qquad (3.25)$$

[14](3.23) becomes (3.24) by adding and subtracting the asset holding and the return on the numeraire, e.g.

$$\frac{K_{zj,t-1}d_{j,t}}{S_{j,t}} = \frac{K_{zj,t-1}}{S_{j,t}}\left(d_{j,t} - (i_{1,t} + f(\hat{w}_{1,t}))\frac{S_{j,t}}{S_{j,t-1}}\right) + \frac{K_{zj,t-1}(i_{1,t} + f(\hat{w}_{1,t}))}{S_{j,t-1}}.$$

where $\bar{P}_j$ and $\bar{P}^T_{j\Omega}$ are calculated using the steady state versions of (3.3) and (3.5) respectively. In (3.25) as well as (3.26), the steady state per capita government expenditure can be expressed as $\bar{G}_z = \frac{\bar{g}_z(\bar{P}^T_z \bar{Y}^T_z + \bar{P}^N_z \bar{Y}^N_z)}{\bar{P}_z}$, where $\bar{g}_z$ is a constant defined in (3.10), which can be obtained from the data.

The steady state must also satisfy $X-1$ steady state versions of the budget constraints (3.24) for $z = 1 \ldots X-1$, which, taking into account that excess returns $\mathbf{r}_{x,t}$ are zero in the steady state, can be written as:

$$\left(\frac{\beta - 1}{\beta}\right)\left(\bar{W}_z\right) = \frac{\left(\bar{P}^T_z \bar{Y}^T_z + \bar{P}^N_z \bar{Y}^N_z - \bar{P}_z(\bar{C}_z + \bar{G}_z)\right)}{\bar{S}_z} \tag{3.26}$$

The last budget constraint for country X is satisfied automatically due to Walras Law. In addition to (3.25) and (3.26), X money market equilibrium conditions derived from (3.32) for any country z must also be satisfied in the steady state:

$$\chi\left(\frac{\bar{M}_z}{\bar{P}_z}\right)^{-\nu} = (1-\beta)\bar{C}_z^{-\theta}, \tag{3.27}$$

From (3.27), one can then express consumption as $\bar{C}_z = \left(\frac{\chi}{1-\beta}\right)^{\frac{-1}{\theta}}\left(\frac{\bar{M}_z}{\bar{P}_z}\right)^{\frac{\nu}{\theta}}$, which can then be plugged into (3.25) and (3.26) thus reducing the number of equations that one needs to solve to $3X-1$. In this paper, it is assumed that $\chi = 1-\beta$ to simply the system of equations. This assumption has no effect on real allocations, since a change in χ, simply changes nominal prices of endowments, but does not affect relative magnitudes. Note that in the steady state above all consumption levels are effectively determined by the money market equilibrium. This means that one can only impose $X-1$ levels of net external asset holdings, $\bar{W}_z$, which can be taken from the data (specifically, Table A.2 in Appendix A) and the remaining net external asset position for country X is determined from the world asset market clearing constraint.

The model described so far in Sections 3.1, 3.2 and 3.3 does not have a closed form solution even for the steady state, except in very special case when, $\lambda_z = 1$, $\gamma_z = 1$, and $\bar{W}_z = 0$ for every country z. One can briefly consider some intuition behind such a nonstochastic steady state with homogeneous consumption tastes, where, for further simplification, one can let for all z: the traded and nontraded baskets to have equal weight, $h_z = \frac{1}{2}$, the degree of home bias be the same for all countries, $\omega_{zz} = \kappa$, and, finally, for all $z \neq j$, $\omega_{zj} = \frac{1-\kappa}{X-1}$. Consumer consumption baskets specified in (3.2) and (3.4) in this case become identical Cobb-Douglas aggregates for every region z. Appendix B derives the terms of trade, real exchange rate and nominal exchange rates for such a steady state.

The steady state terms of trade of country z with country j in (3.28) are determined by the relative abundance of traded endowments (e.g. the aggregate amount of the traded good from country j is $L_j \bar{Y}^T_j$):

$$\frac{\bar{P}^T_z \bar{S}_j}{\bar{S}_z \bar{P}^T_j} = \frac{L_j \bar{Y}^T_j}{L_z \bar{Y}^T_z}. \tag{3.28}$$

The steady state real exchange rate between country z and country j is determined by the relative abundance of both traded and nontraded endowments:

$$\frac{\bar{P}_z \bar{S}_j}{\bar{S}_z \bar{P}_j} = \left(\frac{L_z \bar{Y}_z^T}{L_j \bar{Y}_j^T} \right)^{\frac{1-\kappa X}{2(X-1)}} \left(\frac{L_j \bar{Y}_j^N}{L_z \bar{Y}_z^N} \right)^{\frac{1}{2}} . \tag{3.29}$$

In (3.29), $0 < \kappa < 1$ and $X > 1$. A rise in the aggregate nontraded endowment unambiguously causes a real depreciation, lowering the price of the domestic consumption basket with respect to the foreign consumption basket. The effect of a rise in the aggregate traded endowment depends on whether there is home bias in consumption.

A rise in the aggregate traded endowment lowers the domestic price of the traded endowment, but also increases relative prices of imports, worsening the terms of trade. This can change the price level for the traded goods basket and hence aggregate price level in both directions. If $\kappa < \frac{1}{X}$, that is consumption of traded goods is biased towards foreign goods then a rise in the traded endowment increases the aggregate price level for traded goods, which causes a real appreciation, reflecting Harrod-Balassa-Samuelson effect. If $\kappa > \frac{1}{X}$ then a rise in the aggregate traded endowment lowers the aggregate price level for traded goods, which causes a real depreciation.

If $\kappa = \frac{1}{X}$, that is there is no home bias in consumption, the two effects cancel out and the real exchange rate does not depend on the aggregate endowment of traded goods. As the number of countries becomes very large ($X \to \infty$), the effect on the terms of trade with any individual country becomes negligible $\left(\lim_{X\to\infty} \frac{1-\kappa X}{2(X-1)} = -\frac{\kappa}{2} \right)$ and a rise in traded endowment causes a real depreciation.

The nominal exchange rate between region z and region j is determined by the relative money supply, the relative abundance of traded and nontraded endowments and the relative size of government:

$$\frac{\bar{S}_z}{\bar{S}_j} = \left(\frac{(1-\bar{g}_j)L_z}{(1-\bar{g}_z)L_j} \right)^{\frac{\theta}{\nu}} \left(\frac{\bar{M}_z}{\bar{M}_j} \right) \left(\frac{L_z \bar{Y}_z^T}{L_j \bar{Y}_j^T} \right)^{\frac{(\theta-\nu)(1-\kappa X)}{2\nu(X-1)}} \left(\frac{L_j \bar{Y}_j^N}{L_z \bar{Y}_z^N} \right)^{\frac{\theta-\nu}{2\nu}} \tag{3.30}$$

A rise in the steady state per capita money supply unambiguously leads to a nominal depreciation as shown in (3.30). Similarly, a rise in the steady state level of government spending $\bar{g}_z$ leads to a fall in the demand for currency by the private sector of country z and hence to a nominal depreciation. The effects of aggregate endowment changes on the nominal exchange rate are ambiguous and depend on the relative magnitudes of θ and ν as well as κ and X. If $\theta > \nu$ then a rise in the aggregate nontraded endowment unambiguously leads to a nominal appreciation and the effects of a rise in the traded endowment depend on whether $\kappa > \frac{1}{X}$ as in the case of the real exchange rate. If $\theta < \nu$, then the effects are reversed. The absolute value of the elasticity of real and nominal exchange rate to changes in traded endowments is a decreasing function of X, the more countries there are, the smaller is the effect on the exchange rate.

Even with simplified preferences, the steady state described above in (3.28), (3.29) and (3.30) is unambiguously unique only when $X \leq 3$. When $X > 3$, (3.28), (3.29) and (3.30) continue to describe the unique symmetric equilibrium, which is a solution to an overdetermined system of equations, which could also have other solutions that are not symmetric.

3.4.2 First-Order Linearisation

To begin deriving the portfolio selection condition, note that maximising (3.1) subject to (3.22) yields $2X$ Euler equations for gross asset returns. In other words, for every country $z = 1 \ldots X$ and $j = 1 \ldots X$, one can write:

$$\frac{C_{z,t}^{-\theta}}{P_{z,t}} = \beta E_t \left[\frac{d_{j,t+1} S_{zj,t+1} C_{z,t+1}^{-\theta}}{S_{zj,t} P_{z,t+1}} \right] \qquad \frac{C_{z,t}^{-\theta}}{P_{z,t}} = \beta E_t \left[\frac{(i_{j,t+1} + f(\hat{w}_{j,t}) S_{zj,t+1} C_{z,t+1}^{-\theta}}{S_{zj,t} P_{z,t+1}} \right]. \tag{3.31}$$

Solving the consumer problem also yields an Euler equation for money demand in country z:

$$\frac{C_{z,t}^{-\theta}}{P_{z,t}} = \frac{\chi}{P_{z,t}} \left(\frac{M_{z,t}}{P_{z,t}} \right)^{-\nu} + \beta E_t \left[\frac{C_{z,t+1}^{-\theta}}{P_{z,t+1}} \right]. \tag{3.32}$$

As usual, (3.32) means that the value of converting money to consumption in period t must be equal to the marginal utility derived from the transactions role of money and the discounted value of converting money to consumption in period $t+1$.

Let country 1 bond be the numeraire asset. Combining (3.31) for country z and the numeraire asset with (3.31) for country z and country j equity[15] and linearising the result to the first order for consumption and price level and to the second order for net external asset position, returns and exchange rates yields:

$$\begin{aligned} E_t \Bigg[& \Big(\hat{d}_{j,t+1} - (\hat{i}_{1,t+1} - \beta\varsigma \hat{w}_{1,t}) - (\hat{s}_{j,t+1} - \hat{s}_{j,t}) \Big) - \hat{d}_{j,t+1}(\hat{s}_{j,t+1} - \hat{s}_{j,t}) - \hat{s}_{j,t+1}\hat{s}_{j,t} \\ & + \Big(-\theta \hat{c}_{z,t+1} - \hat{p}_{z,t+1} + (\hat{s}_{z,t+1} - \hat{s}_{z,t}) \Big) \Big(\hat{d}_{j,t+1} - (\hat{i}_{1,t+1} - \beta\varsigma \hat{w}_{1,t}) - (\hat{s}_{j,t+1} - \hat{s}_{j,t}) \Big) \\ & + \frac{1}{2} \Big((\hat{d}_{j,t+1})^2 - \hat{i}_{1,t+1}^2 + \hat{s}_{j,t}^2 + \hat{s}_{j,t+1}^2 - \beta\varsigma \hat{w}_{1,t}^2 \Big) \Bigg] = 0 + O(\epsilon^3), \end{aligned} \tag{3.33}$$

where $\hat{x}_t = \log X_t - \log \bar{X}$ denotes the log-deviation of the variable X from its steady state for all variables, except when otherwise specified.

The linearised combination of Euler equations in (3.33) gives the relationship between the first and second moments of excess return and certain covariances of model variables. Most importantly, (3.33) means that excess return is decreasing in the covariance between the excess return and the linearised stochastic discount factor, which is a measure of appetite for receiving nominal income at time $t+1$: $\Big(-\theta \hat{c}_{z,t+1} - \hat{p}_{z,t+1} + (\hat{s}_{z,t+1} - \hat{s}_{z,t}) \Big)$. Stochastic discount factor (see Benigno (2007) or Campbell (2000)) is higher, when consumption falls below the steady state

[15] Naturally, country j bonds could be used as well.

value (hence the marginal utility of consumption rises), prices fall below the steady state value (hence one can buy more consumption for a unit of nominal income) or there is a depreciation of home currency (hence receiving foreign currency denominated returns becomes more valuable). When the stochastic discount factor is high, agents desire nominal income more, hence an asset, which yields more when stochastic discount factor is high, will have a higher price and a lower excess return.

There are $2X-1$ equations like (3.33) for a given country, one for every asset in the model, except the numeraire asset over which excess returns are defined. For convenience, one can group various elements of (3.33) into several vectors. Let $\hat{\mathbf{s}}_t$ be a $2X-1\times 1$ vector of exchange rate deviations from the steady state:

$$\hat{\mathbf{s}}'_t = \begin{bmatrix} 0 & \hat{s}_{2,t} & \hat{s}_{2,t} & \dots & \hat{s}_{X,t} & \hat{s}_{X,t} \end{bmatrix} \rightarrow \Delta\hat{\mathbf{s}}'_{t+1} = \hat{\mathbf{s}}'_{t+1} - \hat{\mathbf{s}}'_t.$$

The first element of $\hat{\mathbf{s}}_t$ is zero because country 1 currency is the numeraire. Let $\hat{\mathbf{r}}_t$ be a $2X-1\times 1$ vector of deviations from the steady state of returns on all assets but the numeraire:

$$\hat{\mathbf{r}}'_t = \begin{bmatrix} \hat{d}_{1,t} & \hat{i}_{2,t} - \beta\varsigma\hat{w}_{2,t-1} & \hat{d}_{2,t} & \dots & \hat{i}_{X,t} - \beta\varsigma\hat{w}_{X,t-1} & \hat{d}_{X,t} \end{bmatrix},$$

and $\hat{\mathbf{r}}_{x,t}$ be a corresponding $2X-1\times 1$ vector of excess returns over the numeraire asset.[16] Finally, one can group the second- order terms into a $2X-1\times 1$ vector $\hat{\mathbf{r}}^2_{x,t}$.[17] The $2X-1\times 1$ equations of the type of (3.33) can now be written in vector form for any country z as:

$$E_t\Bigg[\Big(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1}\Big) + \frac{1}{2}\hat{\mathbf{r}}^2_{x,t+1} - (\hat{\mathbf{s}}_{t+1}\cdot\hat{\mathbf{s}}_t) - (\hat{\mathbf{r}}_{t+1}\cdot\Delta\hat{\mathbf{s}}_{t+1}) \\ - \Big(\hat{p}_{z,t+1} + \theta\hat{c}_{z,t+1} - (\hat{s}_{z,t+1} - \hat{s}_{z,t})\Big) \times \Big(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1}\Big)\Bigg] = 0 + O(\epsilon^3). \tag{3.34}$$

In (3.34), $\cdot$ is the element-wise multiplication operator. Subtracting (3.34) for country z from (3.34) for country 1 yields:

$$E_t\Bigg[\Big(\theta(\hat{c}_{z,t+1} - \hat{c}_{1,t+1}) + (\hat{p}_{z,t+1} - \hat{s}_{z,t+1} - \hat{p}_{1,t+1}) + \hat{s}_{z,t}\Big) \times \Big(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1}\Big)\Bigg] = 0 + O(\epsilon^3). \tag{3.35}$$

The equilibrium condition in (3.35) means that after exhausting all opportunities for arbitrage, the covariances between the appetite for nominal income and excess returns are equal in any country z and country 1, that is they are equal in all countries. Combining (3.34) and (3.35)[18]

[16] The elements of $\hat{\mathbf{r}}_{x,t}$ are e.g. $\hat{d}_{1,t} - (\hat{i}_{1,t+1} - \beta\varsigma\hat{w}_{1,t})$, etc.

[17] The elements of $\hat{\mathbf{r}}^2_{x,t}$ are $\hat{d}^2_{1,t+1} - \hat{i}^2_{1,t+1} - \beta\varsigma\hat{w}^2_{1,t}$, $\hat{i}^2_{2,t+1} + \beta\varsigma\hat{w}^2_{2,t} + \hat{s}^2_{2,t} + \hat{s}^2_{2,t+1} - \hat{i}^1_{1,t+1} - \beta\varsigma\hat{w}^2_{1,t}$.

[18] To see how (3.36) is obtained note that from (3.35) it follows that:

$$E_t\Bigg[\Big(\theta\hat{c}_{z,t+1} + \hat{p}_{z,t+1} - (\hat{s}_{z,t+1} - \hat{s}_{z,t})\Big)(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1})\Bigg] = E_t[(\theta\hat{c}_{1,t+1} + \hat{p}_{1,t+1})(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1})].$$

yields (3.36) - the equation for excess returns, adjusted for the changes in the exchange rate:

$$E_t[\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1}] = \frac{1}{2}E_t\Bigg[\Big(\theta(\hat{c}_{1,t+1} + \hat{c}_{z,t+1}) + \hat{p}_{z,t+1} + \hat{p}_{1,t+1} - (\hat{s}_{z,t+1} - \hat{s}_{z,t})\Big)\times \Big(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1}\Big)\Bigg] - \frac{1}{2}E_t[\hat{\mathbf{r}}^2_{x,t+1}] + O(\epsilon^3). \tag{3.36}$$

The equilibrium condition in (3.35) and the definition of excess returns in (3.36) are the central equations necessary to solve the model, since (3.35) pins down asset allocation and (3.36) shows that expected excess returns are zero to a first-order approximation.

The three key properties of the solution method proposed by Devereux and Sutherland (2008) are applicable in this case. First, in order to evaluate the left hand side of (3.35), one only needs to derive expressions for the first-order accurate behaviour of consumption, real exchange rate (price levels and nominal exchange rate) and excess returns. Second, the only aspect of the portfolio allocation decision affecting the first-order accurate behaviour of consumption and excess returns is the steady-state portfolio allocation - $\bar{\mathbf{a}}_z$. The reason is that portfolio decision only enters via the term $\mathbf{a}'_{z,t}\mathbf{r}_{x,t}$ in (3.24). Since the steady state returns are equal, the first order expansion of this term is

$$\mathbf{a}'_{z,t-1}\mathbf{r}_{x,t} = \frac{\bar{\mathbf{a}}_z(\hat{\mathbf{r}}_{x,t} - \Delta\hat{\mathbf{s}}_t)}{\beta} + O(\epsilon^2), \tag{3.37}$$

where $\bar{\mathbf{a}}_z$ is a $2X - 1 \times 1$ vector of steady state asset holdings of country z.

Third, to a first-order approximation the term $\bar{\mathbf{a}}'_z(\hat{\mathbf{r}}_{x,t} - \Delta\hat{\mathbf{s}}_t)$ can be considered zero mean i.i.d. random variable. This follows from (3.36), which only contains second order terms. The first-order approximation $\bar{\mathbf{a}}'_z(\hat{\mathbf{r}}_{x,t} - \Delta\hat{\mathbf{s}}_t)$ is therefore a linear combination of zero-mean i.i.d. variables and is itself a zero-mean i.i.d. random variable.

The first-order linearisation of the per capita budget constraint in (3.24) can be written as:

$$\begin{aligned}\hat{w}_{z,t} =& \bar{y}^T_z(\hat{p}^T_z + \hat{y}^T_{z,t} - \hat{s}_{z,t}) + \bar{y}^N_z(\hat{p}^N_z + \hat{y}^N_{z,t} - \hat{s}_{z,t}) - \bar{c}_z(\hat{c}_{z,t} + \hat{p}_{z,t} - \hat{s}_{z,t}) \\ & - \bar{g}_z(\hat{g}_{z,t} + \hat{p}_{z,t} - \hat{s}_{z,t}) + \left(\frac{\bar{w}_z}{\beta}\right)\left(\hat{i}_{1,t} - \beta\varsigma\hat{w}_{1,t-1}\right) + \frac{1}{\beta}\hat{w}_{z,t-1} + \xi_{z,t} + O(\epsilon^2).\end{aligned} \tag{3.38}$$

In (3.38) $\bar{y}^T_z$, $\bar{y}^N_z$, $\bar{c}_z$ and $\xi_{z,t}$ are defined as:

$$\bar{y}^T_z = \frac{\bar{P}^T_z\bar{Y}^T_z}{\bar{P}^T_z\bar{Y}^T_z + \bar{P}^N_z\bar{Y}^N_z} \qquad \bar{y}^N_z = \frac{\bar{P}^N_z\bar{Y}^N_z}{\bar{P}^T_z\bar{Y}^T_z + \bar{P}^N_z\bar{Y}^N_z}$$

$$\bar{c}_z = \frac{\bar{P}_z\bar{C}_z}{\bar{P}^T_z\bar{Y}^T_z + \bar{P}^N_z\bar{Y}^N_z} \qquad \xi_{z,t} = \frac{\bar{S}_z\bar{\mathbf{a}}_z(\hat{\mathbf{r}}_{x,t} - \Delta\hat{\mathbf{s}}_t)}{\beta(\bar{P}^T_z\bar{Y}^T_z + \bar{P}^N_z\bar{Y}^N_z)},$$

which shows how portfolio allocation enters the model via the zero-mean i.i.d. variable $\xi_{z,t}$ in the budget constraint of every country.

To solve the model it is necessary to find the log-deviations of per capita consumption in

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all countries (X variables), the prices of every traded and nontraded good in the currency of own country ($2X$ variables), nominal exchange rates with respect to the numeraire ($X-1$ variables), net external position in bonds and equity for $X-1$ regions and, finally, the riskless interest rate for the numeraire region. The net external position in bonds and equity for the remaining, numeraire region can be obtained using (3.39), which follows from asset market clearing conditions (3.16):

$$\sum_{j=1}^{X}(k_j L_j \hat{w}_{j,t}) = 0, \tag{3.39}$$

where k_j is a constant defined as:

$$k_j = \frac{\bar{P}_j^T \bar{Y}_j^T + \bar{P}_j^N \bar{Y}_j^N}{\bar{P}_1^T \bar{Y}_1^T + \bar{P}_1^N \bar{Y}_1^N}$$

Thus, there are $5X-1$ variables to solve for, which requires $5X-1$ equations. Subtracting (3.38) for country 1 from (3.38) for country z for $z = 2 \ldots X$ allows obtaining $X-1$ of the necessary equations as shown in Appendix C.

Another X of the required equations are provided by the money market equilibrium conditions for country $z = 1 \ldots X$, (3.32), which are linearised as:

$$\beta \hat{p}_{z,t} + \theta \hat{c}_{z,t} = \nu(1-\beta)(\hat{m}_{z,t} - \hat{p}_{z,t}) + \beta E_t[\theta \hat{c}_{z,t+1} + \hat{p}_{z,t+1}] + O(\epsilon^2) \tag{3.40}$$

The $2X$ linearised goods market clearing conditions given in (3.41) and (3.42) provide a further $2X$ of the necessary equations. The linearised market clearing condition for traded goods from country z is:

$$\begin{aligned} \hat{y}_{z,t}^T = \sum_{j=1}^{X} \Bigg[& (1-h_j)\omega_{jz} \left(\frac{L_j}{L_z}\right) \left(\frac{\bar{S}_{jz}\bar{P}_z^T}{\bar{P}_{j\Omega}}\right)^{-\gamma_j} \left(\frac{\bar{P}_{j\Omega}}{\bar{P}_j}\right)^{-\lambda_j} \times \left(\left(\frac{\bar{C}_j + \bar{G}_j}{\bar{Y}_z^T}\right) \times \right. \\ & \left. \Big(-\lambda_j(\hat{p}_{j\Omega,t} - \hat{p}_{j,t}) - \gamma_j(\hat{s}_{zj,t} + \hat{p}_{z,t}^T - \hat{p}_{j\Omega,t})\Big) + \left(\frac{\bar{C}_j}{\bar{Y}_z^T}\right)\hat{c}_{j,t} + \left(\frac{\bar{G}_j}{\bar{Y}_z^T}\right)\hat{g}_{j,t} \right) \Bigg] + O(\epsilon^2), \end{aligned} \tag{3.41}$$

and, similarly, the linearised market clearing condition for nontraded good is:

$$\hat{y}_{z,t}^N = h_z \left(\frac{\bar{P}_z^N}{\bar{P}_z}\right)^{-\lambda_z} \left(-\lambda_z(\hat{p}_{N,t} - \hat{p}_t)\left(\frac{\bar{C}_z + \bar{G}_z}{\bar{Y}_z^N}\right) + \hat{c}_t \frac{\bar{C}_z}{\bar{Y}_z^N} + \hat{g}_t \frac{\bar{G}_z}{\bar{Y}_z^N} \right) + O(\epsilon^2). \tag{3.42}$$

Appendix C also derives $X-1$ linearised risk sharing conditions for country $z = 2 \ldots X$:

$$\begin{aligned} & \theta(\hat{c}_{z,t} - \hat{c}_{1,t}) + (\hat{p}_{z,t} - \hat{p}_{1,t} - \hat{s}_{z,t}) \\ & \quad = E_t\Big[\theta(\hat{c}_{z,t+1} - \hat{c}_{1,t+1}) + (\hat{p}_{z,t+1} - \hat{p}_{1,t+1} - \hat{s}_{z,t+1}) + \beta\varsigma(\hat{w}_{z,t} - \hat{w}_{1,t})\Big] + O(\epsilon^2). \end{aligned} \tag{3.43}$$

The final equation, which incorporates the riskless rate on the numeraire bond[19] is simply the

[19] Note, that this value is already known at time t.

linearised Euler equation for country 1:

$$-\theta\hat{c}_{1,t} - \hat{p}_{1,t} = E_t[\hat{i}_{1,t+1} - \beta\varsigma\hat{w}_{1,t} - \theta\hat{c}_{1,t+1} - \hat{p}_{1,t+1}] + O(\epsilon^2). \tag{3.44}$$

Thus, there are $5X-1$ equations that one can solve for $5X-1$ unknowns. In (3.38), (3.41), (3.42), (3.43) and (3.44) the deviation from the steady state of the aggregate price index is given by:

$$\hat{p}_{z,t} = h_z \left(\frac{\bar{P}_z^N}{\bar{P}_z}\right)^{1-\lambda_z} \hat{p}_t^N + (1-h_z)\left(\frac{\bar{P}_{z\Omega}^T}{\bar{P}_z}\right)^{1-\lambda_z} \hat{p}_{z\Omega,t}^T + O(\epsilon^2), \tag{3.45}$$

and the linearised price index for the aggregator of traded goods (3.4) is:

$$\hat{p}_{z\Omega,t}^T = \sum_{j\neq z}\left[(\omega_{zj})\left(\frac{\bar{P}_{zj}^T}{\bar{P}_{z\Omega}^T}\right)^{1-\gamma_z} \hat{p}_{zj,t}^T\right] + (\omega_{zz})\left(\frac{\bar{P}_z^T}{\bar{P}_{z\Omega}^T}\right)^{1-\gamma_z} \hat{p}_{z,t}^T + O(\epsilon^2). \tag{3.46}$$

The linearised price of a good from region j in the currency of region z in (3.46) can be obtained from (3.15) and (3.14):

$$\hat{p}_{zj,t}^T = \hat{p}_{j,t}^T + \hat{s}_{z,t} - \hat{s}_{j,t} + O(\epsilon^2).$$

The first-order approximate solution for consumption, prices and exchange rates is obtained by applying the method of Christiano (2002) described in Appendix C. Using this solution, one can express the relationship between excess returns, after adjustment for exchange rate changes, exogenous innovations in the model as well as innovations to wealth of $X-1$ regions (i.e. all regions, but the numeraire), which is given by:

$$\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1} = \mathbf{R}_1\hat{\mathbf{e}}_{t+1} + \mathbf{R}_2\mathbf{u}_{t+1} + O(\epsilon^2), \tag{3.47}$$

where $\hat{\mathbf{e}}_t$ is a $4X \times 1$ vector, containing exogenous shocks stacked by country:

$$\hat{\mathbf{e}}'_{t+1} = \begin{bmatrix}\hat{\varepsilon}_{1,t+1}^T & \hat{\varepsilon}_{1,t+1}^N & \hat{\varepsilon}_{1,t+1}^G & \hat{\varepsilon}_{1,t+1}^M & \dots & \hat{\varepsilon}_{X,t+1}^T & \hat{\varepsilon}_{X,t+1}^N & \hat{\varepsilon}_{X,t+1}^G & \hat{\varepsilon}_{X,t+1}^M\end{bmatrix},$$

and $\mathbf{u}_{t+1}$ is an $X-1\times 1$ vector, containing innovations to net external assets to all regions except the numeraire:

$$\mathbf{u}'_{t+1} = \begin{bmatrix}\xi_{2,t+1} & \dots & \dots & \xi_{X-1,t+1} & \xi_{X,t+1}\end{bmatrix}.$$

$\mathbf{R}_1$ in (3.47) is a $2X-1\times 4X$ matrix and $\mathbf{R}_2$ is a $2X-1\times X-1$ matrix, whose elements are obtained from the first-order approximate solution in Appendix C. Applying the original definition of $\xi_{z,t}$, yields $\mathbf{u}_{t+1} = \mathbf{H}'(\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1})$, where $\mathbf{H}'$ is an $X-1\times 2X-1$ matrix containing the ratios of asset holdings to nominal GDP:

$$\mathbf{H}' = \begin{bmatrix}\frac{\bar{S}_2\bar{\mathbf{a}}_2}{\bar{P}_2^N\bar{Y}_2^N + \bar{P}_2^T\bar{Y}_2^T} & \dots & \frac{\bar{S}_X\bar{\mathbf{a}}_X}{\bar{P}_X^N\bar{Y}_X^N + \bar{P}_X^T\bar{Y}_X^T}\end{bmatrix}.$$

One can multiply (3.47) by $\mathbf{H}'$ and rearrange to obtain:

$$\mathbf{u}_{t+1} = (\mathbf{I}_{X-1} - \mathbf{H}'\mathbf{R}_2)^{-1}\mathbf{H}'\mathbf{R}_1\hat{\mathbf{e}}_{t+1} = \tilde{\mathbf{R}}\hat{\mathbf{e}}_{t+1}, \tag{3.48}$$

where $\tilde{\mathbf{R}}$ is a $X-1 \times 4X$ matrix. Combining (3.47) and (3.48) yields:

$$\hat{\mathbf{r}}_{x,t+1} - \Delta\hat{\mathbf{s}}_{t+1} = \left(\mathbf{R}_1 + \mathbf{R}_2\tilde{\mathbf{R}}\right)\hat{\mathbf{e}}_{t+1} + O(\epsilon^2).$$

The next step is to express another part of (3.35) as the function of shocks in the model. The solution in Appendix C allows writing:

$$\theta(\hat{c}_{z,t+1} - \hat{c}_{1,t+1}) + (\hat{p}_{z,t+1} - \hat{s}_{z,t+1} - \hat{p}_{1,t+1}) + \hat{s}_{z,t} = (\mathbf{q}_{z,1} + \mathbf{q}_{z,2}\tilde{\mathbf{R}})\hat{\mathbf{e}}_{t+1} + \mathbf{q}_{z,3}\mathbf{x}_t + O(\epsilon^2), \tag{3.49}$$

where $\mathbf{q}_{z,1}$ is $1 \times 4X$ vector, $\mathbf{q}_{z,2}$ is a $1 \times X-1$ vector and $\mathbf{x}_t$ is a $5X-1 \times 1$ vector containing values of exogenous variables and portfolio innovations at time t and $\mathbf{q}_3$ is a corresponding $1 \times 5X-1$ vector. Combining (3.47) and (3.49) allows rewriting (3.35) for regions z and 1 as the following matrix equation:

$$E_t\left[\left((\mathbf{q}_{z,1} + \mathbf{q}_{z,2}\tilde{\mathbf{R}})\hat{\mathbf{e}}_{t+1} + \mathbf{q}_{z,3}\mathbf{x}_t\right) \times \left(\mathbf{R}_1 + \mathbf{R}_2\tilde{\mathbf{R}}\right)\hat{\mathbf{e}}_{t+1}\right] = \mathbf{0}_{2X-1\times 1} + O(\epsilon^3). \tag{3.50}$$

Given that $\left((\mathbf{q}_{z,1} + \mathbf{q}_{z,2}\tilde{\mathbf{R}})\hat{\mathbf{e}}_{t+1} + \mathbf{q}_{z,3}\mathbf{x}_t\right)$ is a scalar, (3.50) is a system of $2X-1$ equations in $(2X-1)(X-1)$ unknowns. However, stacking (3.50) for all regions except the numeraire (that is $X-1$ regions), one obtains enough equations to determine the solution for $2X-1$ elements of the portfolio for $X-1$ countries. The remaining element of these portfolios can be found from the fact that $\bar{W}_j$ is known from the parametrization of the steady state. Finally, the portfolio of the numeraire country can be found using the asset market clearing conditions given by (3.16).

If shocks are independent across time, then $E_t[\hat{\mathbf{e}}_{t+1}\hat{\mathbf{e}}'_{t+1}] = \mathbf{\Sigma}$, where $\mathbf{\Sigma}$ is the $4X \times 4X$ covariance matrix of the exogenous shocks and $E_t[\hat{\mathbf{e}}_{t+1}\mathbf{x}'_{t+1}] = 0$. Taking expectations and stacking (3.50) for $X-1$ regions yields:

$$\left(\mathbf{R}_1 + \mathbf{R}_2\tilde{\mathbf{R}}\right)\mathbf{\Sigma}\left(\mathbf{Q}_1 + \mathbf{Q}_2\tilde{\mathbf{R}}\right)' = \mathbf{0}_{2X-1\times X-1} + O(\epsilon^3). \tag{3.51}$$

In (3.51) $\mathbf{Q}_1$ is an $X-1\times 4X$ matrix given by $\mathbf{Q}'_1 = \begin{bmatrix}\mathbf{q}_{2,1} & \dots & \mathbf{q}_{X,1}\end{bmatrix}$ and $\mathbf{Q}_2$ is an $X-1\times X-1$ matrix given by $\mathbf{Q}'_2 = \begin{bmatrix}\mathbf{q}_{2,2} & \dots & \mathbf{q}_{X,2}\end{bmatrix}$.

In general (3.51) may have more than one solution, which is also acknowledged by Devereux and Sutherland (2008). A specific solution, which always exists and has appealing theoretical interpretation as shown in Section 4, is derived by assuming that the matrix $(\mathbf{I}_{2X-1} + \mathbf{R}_2(\mathbf{I} - \mathbf{H}'\mathbf{R}_2)^{-1}\mathbf{H}')$ is invertible and that other appropriate inverses exist in which case a solution to (3.51) is:[20]

$$\mathbf{H} = [\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1'(\mathbf{Q}_2')^{-1}\mathbf{R}_2' - \mathbf{R}_1\boldsymbol{\Sigma}\mathbf{R}_1']^{-1}\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1'(\mathbf{Q}_2')^{-1} + O(\epsilon). \tag{3.52}$$

When there are only two countries, $\mathbf{Q}_2$ is a scalar and (3.52) can be written as:

$$\mathbf{H} = \left[\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1'\mathbf{R}_2' - \mathbf{Q}_2\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{R}_1'\right]^{-1}\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1' + O(\epsilon), \tag{3.53}$$

which is the formula provided in Devereux and Sutherland (2008). Note that (3.52) means that the scale of the covariance matrix $\boldsymbol{\Sigma}$ does not matter because multiplying $\boldsymbol{\Sigma}$ by a scalar will not change the optimum steady state asset allocation.

The seeming possibility of multiple solutions, however, is less troubling when one realises that another way to write (3.51) is in the form $\mathbf{AH} + \mathbf{HB} = \mathbf{C}$, where:

$$\mathbf{A} = -[\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{R}_1']^{-1}\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1'(\mathbf{Q}_2')^{-1}\mathbf{R}_2' \quad \mathbf{B} = \mathbf{I}_{X-1} \quad \mathbf{C} = -[\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{R}_1']^{-1}\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{Q}_1'(\mathbf{Q}_2')^{-1}$$

This formulation, which is known as Sylvester equation, is advantageous because it is possible to verify whether multiple solutions exist. A Sylvester equation has a unique solution if and only if matrices $\mathbf{A}$ and $\mathbf{B}$ have no common eigenvalues. In numerical simulations in Sections 4 and 5 this always turned out to be the case suggesting that the solution in (3.52) is the most relevant one.

4 Theoretical Properties of Portfolio Allocation

To build an intuitive understanding of the model described and solved in Section 3, in this section some simplified calibrations are considered. In particular, the focus is on analysing conditions under which home bias in equity holdings can arise, conditions when one country has net assets in equity and net liabilities in bonds, whereas another country has an opposite situation (similar to the US as 'a venture capitalist of the world' described by Gourinchas and Rey (2005)) and conditions when external asset holdings of more than one country are biased towards a particular country. Unless otherwise noted the calibrations presented below are sensitive to the assumed values of preference parameters. The calibrations are meant to illustrate some of the driving factors behind the model and illustrate that the model can match empirically relevant situations. Section 5 provides a more empirical application of the model.

For benchmark calibrations, unless it is specified otherwise, it will be assumed that the world consists of two countries 'Home', whose currency is the numeraire, and 'Foreign'. Both countries have equal steady state traded and nontraded endowments as well as money supply (that is $\bar{Y}_z^T = \bar{Y}^T$, $\bar{Y}_z^N = \bar{Y}^N$ and $\bar{M}_z = \bar{M}$ for all z), both countries have zero steady state net external assets, $\bar{w}_z = 0$ for all z and there is no government spending in the steady state

[20] To see how (3.52) is derived, one can first expand (3.51) using (3.48) and define two matrices $\mathbf{R}^1 = \mathbf{R}_2(\mathbf{I} - \mathbf{H}'\mathbf{R}_2)^{-1}\mathbf{H}'$ and $\mathbf{R}^2 = \mathbf{H}(\mathbf{I} - \mathbf{R}_2'\mathbf{H})^{-1}\mathbf{Q}_2'$. Further rearranging yields (3.52). Note that the case when $(\mathbf{I}_{2X-1} + \mathbf{R}_2(\mathbf{I} - \mathbf{H}'\mathbf{R}_2)^{-1}\mathbf{H}')$ is a matrix of zeros is another solution.

$\bar{G}_z = \bar{G} = 0$. Assume also that preferences are the same across countries ($h_z = h$, $\lambda_z = \lambda$, $\omega_{zz} = \omega$ and $\gamma_z = \gamma$ for all z), that there is no home bias ($\omega = \frac{1}{2}$ for all j). Finally, assume that all shocks are white noise ($\rho_z^x = 0$ for all z and $x \in \{T, N, G, M\}$) and that the variance of all shocks is the same in Home and Foreign country and equal to σ^2.

The reason it is necessary to consider numerical calibrations is that an analytical solution for the steady state is available only under restrictive assumptions described in Section 3.4.1 and the solution for the first-order approximation of the model is available only when there is no risk premium, that is $\varsigma = 0$. A model with $\varsigma = 0$ would suffer from the nonstationarity problem described in Section 3.4.2 and therefore the focus is on numerical calibration. In any case, an analytical solution to the model with more than two countries and potentially heterogeneous preference parameters is too complex to be of any practical use.

The solution to the asset allocation problem given by (3.52) depends on the invertibility of the matrix $[\mathbf{R}_1 \mathbf{\Sigma} \mathbf{Q}_1' (\mathbf{Q}_2')^{-1} \mathbf{R}_2' - \mathbf{R}_1 \mathbf{\Sigma} \mathbf{R}_1']$. An interesting case for which this matrix is not invertible is when the elasticities of substitution are $\lambda = \gamma = 1$ there is equal preference for traded goods basket and nontraded goods $h = \frac{1}{2}$ and some shocks in the model have their variance set to zero.

If the variance of shocks to nontraded endowments is set to zero, financial assets are redundant and the equilibrium portfolio involves the Home country exchanging any amount of its equity for the same amount of Foreign equity and taking a zero position in bonds. The reason for such an equilibrium is that the terms of trade channel is sufficient to hedge risks to traded endowment. Following a positive transitory shock to Home traded endowment, Home terms of trade decline proportionately. Due to the increased relative abundance of Home traded endowment, the relative price of nontraded goods to the traded goods basket rises in both cases and consumption also rises in both countries.[21] There is no change in net external asset position of both countries. Thus, international trade in goods alone can ensure perfect risk sharing and financial assets are redundant. The same result occurs when one eliminates nontraded goods from the model entirely by removing nontraded goods market clearing conditions and adjusting the model accordingly. This interaction between trade in goods and trade in financial assets has been noted first by Cole and Obstfeld (1991) and is also underscored by Corsetti et al. (2008).

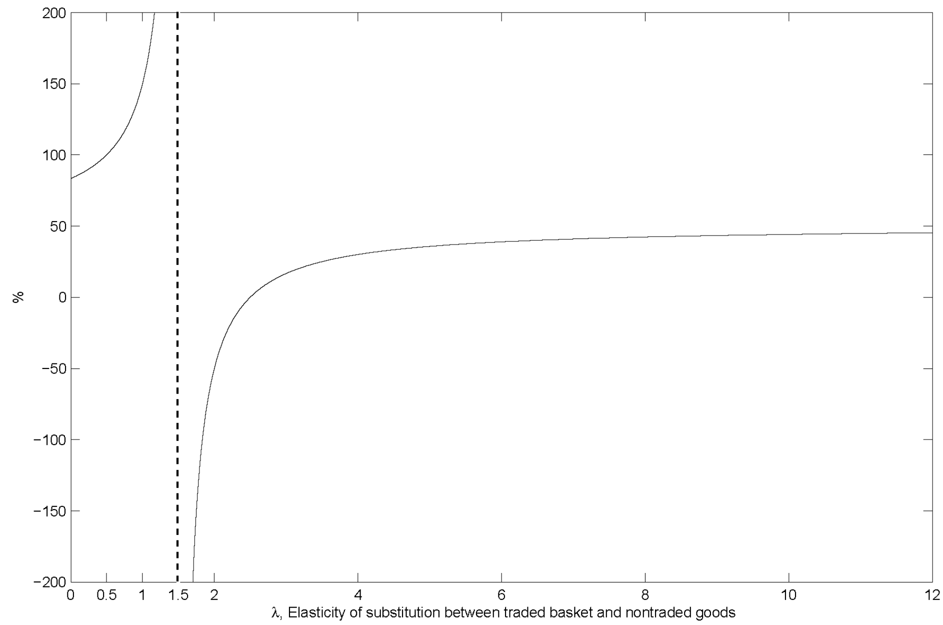
4.1 Home Bias in Equity Holdings

Suppose one further restricts our benchmark preferences to the case when the elasticity of substitution between traded goods $\gamma = 1$. Portfolio allocation of equities will then depend on the interaction between two preference parameters: the elasticity of substitution between traded and nontraded goods, λ, and the intertemporal elasticity of substitution, $\frac{1}{\theta}$.

Figure 4.1 shows how home share of home equity holdings depends on the λ parameter.

[21] Specifically a 1% shock to traded endowment leads to 1% decline in the terms of trade, an 0.5% rise in the relative price of nontraded goods in both country and an 0.25% rise in consumption in both countries, assuming there's no government spending.

Figure 4.1: Home Share of Home Equity as Function of λ



Notes: Other assumptions are $\theta = \nu = 2$, $h = \frac{1}{2}$, $\gamma = 1$ and $\omega = \frac{1}{2}$ $\beta = 0.98$ and $\varsigma = 0.0015$.

Note that to interpret external portfolio allocation in terms of home bias in equity holdings it is assumed that every country initially starts owning 100 percent of its equity, that is 100 percent of the claims to its future nominal GDP, which, in the steady state, has the value of $\beta(\bar{P}_z^T \bar{Y}_z^T + \bar{P}_z^N \bar{Y}_z^N)$, and no bonds. Home share of home equity holdings for country z can then be calculated as:

$$H_z = 1 + \frac{\bar{K}_{zz}}{\bar{P}_z^T \bar{Y}_z^T + \bar{P}_z^N \bar{Y}_z^N}$$

Recall that $\bar{K}_{zz} < 0$ in the data. Home bias in equity holdings implies that $H_z > 1/X$ with X being the number of countries. In other words a small external liability in equity as a share of GDP corresponds in the model to high levels of home bias in equity holdings.

The graph in Figure 4.1 is similar to the one originally obtained by Baxter et al. (1998), who studied optimal asset allocation with separate equity for traded and nontraded goods industries in a model with complete markets. In this paper, this result is replicated in a stochastic setting with incomplete markets and equity modelled as a claim on the country's aggregate mutual fund. The allocation in Figure 4.1 is caused by two effects: the intratemporal substitution between the nontraded good and the traded goods basket, and the intertemporal substitution between consumption today and in the future.

When $\lambda < \frac{1}{\theta}$, households are more inclined to substitute consumption over time than substitute between traded and nontraded endowments. In particular, this means that the marginal utility of consumption of additional traded endowment is very high when there's a high amount of nontraded endowment. Hence, households desire a portfolio that delivers a high pay-out when the endowment of nontraded goods is high. However, in such an environment the payoff to the domestic equity portfolio is low when nontraded goods endowment is high, because the nominal price of nontraded endowment falls more than proportionately. Hence, the desired hedging of endowment risk is obtained by holding less than 100 percent of domestic equity.

As the intratemporal elasticity of substitution λ increases above $\frac{1}{\theta}$, households are more inclined to substitute between traded and nontraded endowments than across time. Hence, households now desire a portfolio that will have a higher payoff when the endowment of the nontraded good is low. For low values of the elasticity of intratemporal substitution, the price effect from the change in quantity of the nontraded endowment will still outweigh the quantity effect, but will be decreasing, so the optimal share of Home equity in Home portfolio increases. At the asymptote point in Figure 4.1, which is marked by the dotted line, the price and quantity effects of the rise in the nontraded goods endowment exactly offset and hence equity cannot be used to hedge risks.

As λ rises further, households are still more inclined to substitute across goods than across time and hence still desire a portfolio that achieves a low payoff when the endowment of nontraded good is high. However, since now quantity effect from the change in endowment outweighs the price effect, home nontraded equity delivers a higher payoff when the endowment of the home nontraded good is high, so it becomes optimal to take short positions in own nontraded goods sector equity. As the intratemporal elasticity of substitution rises, the price effect becomes weaker for both home and foreign holdings of equity in nontraded industries and in the limit it simply becomes optimal to hold pooled portfolios of home and foreign nontraded industry equity.

The shape of Figure 4.1 remains robust to varying the coefficient of relative risk aversion for real money balances ν, risk premium ς, and the discount factor β within their reasonable values.[22] The shape of the graph in Figure 4.1 is also robust to changing θ so long as $\theta > 1$, only the location of the asymptote and the point where it becomes optimal to hold more than 100% of Home equity portfolio change accordingly. Figure 4.1 is sensitive to changing the values of γ, the elasticity of substitution between different traded goods, h, the weight on nontraded goods in the overall consumption index and ω, the weight on own traded goods in the traded goods consumption index.

One of the reasons for the discontinuity observed in Figure 4.1 is the fact that in this calibration terms of trade allow perfect sharing of the risks to traded endowment. Relaxing this assumption introduces additional reasons for holding equity.[23]

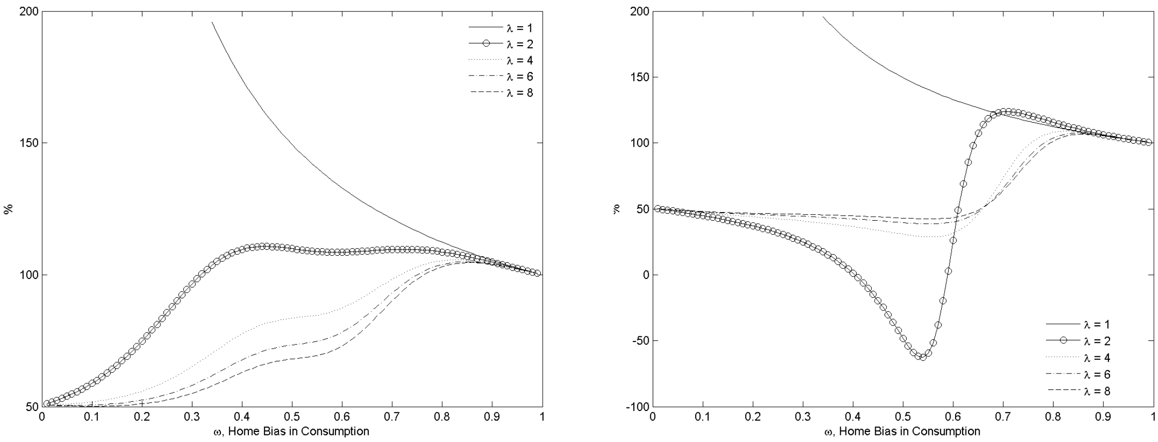
Figure 4.2 shows the home share of home equity as a function of ω, which controls home bias, in consumption for various values of λ. In particular, Figure 4.2(a) shows how the Home share of Home equity changes when $\frac{1}{\theta} = 2.5$, which, as shown in Section 5, seems to be the empirically relevant case and Figure 4.2(b) shows how the behaviour changes when $\frac{1}{\theta} = 0.5$, which is a more commonly accepted value in the literature.

For $\lambda = 1$, the behaviour of Home share of Home equity appears similar whether the

[22] Robustness is verified by letting $1 \leq \nu \leq 10$ with a step of one, $0 \leq \varsigma \leq 1$ with a step of 0.01, $0.95 \leq \beta \leq 0.995$ with a step of 0.01.

[23] One can also reformulate the model so that separate asset classes are available for equity of traded and nontraded industries. In that case, the amount of home bias is actually smaller *ceteris paribus*, since claims to nontraded goods industry are more responsive to shocks to nontraded endowment than claims to the aggregate mutual fund.

Figure 4.2: Home Share of Home Equity as Function of ω



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\gamma = 1$ and $\varsigma = 0.0015$.

elasticity of intertemporal substitution is greater or less than one. For all values of $\omega < 1$ it is optimal for Home residents to hold more than 100% of Home equity, with the optimal share approaching 100% (in other words, no asset trade in equity), when $\omega = 1$. If $\omega = 1$ households only value their own traded and nontraded endowment. Given that endowment shocks are independent and Home prices (and equity returns) do not respond to Foreign shocks, trade in equity (or even goods) cannot generate any risk sharing benefits and hence there are no external assets or liabilities. This fact does not depend on the values of λ or $\frac{1}{\theta}$ as seen in Figures 4.2(a) and Figures 4.2(b).

To understand intermediate cases in Figures 4.2(a) and Figures 4.2(b), consider three possible intratemporal elasticities of substitution: between Home traded endowment and Home nontraded endowment, between Foreign traded endowment and Home nontraded endowment and, finally, Home traded and nontraded endowment. The intratemporal elasticity of substitution is defined as:

$$e(C_X, C_Y) = \frac{d \ln \left(\frac{C_X}{C_Y} \right)}{d \ln \left(\frac{U_{C_Y}}{U_{C_X}} \right)} \tag{4.1}$$

In the symmetric steady state with equal endowments and money supply, all relative prices are equal to one and do not depend on preferences (so long as preferences are the same across countries). Hence, imposing the simplifying assumptions $h = \frac{1}{2}$ and $\gamma = 1$, and noting that at the steady state: $\frac{\bar{C}_{HH}}{\bar{C}_H^N} = \omega$ and $\frac{\bar{C}_{HF}}{\bar{C}_H^N} = (1 - \omega)$ one can apply (4.1) to write the intratemporal elasticities of substitution as:

$$e(\bar{C}_{HF}, \bar{C}_H^N) = \frac{\lambda\omega}{1 - \omega - \lambda(1 - 2\omega)} \quad e(\bar{C}_{HH}, \bar{C}_H^N) = \frac{\lambda(1 - \omega)}{\lambda(1 - 2\omega) + \omega} \quad e(\bar{C}_{HF}, \bar{C}_{HH}) = 1 \tag{4.2}$$

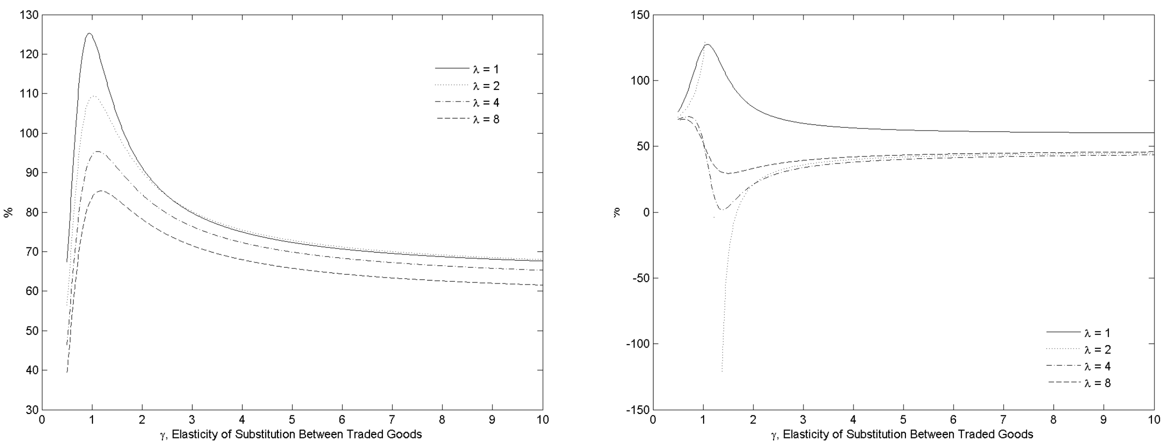
The elasticity of substitution between foreign and home traded endowments is equal to one, because $\gamma = 1$ and it is not affected by being nested in the additional CES aggregator (3.2). Notice that when $\omega = \frac{1}{2}$, $e(\bar{C}_{HF}, \bar{C}_H^N) = e(\bar{C}_{HH}, \bar{C}_H^N) = \lambda$, which is consistent with the intuition behind the result by Baxter et al. (1998) described previously. When $\lambda = 1$, all three elasticities are equal to one. In Figure 4.2(a), this means that the intertemporal substitution effect dominates the intratemporal one. Hence, Home households desire equity portfolio with high pay off when Home traded or nontraded endowments are high. In Figure 4.2(b), on the other hand, households desire a portfolio with low payoff when Home endowments are high, since the intratemporal substitution effect is now stronger. For low values of elasticities of intratemporal substitution, $\lambda = 1$ and $\gamma = 1$, however, the price effect from the change in quantity of the nontraded endowment outweighs the quantity effect, hence Home equity provides a better hedge against Home endowment risks.[24]

When $\frac{1}{\theta} = 1$, neither λ, nor ω affect the optimal Home share of Home equity and it is equal to 50%, that is full diversification in equities is optimal. Varying the values of ν does not change asset allocation in Figure 4.2, instead it only affects the responsiveness of bonds to macroeconomic shocks and changes the bond position that also arise when varying the home bias parameter ω.

The last two details to notice about Figure 4.2 is, first, that for $\lambda > 1$, and extreme values of $\omega = 0$ or $\omega = 1$, optimal equity allocations do not depend on λ. This is due to the fact elasticities in (4.2) do not depend on λ for extreme values of ω. Second, as λ increases there is less difference between optimum asset allocations. This is due to the fact that the changes in equity allocation arise partly due to the price effect of changes in endowment, which becomes smaller as elasticity of substitution rises.

It is also worth emphasizing that allowing for home or foreign bias in consumption (setting $\omega = \frac{1}{2}$, makes nominal variables responsive to fluctuations in endowments and hence makes bonds useful as a hedging instrument as well. This was not the case for the result of Baxter et al. (1998).

Figure 4.3: Home Share of Home Equity as Function of γ



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\omega = \frac{1}{2}$, $\gamma = 1$ and $\varsigma = 0.0015$.

Figure 4.3 shows the Home share of Home equity in Home portfolio as a function of γ, the

[24]Only endowment shocks can be considered here,because steady state government spending is assumed to be zero and money supply shocks do not affect the real exchange rate or consumption differential in (3.35).

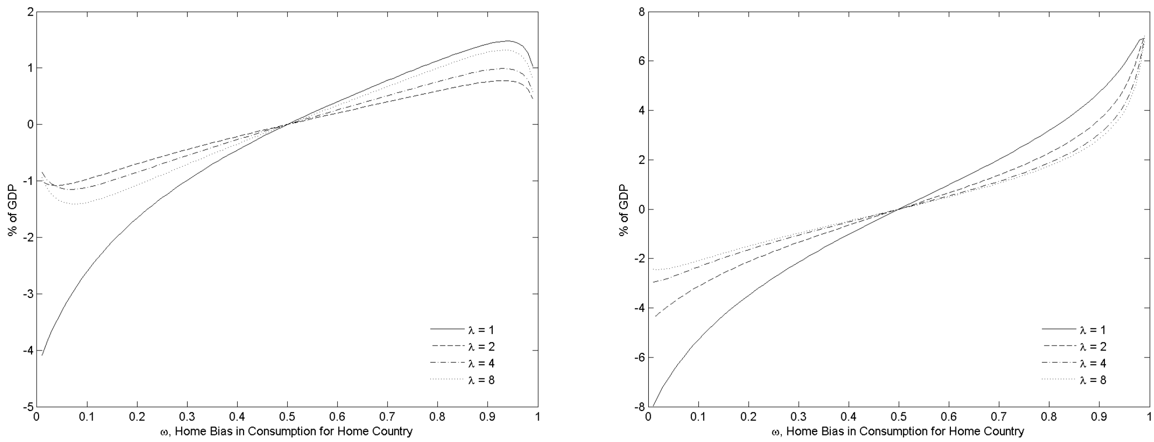
elasticity of substitution between different traded goods, for various values of λ. There are some differences in optimum asset allocation depending on whether $\frac{1}{\theta} < 1$ or $\frac{1}{\theta} > 1$, but for high enough elasticities γ and λ, plausible values for Home bias in equity holdings can be obtained.

Thus, Section 4.1 shows that home bias in equity holdings can be easily obtained simply by appropriate interactions between intertemporal and intratemporal substitution effects and modelling equity as a claim on the nominal GDP of the country. The intuition of the equilibrium condition (3.35) of equating the covariance between the stochastic discount factor and excess returns underlies more elaborate explanations for particular combinations of preference parameters.

4.2 Net Assets in Different Asset Categories

In Section 4.1 Home and Foreign countries were entirely symmetric in all relevant characteristics and, as a result, the optimal asset allocation was also symmetric that is Home holdings of Foreign equity were equal to Foreign Holdings of Home Equity and the same applied to bond holdings. In order to replicate the empirically relevant case of countries having asymmetric asset allocations it is necessary to postulate some heterogeneity in country preferences and/or macroeconomic fundamentals.

Figure 4.4: Home Net Equity Exposure as a Function of ω_H



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\gamma = 4$, $\omega_F = \frac{1}{2}$ and $\varsigma = 0.0015$.

Figure 4.4 shows Home net equity exposure in percent of GDP as a function of Home parameter regulating home bias in consumption ω_H, while keeping $\omega_F = \frac{1}{2}$. Home net equity exposure is defined as:[25]

$$\bar{K}_{net} = \frac{\bar{K}_{HH} + \frac{\bar{K}_{HF}}{\bar{S}_F}}{\bar{P}_H^T \bar{Y}_H^T + \bar{P}_H^N \bar{Y}_H^N}$$

Market clearing conditions mean that Foreign net equity exposure is equal and opposite to Home. It is easiest to see some intuition behind Figure 4.4 if one focuses on departures from

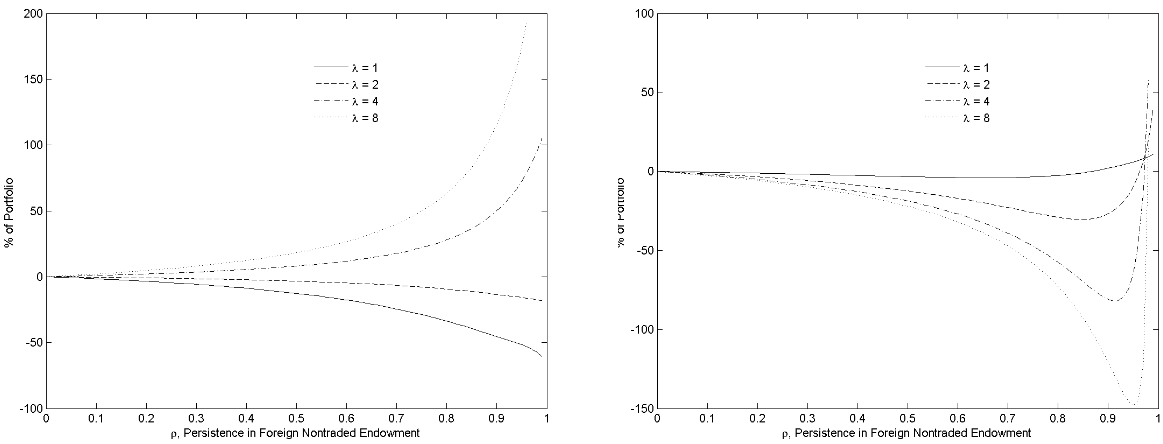
[25] Recall, that home holdings of Foreign equity are measured in Foreign currency, hence the exchange rate term in the equation below.

the case of symmetric preferences when $\omega_H = \frac{1}{2}$ and Home net equity exposure is equal to zero independently of the values of λ.

Setting ω_H different from $\frac{1}{2}$ effectively leads to excess demand for equity of either Home or Foreign country, and the Home or Foreign country acquires a positive bond position in exchange for additional external liabilities in equity. Figures 4.4(a) and Figures 4.4(b) are quite similar in a sense that Home moves from having net liabilities to net assets in equity (becoming a 'venture capitalist' for the Foreign country) as it moves to home bias in consumption. The lower the intratemporal elasticity of substitution the more pronounced are the changes, because equity is more responsive to endowment shocks.

As we shall see in Section 5 preference heterogeneity is not essential to replicating non-zero net aggregate external asset liabilities in bonds or equity. Figure 4.5 highlights the impact

Figure 4.5: HOME NET EQUITY EXPOSURE AS A FUNCTION OF ρ_F^N



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\gamma = 4$, $\omega = \frac{1}{2}$ and $\varsigma = 0.0015$.

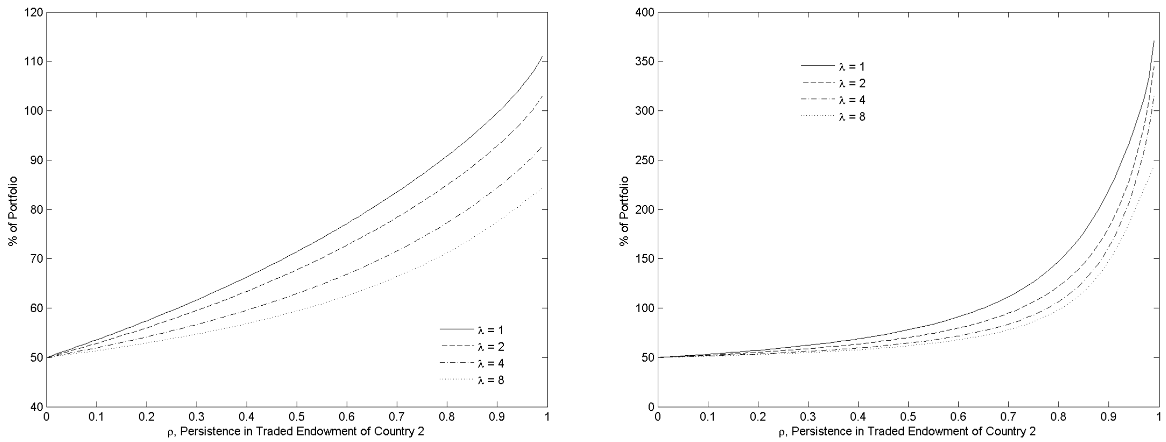
of heterogeneity in macroeconomic fundamentals on Home net equity exposure: varying the persistence coefficient in Foreign nontraded endowment, while keeping all the other shocks white noise. It is worth noting that the model predicts very high aggregate net equity exposure when shocks to Home nontraded endowment become very persistent. The intuition behind this is simple, asset holdings should increase *ceteris paribus* as shocks become more persistent, because portfolio excess returns are delivered only once and need to compensate for the impact of persistent shock on future macroeconomic variables. However, given that such high net positions are rare in Table A.3 in Appendix A, it is comforting that empirically persistence of shocks is estimated to be rather low (as shown in Tables A.13 through to A.16).

In Figures 4.5(a) and 4.5(b) the responsiveness of the net equity exposure is now higher as λ increases, as opposed to Figure 4.4. The reason is that as λ increases, quantity effects of changes in endowments matter more in determining the price and equity returns and since endowment shocks are now persistent, their influence becomes even stronger. A combination of substitution effects also ensures that Figures 4.5(a) and 4.5(b) provide different predictions for net equity exposure depending on whether the elasticity of intertemporal substitution is larger than unity.

4.3 Concentrated External Asset Holdings

One of the stylised facts described in Section 2 and visible in Tables A.4 through to A.12 is that external asset holdings are concentrated in assets issued by particular countries. This phenomenon cannot be replicated by conventional two - country models, however, the model proposed in this paper is rich enough to attempt some calibration exercises. In this section, therefore it is assumed that the world consists of three countries, which are completely symmetric in preferences and macroeconomic fundamentals unless specified otherwise. Figure 4.6

Figure 4.6: Share of Country 2 in Equity Portfolio of Country 1 as a Function of ρ_2^T



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\gamma = 4$, $\omega = \frac{1}{3}$ and $\varsigma = 0.0015$.

shows the share of Country 2 equity in Country 1 equity portfolio defined as:[26]

$$\bar{k}_{12} = \frac{\frac{\bar{K}_{12}}{\bar{S}_2}}{\frac{\bar{K}_{12}}{\bar{S}_2} + \frac{\bar{K}_{13}}{\bar{S}_3}},$$

for various values of the persistence in traded endowment of Country 2 (as usual all other shocks are assumed to be white noise). Regardless of the elasticity of intertemporal substitution or λ, when $\rho_2^T = 0$, that is all countries are actually the same, the optimal share of Country 2 equity in Country 1 equity portfolio is $\frac{1}{2}$. This is to be expected as Country 1 (as well as Country 3) fully diversify their equity holdings.

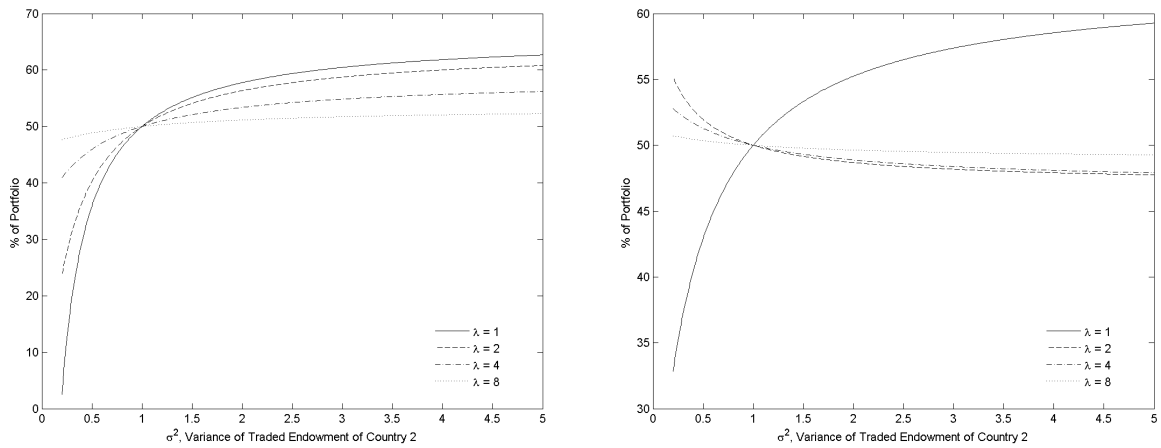
As persistence of the traded endowment shock in Country 2 increases, its share in Country 1 portfolio begins to increase reaching very high values for high ρ. The dynamics are the same in Figure 4.6(a) and 4.6(b), however the growth of portfolio share is faster for lower elasticity of intertemporal substitution. Country 2 portfolio share of above 100% implies that Country 1 actually sells short some of the equity of Country 3 to buy additional equity of Country 2. Interestingly, the United States, a central destination for equity investment, is estimated to

[26]Here the currency of Country 1 is the numeraire.

have very persistent traded endowment from 1990 to 1995 in Table A.13, although the estimates becomes close to zero for the latest period in 2001 to 2005. With more countries, the share of Country 2 equity would not rise as quickly, because countries would be diversifying across more counterparties.

The reason Country 2 share in Country 1 equity portfolio is rising with the persistence of traded endowment of Country 2 is simply the fact that Country 2 equity is a better hedge against the shock that now acquires greater importance due to its persistence.

Figure 4.7: SHARE OF COUNTRY 2 IN EQUITY PORTFOLIO OF COUNTRY 1 AS A FUNCTION OF σ_T^2



(a) $\frac{1}{\theta} = 2.5$ (b) $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $h = \frac{1}{2}$, $\gamma = 4$, $\omega = \frac{1}{3}$ and $\varsigma = 0.0015$.

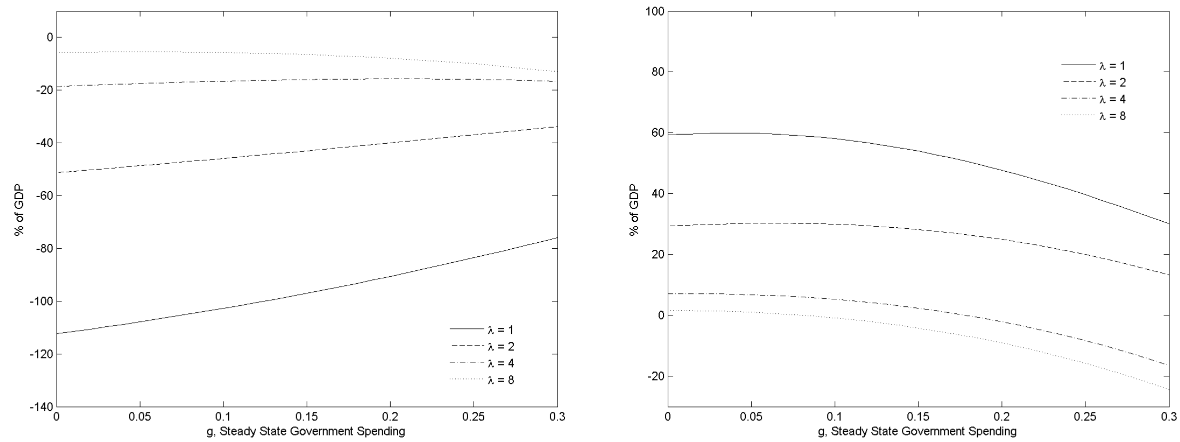
Figure 4.7 shows the share of equity issued by Country 2 in Country 1 portfolio as a function of the variance of traded endowment of Country 2. Recall that variances are identified up to the scale parameter. The variance on the horizontal axis is therefore interpreted as relative magnitude, when $\sigma_T^2 = 1$, all variances are equal, when $\sigma_T^2 < 1$, shocks to traded endowment for Country 2 are less volatile than other shocks and the reverse is true when $\sigma_T^2 > 1$. As expected, when $\sigma_T^2 = 1$, all countries fully diversify their equity holdings. The responsiveness of Country 2 portfolio share decreases with the rise in λ.

The dynamics in Figure 4.7 are similar for the case when $\lambda = 1$, but different for $\lambda > 1$. When elasticity of intertemporal substitution is high as in Figure 4.7(a), the share of Country 2 in Country 1 portfolio decreases when its shocks are less volatile and increases otherwise presumably due to the fact that Country 2 equity is the best hedge against the more relevant risk. However, when elasticity of intertemporal substitution is low, Country 2 portfolio share is actually increases when its shocks are less volatile. This reflects increased desire for constant income stream by agents with low elasticity of intertemporal substitution.

4.4 Bond Allocation

The focus of the previous discussion has been on equity allocation decisions. However, with the exception of the first result considered in this session, different equity allocation decisions also give rise to complementary bond allocation decisions. This topic has so far been much less explored in the literature, where a single riskless, often real bond is assumed. In this model the main driver for bond allocation decisions is different responsiveness of exchange rate of different countries to different shocks. It is assumed that both countries initially start with no bonds issued, hence Home position in Home bonds as a ratio of GDP is simply $\frac{\bar{B}_{HH}}{\bar{P}^N\bar{Y}^N+\bar{P}^T\bar{Y}^T}$. In the data $\bar{B}_{HH} < 0$, which means countries typically sell some of their bonds to other countries in exchange for their bonds or equity.

Figure 4.8: Home Position in Home Bonds



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $\gamma = 4$, $\omega = \frac{1}{3}$, $h = \frac{1}{2}$, and $\varsigma = 0.0015$.

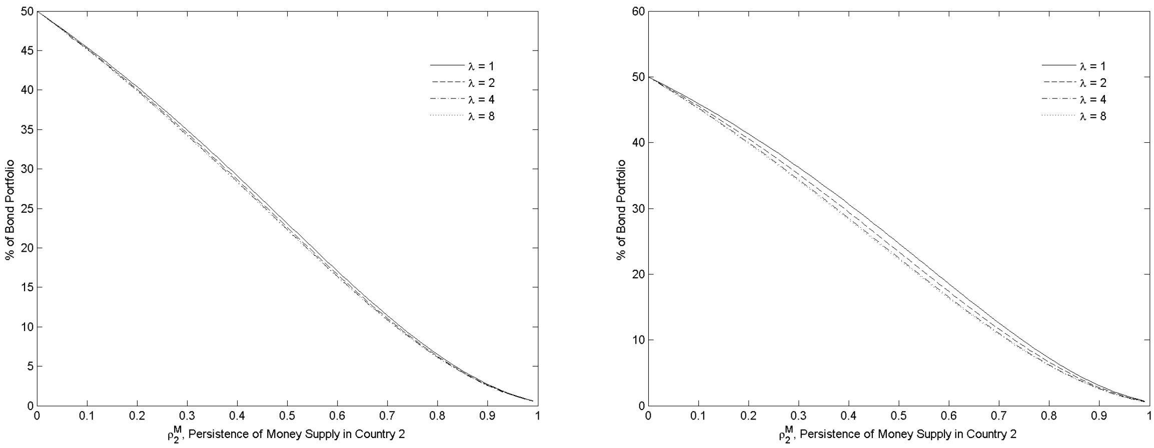
Figure 4.8 shows Home external position in own bonds for different values of the steady state share of government spending, something which was set to zero in previous sections. Returns on bonds and bond holdings are very responsive to the value of g, hence the maximum value considered is $g = 0.3$. Empirically, as shown in Table A.17, this value does not exceed 0.25. There are significant differences between Figure 4.8(a) and Figure 4.8(b).

With high elasticity of intertemporal substitution Home external position in own bonds is negative as expected, but unrealistically high for high values of λ. Low elasticity of intertemporal substitution produces positive external position in own bonds unless steady state share of government spending is fairly high, which contradicts the data. This is one of the driving forces for the fact that empirical applications of the model as outlined in Section 5 tend to predict high elasticity of intertemporal substitution. The behaviour of exchange rates in the model may be quite complicated and is affected not only by changes in endowments, but also by changes in marginal utility of real money balances, which may arise, among other reasons, due to a government spending shock, which crowds out consumption in the economy.

Similarly to equity holdings, bond holdings are also overwhelmingly biased towards particular countries such as the United States. To investigate potential concentration in bond holdings, it is again assumed that the world consists of three almost symmetric countries. As in the case of equity Country 2 share of the external bond portfolio is defined as:

$$\bar{b}_{12} = \frac{\frac{\bar{B}_{12}}{\bar{S}_2}}{\frac{\bar{B}_{12}}{\bar{S}_2} + \frac{\bar{B}_{13}}{\bar{S}_3}}$$

Figure 4.9: Share of Country 2 Bonds in Country 1 Bond Portfolio



(a) $\frac{1}{\theta} = 2.5$ **(b)** $\frac{1}{\theta} = 0.5$

Notes: Other assumptions are $\nu = 6$, $\beta = 0.98$, $\gamma = 4$, $\omega = \frac{1}{3}$, $g = 0.15$ and $\varsigma = 0.0015$.

Figure 4.9 shows how Country 2 share of Country 1 overall bond portfolio varies with changes in persistence of the shock to the money supply in Country 2. In this case there is no difference in the dynamics of Country 2 share in case the elasticity of intertemporal substitution is greater or less than unity. As persistence of money supply shocks increases, the share of Country 2 bonds in Country 1 bond portfolio falls reaching zero when shocks to the money supply approach persistence. Persistent money supply shocks also negatively affect the share of Country 2 equity in Country 1 equity portfolio. With persistent money supply shocks Country 2 currency depreciates by more in response to a positive Country 2 monetary shock than it appreciates in response to a positive money supply shock in another country. Thus, it becomes unattractive for other countries to hold Country 2 bonds.

5 Empirical Results

The model proposed in Section 3 requires two kinds of inputs. First, it requires consumption preferences characterised by the values of the preference weights in consumption aggregates ω_{zz} and h_z, elasticities of substitution λ_z and γ_z, and the coefficients of relative risk aversion θ and ν. Second, it requires what could be termed 'macroeconomic fundamentals', that is the values of the autoregressive parameters and the variance of shocks to the stochastic processes described in (3.7), (3.8) and (3.13), which characterise the deviations from the steady state. The purpose of this section is to measure macroeconomic fundamentals from the data and identify the empirically relevant values of consumption preferences. Choosing consumption preferences that maximise the fit of the model allows one to assess the empirical performance of the model.

5.1 Macroeconomic Fundamentals

To identify the deviations from the steady state and estimate the values of macroeconomic fundamentals this paper employs a small structural time series model, which is estimated for every country in the dataset and every variable x (with $x \in \{T, N, G, M\}$, that is traded and nontraded endowments, government spending and money supply respectively) given by (5.1) and (5.2):

$$\Delta \ln x_{z,t} = \alpha_z^x + \tau_{z,t}^x, \tag{5.1}$$

$$\tau_{z,t}^x = \rho_z^x \tau_{z,t-1}^x + \hat{\varepsilon}_{z,t}^x, \qquad \hat{\varepsilon}_{z,t}^X \sim N(0, (\sigma_z^x)^2). \tag{5.2}$$

The difference of the series $\ln x_{z,t}$ in (5.1) is assumed to consist of a constant mean α_z and the AR(1) component $\tau_{z,t}$ in (5.2). The parameters of interest are ρ_z^x and $(\sigma_z^x)^2$. All variables are considered on a per-capita basis. The model in (5.1) and (5.2) is estimated using quarterly data for the same three separate time periods discussed in Section 2.

The model is estimated for 18 countries in the data set and in addition for three other countries: Indonesia, Russia and South Africa. These countries were chosen because they account for a sizeable share of output outside the 18 countries and have reasonable data availability for production of various sectors of the economy, government spending and money supply. The simple averages of estimated values of ρ_z^x and $(\sigma_z^x)^2$ are set to be the macroeconomic fundamentals for the notional Rest of the World country. The simple average is appropriate if one treats observations for Indonesia, Russia and South Africa as equally likely observations of the unknown parameters for the Rest of the World. The Rest of the World country is introduced because aggregate external asset positions from Table A.2 in parametrising the steady state. This allows asset holdings for all 18 countries to be the outcome of the optimisation procedure and avoids the need to use market clearing conditions to obtain asset allocation for one of the 18 countries. Asset allocation for the notional Rest of the World country is ignored.

It should be stressed that no particular theoretical interpretation is placed on the shocks $\hat{\varepsilon}_{z,t}^x$ in (5.2). Instead they should be seen as reduced form shocks reflecting all possible structural reasons for the change in productivity, government spending or money supply. The simple model in (5.1) and (5.2) is assumed first, for tractability, and second, to not rely on potentially different structural models for different countries. Such an interpretation is less demanding to the knowledge and rationality of households, who need only take into account reduced form shocks when forecasting the development of macroeconomic fundamentals that is relevant for their asset allocation decisions. At the same time, it should be acknowledged that forecasting not based on structural model can be suboptimal, assuming that the Lucas critique is true.

The estimated variances of the shocks in (5.2) provide the diagonal elements for the covariance matrix of the shocks $\mathbf{\Sigma}$ that can be used in empirical application. Theoretically residuals from these equations could also be used to estimate the off-diagonal elements of $\mathbf{\Sigma}$. However, because the number of observations is not the same for all models, i.e. there is some missing data, $\mathbf{\Sigma}$ estimated in such a way will not be positive semi-definite. Such a problem is frequently encountered in finance literature and one possible solution is to compute a positive semi-definite matrix that is in some sense closest to the given $\mathbf{\Sigma}$ using, for example, the method of Higham (2002). However, that method does not always converge and did not converge for any of the three time periods considered in this paper.

In this paper $\mathbf{\Sigma}$ is therefore restricted to be a diagonal matrix. There are two reasons for that. First, there is some degree of uncertainty about the data used to calculate macroeconomic fundamentals. They have missing observations and there is a possibility of substantial measurement error in some cases as well. Basing the empirical results on the full matrix $\mathbf{\Sigma}$ might therefore make them too dependent on some potentially spurious parameters. Second, the assumption of no correlation between various time series may not be that inappropriate given the large number of series involved, for example, the correlation between government spending in Argentina and money supply in Singapore is likely to be insignificant. Importantly, the restriction that $\mathbf{\Sigma}$ is diagonal is not in any sense weaker than allowing the off-diagonal elements to be filled by estimated covariances.

The estimated values for the parameters of (3.7), which describes the law of motion for the deviations from the steady state of traded and nontraded endowments for the countries considered in this paper are given in Tables A.13 and A.14. The constant mean of the growth rate α_z^x is assumed to reflect trendline growth and is not provided in Tables A.13 and A.14, since the model in this paper does not feature trendline growth.

The identification of suitable proxies for the activity of the traded and nontraded sector is an important issue in empirical analysis. A frequently used approach is to look at the sectoral output data and classify some sectors as traded and others as nontraded (e.g. Stockman and Tesar (1995) and MacDonald and Ricci (2007)). In particular, it is often assumed that services are nontraded goods (e.g. Dutton and Strauss (1997), Coeudacier (2009)). Manufacturing output is typically considered to be traded. For example, Betts and Kehoe (2006) classify agriculture, mining and petroleum and manufacturing as traded goods, whereas services, utilities and construction are classified as nontraded goods. Where available quarterly data on production in different sectors of the economy is obtained from the OECD Main Economic Indicators database OECD (2009). For the countries, where OECD (2009) did not have the data on quarterly production in different sectors the author relied on annual estimates provided by Timmer and De Vries (2007) and for the few case in which those were unavailable as well, the annual series on industry and services value added in constant dollars from the World Development Indicators (World Bank, 2009) were used.

In order to convert the annual series to quarterly data this paper used the temporal disaggregation methods proposed by Silva and Cardoso (2001), which involves interpolating the series using related series that is available on the quarterly basis - GDP.[27] Similar methods are used to generate quarterly estimates of macroeconomic variables by statistical agencies.[28]

Table A.13 shows that the most persistent shocks to traded endowments in 1990 to 1995 are in the United States and Germany. On average the traded endowment is estimated to be the most persistent from 1990 to 1995 and the least persistent from 2001 to 2005. Brazil has the most volatile shocks to traded endowments in all time periods and the United States and United Kingdom have the lowest volatilities of shocks to traded endowments. The average

[27]For the estimates of Chinese GDP at quarterly frequency this paper uses the estimates of Abeysinghe and Rajaguru (2004).

[28]This paper uses software proposed in Abad and Quilis (2005) for temporal disaggregation.

estimated volatility of shocks to traded endowment is almost identical from 1990 to 1995 as well as from 2001 to 2005 and slightly lower from 1996 to 2000.

Table A.14 shows that Germany also has the most persistent shock to nontraded endowment in the first time period from 1990 to 1995. The average estimated persistence for nontraded shocks declines over time, while the average estimated volatility stays remarkably constant. Argentina is the only country with the estimated persistence of nontraded endowment from 2001 to 2005 being significantly different from zero.

Data on government spending and money supply is collected from the OECD Main Economic Indicators OECD (2009) (series on government spending in constant prices as well as M2 or M3 aggregates) and IMF International Financial Statistics IMF (2009b) (deflated government spending and the sum of money and quasi-money series).

As seen from Table A.15, for the majority of countries the persistence of shocks to government spending is estimated to be not significantly different from zero. The exception is Portugal, Italy (from 1990 to 1995) and France. The persistence of money supply shocks in Table A.16 is also often estimated to be low and not significantly different from zero. Both government spending shocks and money supply shocks are estimated to have approximately constant levels of volatility over time.

In order to define the steady state for any given value of preferences and macroeconomic fundamentals one also needs to define the steady state traded and nontraded endowments as well as the steady state money supply. The steady state money supply is normalised to one in all of the countries. This is an innocuous assumption, since changes in steady state money supply will affect absolute, but not relative nominal prices.

As for the steady state endowments, two facts from the data are used to identify those. First, the ratio between nontraded and traded endowments is set to the average ratio between the value added in nontraded and traded sector over the corresponding period. Second, the world real GDP is set to be 1 and the real GDP in individual countries is set to correspond to their share in the total real output of 18 countries. Lettin $\bar{Y}_z^N/\bar{Y}_z^T = \bar{\eta}_z$ and the share of total output be $\bar{\varphi}_z$, one can write the traded and nontraded endowments for region z as:

$$\bar{Y}_z^T = \frac{\bar{\varphi}_z}{1+\bar{\eta}_z}, \qquad \bar{Y}_z^N = \frac{\bar{\eta}_z\bar{\varphi}_z}{1+\bar{\eta}_z}.$$

and hence $\bar{Y}_z^T$ and $\bar{Y}_z^N$ can be plugged into (3.25) and (3.26) as functions of prices, $\bar{\eta}_z$ and $\bar{\varphi}_z$. The values of $\bar{\varphi}_z$ and $\bar{\eta}_z$ as well as the steady state weight of government expenditure $\bar{g}_z$ for each time period are reported in Table A.17 in Appendix A.

5.2 Implied Consumption Preferences

The primary objective of the empirical application of the model is to replicate external asset allocation across asset classes and across countries in three time periods. The allocations between different asset classes is provided in Table A.3 in Appendix A, whereas the allocation between different countries is given in Tables A.7 through to A.12 in Appendix A. For any given values of preference parameters, the resulting matrix of steady state asset allocations $\mathbf{H}$ can be assessed against the following goodness-of-fit criterion:

$$\begin{aligned} Q(\mathbf{H}) = r\left(\sum_{j=1}^{X}\bar{B}_{zj}\right) + r\left(\sum_{j=1}^{X}\bar{K}_{zj}\right) + r\left(\bar{B}_{zj}\right) + r\left(\bar{K}_{zj}\right) \\ + \mathbf{1}\left(\sum_{j=1}^{X}\bar{B}_{zj}\right) + \mathbf{1}\left(\sum_{j=1}^{X}\bar{K}_{zj}\right). \end{aligned} \tag{5.3}$$

The first two terms in (5.3) denote root mean square errors of predicted aggregate holdings of bonds and equity respectively. The second two terms are the root mean square errors of predicted bond and equity portfolio allocations between countries. The final two terms of (5.3) are additional penalties for the wrong signs of external aggregate holdings of bonds and equity. In other words, for each instance when the model fails to predict the sign of aggregate exposure in bonds or equity (i.e. whether a country has net assets or net liabilities in this asset class), the penalty functions is increased by one.

There are two reasons for introducing such a correction. First, external asset holdings contain some outliers that is some countries have extremely high external asset holdings relative to GDP (e.g. Singapore's external position in bonds ranging from net liabilities of 584.7% of GDP in 1990-1995 to net liabilities of 79.5% of GDP in 2001-2005) and the reliance solely on root mean square errors might bias the model into trying to match these high levels over the qualitatively significant prediction of the signs of net exposures in bonds and equity. Second, there may be more uncertainty over the size of net external holdings of bonds and equities than over their sign. Therefore it is especially important that the model captures the sign of net external holdings to replicate the heterogeneity observed in the data e.g. the US having net assets in equity and net liabilities in debt, etc.

As described previously, macroeconomic fundamentals that is the estimated parameters governing stochastic processes for endowments ρ_z^T and ρ_z^N, government spending ρ_z^G and money supply ρ_z^M, as well as the estimated covariance matrix of the shocks, $(\varepsilon_z^T)^2$, $(\varepsilon_z^N)^2$, $(\varepsilon_z^G)^2$ and $(\varepsilon_z^M)^2$ for matrix $\mathbf{\Sigma}$ are taken as given at their values in Tables A.13 through to A.16.

In the parameters that are varied in minimising (5.3), one can distinguish two groups - those that are assumed to be same across countries and those that are allowed to differ across countries. The first group includes θ, the coefficient of relative risk aversion with respect to consumption, ν, the coefficient of relative risk aversion with respect to real money balances. The second group consists of ω_{zz}, the home bias, h_z, the preference for nontraded goods, λ_z, the elasticity of substitution between traded and nontraded consumption baskets, and γ_z, the elasticity of substitution between traded endowments from different regions.

The baseline empirical application of the model considers two specifications - the homogeneous specification, which keeps the parameters of both groups the same across all countries, and the heterogeneous specification, which allows the parameters in the second group to vary across regions. In both specifications, however, it is assumed that the preferences for traded goods from countries other than one's own are equal to each other, that is $\omega_{zj} = \frac{1-\omega_{zz}}{X-1}$ for all

$j \neq z$.

The value of (5.3) is found by first, solving for the steady state of the model for given values of preference parameters as well as the parameters describing the steady state $\bar{\eta}_z$, $\bar{\varphi}_z$ and $\bar{g}_z$ given in Table A.17. The steady state is a solution to the system of $3X - 1$ nonlinear equations. If the steady state has been found successfully,[29] the first-order approximation of the model is obtained using the method of Christiano (2002). Finally, one can find the optimum asset allocation using (3.52).

Minimising (5.3) is not trivial because it is discontinuous and not differentiable. Discontinuities may arise because the first-order solution of the model is not always available, i.e. the eigenvalue condition of Christiano (2002) may not be fulfilled, or because asset holdings change their signs following small changes in preference parameters as described in Section 4. Naturally, the penalty for wrongly determined signs (the last two terms of (5.3)) also introduces discontinuities. This makes the application of gradient based optimisation methods problematic, since they may end up converging to a local rather than global minimum.

In order to minimise (5.3) with a homogeneous specification, this paper applies a combination of simulated annealing algorithm (Kirkpatrick et al., 1983), genetic optimisation algorithm (Conn et al., 1991) and pattern search algorithm.[30] The algorithms are applied in succession with each step taking the output of the previous step as input until no algorithm can generate an improvement in the value of the objective function. Such a procedure above does not guarantee that a global minimum of (5.3) is found, which is, in general, impossible for optimisation problems with discontinuous functions and a large number of parameters. Therefore, the results in Table 5.1 and 5.2 can be seen as conservative in a sense that an even better performance of the model can potentially be achieved if the algorithms are run for longer time or with different starting points.

Table 5.1 illustrates the empirical performance of the model for the specification, where all parameters are assumed to be the same across regions. Several indicators are employed to evaluate the empirical success of the model. First, root mean square errors are reported in four categories: predictions of bond and equity portfolio allocations by country as well as predictions of aggregate net holdings of bonds and equity. Second, the number of correctly predicted signs of net aggregate holdings in bonds and equity (out of 18 for each category) is reported. Third, as an overall measure of fit, one can suggest the fraction of variance unexplained defined as:

$$FVU = \frac{MSE(\mathbf{H})}{Var(\mathbf{H})},$$

where $MSE(\mathbf{H})$ is the mean square error of predicted asset holdings relative to GDP and $Var(\mathbf{H})$ is the variance of observed asset holdings relative to GDP. An $FVU > 1$, means that simply choosing a mean of asset holdings relative to GDP across all countries would have been a

[29] Trust-region reflective and trust-region dogleg methods are used to find the steady state. Different starting values for numerical algorithms are used depending on whether $\theta > \nu$ or vice versa. In more than 95% of cases steady state is located successfully in a maximum of 30000 function evaluations.

[30] These algorithms are accessible in the Global Optimization Toolbox in MatLab.

better predictor than the model. An $FVU < 1$ suggests that the model has better explanatory power than the sample mean.

The FVU measure, as reported in Tables 5.1 and 5.2, has to be interpreted with care because the model calibration procedure trades off some of the improvement in the mean square error in favour of obtaining correct signs of net exposures for different countries. In addition, the objective of the model is not to replicate the ratio of every single asset holding relative to GDP per se, but rather functions of these parameters such as the aggregate net exposure and external asset allocation by countries. Suppose, for example, that the true holdings of two assets, which together form the entire external asset portfolio, are 5% and 10% of GDP respectively whereas the model predicts the holdings of these assets at 10% and 20% of GDP respectively.

The model predictions still correctly reflect the fact that $\frac{2}{3}$ of the external portfolio are allocated to one asset and $\frac{1}{3}$ to the other, yet the FVU measure for such prediction would be equal to 5 suggesting a poor overall fit. Hence an $FVU > 1$ does not mean that the model has no value. One can construct individual FVU measures for various components of the objective function from Tables 5.1 and 5.2 by squaring the ratio of root mean square error to the standard deviation of the particular variable of interest.

Table 5.1: Empirical Performance of the Model, Homogeneous Case

		1990 - 1995		1996 - 2000		2001 - 2005	
Allocation	Asset	Portfolio	Total	Portfolio	Total	Portfolio	Total
Country	Bond	0.147 (0.105)	0.090 (0.105)	1.025 (0.104)	0.092 (0.104)	0.280 (0.158)	0.179 (0.158)
	Equity	0.146 (0.140)	0.193 (0.121)	0.428 (0.133)	0.112 (0.124)	0.143 (0.157)	0.178 (0.164)
Aggregate	Bond	1.321 (1.373)	1.310 (1.373)	0.965 (0.751)	0.793 (0.751)	0.569 (0.478)	0.492 (0.478)
	Equity	0.423 (0.034)	0.122 (0.109)	0.562 (0.135)	0.135 (0.130)	0.130 (0.107)	0.154 (0.145)
Signs	Bond	17	15	15	15	15	15
	Equity	15	16	15	16	14	17
FVU		1.101	0.994	210.92	0.987	99.68	0.924

Notes: The RMSE of country allocation is the root mean square error of predicted portfolio allocations of all 18 countries, that is $18 \times 17 = 306$ holdings. The aggregate allocation refers to the aggregate net holdings of equity and liabilities (36 holdings). The standard deviation of corresponding holdings is given in brackets.

Table 5.1 suggests that with homogeneous parameters the overall fit of the model is better when FDI is included in overall equity position (total equity case). The model is able to replicate from 29 to 32 out of 36 total signs of net asset exposures, however in case of portfolio equity from 1996 to 2005 it does so at the expense of considerable increase in mean square error as indicated by the high FVU values.

Table 5.2 shows how the performance of the model improves when taste parameters are allowed to vary across countries. The best result in terms of the number of signs matched (35

Table 5.2: Empirical Performance of the Model, Heterogeneous Case

		1990 - 1995		1996 - 2000		2001 - 2005	
Allocation	Asset	Portfolio	Total	Portfolio	Total	Portfolio	Total
Country	Bond	0.099 (0.105)	0.093 (0.105)	0.282 (0.104)	0.093 (0.104)	0.185 (0.158)	0.156 (0.158)
	Equity	0.121 (0.140)	0.112 (0.121)	0.245 (0.133)	0.109 (0.124)	0.160 (0.157)	0.103 (0.164)
Aggregate	Bond	1.255 (1.373)	1.125 (1.373)	0.933 (0.751)	0.821 (0.751)	0.275 (0.478)	0.463 (0.478)
	Equity	0.125 (0.034)	0.077 (0.109)	0.434 (0.135)	0.119 (0.130)	0.097 (0.107)	0.122 (0.145)
Signs	Bond	17	17	15	15	17	17
	Equity	17	17	15	18	17	18
FVU		0.945	0.912	113.45	0.947	1.939×10^3	0.770

Notes: The RMSE of country allocation is the root mean square error of predicted portfolio allocations of all 18 countries, that is $18 \times 17 = 306$ holdings. The aggregate allocation refers to the aggregate net holdings of equity and liabilities (36 holdings). The standard deviation of corresponding holdings is given in brackets.

out of 36) and the overall FVU value is the case of total equity from 2001 to 2005. This is significant because, in the data by Kubelec and Sá (2009) this period has the least amount of estimated data. Similarly to the homogeneous case, the overall fit of the model is better when FDI is included in overall equity position. Heterogeneous case also fits equities better than bonds both in terms of allocation and aggregate exposures. For example, the FVU value only for equity allocation in case of total equity from 2001 to 2005 is $\left(\frac{0.103}{0.164}\right)^2 \approx 0.394$.

Table 5.3 shows the implied values for the homogeneous specification of consumption preferences. Several findings emerge from Table 5.3. First, the homogeneous specification indicates

Table 5.3: Implied Homogeneous Preferences

Parameter	1990 - 1995		1996 - 2000		2001 - 2005	
	Portfolio	Total	Portfolio	Total	Portfolio	Total
ω_{zz}	0.106	0.945	0.299	0.967	0.509	0.941
h_z	0.720	0.894	0.912	0.001	0.043	0.320
λ_z	13.471	11.688	6.145	6.557	2.845	13.928
γ_z	4.296	5.569	2.103	1.371	2.805	4.723
θ	0.122	0.157	7.662	0.552	0.189	0.212
ν	10.005	12.000	3.095	9.445	6.130	8.247

the implied weight of own traded goods to be more than proportional to other countries $\omega_{zz} > \frac{1}{X}$ and higher in the case of total equity (where the model performs better) than in the case of portfolio equity. Second, the implied weight on nontraded goods in the overall consumption aggregate varies greatly across time periods and depending on whether total or portfolio equity is considered. For the case of total equity between 1960 to 2000, the implied value actually hits a lower bound of $h_z = 0.001$. One should note, however, that this does not mean that the share of total consumption expenditure on nontraded goods is only 0.1%. For example, the steady state share of consumption expenditure on nontraded goods is given by:

$$\frac{\bar{P}_z^N \bar{C}_z^N}{\bar{P}_z \bar{C}_z} = h_z \left[\frac{\bar{P}_z^N}{\bar{P}_z} \right]^{1-\lambda_z} .$$

This means that even with preference parameters restricted to be the same across regions, expenditure shares on goods from different regions will not be the same. Third, for all cases and all time periods the implied elasticity of substitution between the basket of traded goods and nontraded goods is higher than the elasticity of substitution between traded goods from various countries.

Table 5.3 also shows that implied values of the coefficient of relative risk aversion θ are fairly low with the exception of portfolio equity for the period between 1996 to 2000, which can be considered an outlier due to the overall poor performance of the model as shown in Table 5.2. This result is best understood in terms of the elasticity of intertemporal substitution (EIS), which is the reciprocal of the coefficient of relative risk aversion. The implied value of the elasticity of intertemporal substitution is high, ranging from 1.8 to 8.2 (excluding the outlier for which $\frac{1}{\theta_z} \approx 0.131$. This value is higher than most others reported in the literature although the higher values of implied θ_z are consistent with some evidence provided by Chetty (2006), who uses labour supply data to identify θ_z. Guvenen (2006) constructs a model in which stockholders (wealthy agents) have a higher EIS than poor agents. Given that the model aims to explain the data on external asset holdings, which are likely to be owned by the richest agents in the economy, this might explain the high level of EIS implied. Vissing-Jørgensen (2002) also provides some evidence of EIS being larger than one for the groups of asset holders with the largest asset holdings.

There is less empirical guidance on the plausible values of ν, the coefficient of relative risk aversion with respect to money balances. Dutkowsky and Atesoglu (2001) estimate a similar parameter to be approximately 2. The implied interest rate elasticity of money demand, which is equal to $\frac{\beta}{\nu}$ in this model ranges between 0.1 and 0.3[31] (in absolute value), which is lower than the range reported in Hoffman and Rasche (1991), who suggest a range between 0.4 and 0.5.

The number of signs of net asset exposures correctly predicted by the model reflect how well the model replicates heterogeneity in aggregate net exposures, which is the first stylised fact described in Section 2. The overall FVU measure as well as individual ratios between root mean square error in different predictions measure how well the model reflects heterogeneity in country portfolio allocations across different countries. Finally, it is worth noting that the amount of correctly predicted signs of asset holdings relative to GDP, which is the third stylised fact described in Section 2 generally ranges between 60% and 70% of the total. Thus, the model is arguably the most successful in replicating the first stylised fact.

[31]Note that this is elasticity with respect to the net interest rate. To derive it, one can plug in the linearised Euler equation into the linearised money market equilibrium condition. The elasticity of money demand with respect to changes in the gross interest rate is $\frac{\beta}{(1-\beta)\nu}$.

Table 5.4 shows the implied values for the heterogeneous specification of consumption preferences and only portfolio equity included in the data. In Table 5.4 one can see that heterogeneity

Table 5.4: Implied Heterogeneous Preferences, Portfolio Equity

	1990 - 1995				1996 - 2000				2001 - 2005			
Country	ω_{zz}	h_z	λ_z	γ_z	ω_{zz}	h_z	λ_z	γ_z	ω_{zz}	h_z	λ_z	γ_z
Argentina	0.241	0.960	10.206	3.980	0.553	0.919	14.651	2.103	0.911	0.084	2.860	2.749
Australia	0.725	0.724	14.098	3.652	0.301	0.919	14.145	2.103	0.970	0.135	3.341	2.063
Brazil	0.332	0.998	14.160	4.628	0.553	0.653	8.145	2.131	0.907	0.037	1.486	2.841
Canada	0.240	0.701	9.295	4.628	0.549	0.434	14.203	2.108	0.731	0.527	4.721	2.872
China	0.283	0.590	14.954	4.275	0.049	0.919	3.004	2.140	0.702	0.508	2.721	2.848
France[a]	0.066	0.759	9.953	2.800	0.305	0.690	6.145	2.146	0.482	0.846	3.113	2.747
Germany	0.641	0.222	14.981	4.433	0.303	0.919	6.145	2.108				
Hong Kong	0.838	0.558	13.631	4.650	0.299	0.172	2.145	2.611	0.853	0.035	2.852	2.816
India	0.234	0.821	13.415	5.645	0.299	0.938	6.289	2.104	0.930	0.536	3.845	6.332
Italy	0.361	0.080	12.673	3.560	0.299	0.923	6.145	2.103				
Japan	0.250	0.414	8.869	4.189	0.424	0.106	1.159	2.106	0.665	0.925	3.533	2.809
Korea	0.626	0.669	14.590	4.111	0.058	0.977	3.146	2.114	0.854	0.118	1.228	2.670
Mexico	0.578	0.291	11.680	2.900	0.807	0.169	2.154	2.103	0.942	0.058	1.104	2.905
Portugal	0.585	0.869	11.680	2.987	0.302	0.419	6.147	0.103				
Singapore	0.012	0.004	14.919	4.806	0.799	0.676	1.688	3.106	0.002	0.007	14.982	4.934
Spain	0.711	0.984	10.158	3.775	0.299	0.170	7.149	2.110				
United Kingdom	0.356	0.986	12.236	3.653	0.361	0.419	14.145	2.142	0.801	0.182	1.853	3.162
United States	0.189	0.492	14.872	3.824	0.300	0.669	7.145	2.103	0.334	0.987	14.969	2.846
Rest of the World	0.222	0.850	14.223	4.766	0.305	0.928	6.166	2.111	0.518	0.026	2.962	2.949
Mean	0.394	0.630	12.787	3.994	0.377	0.633	6.838	2.087	0.707	0.334	2.431	2.742
Standard Deviation	0.239	0.309	2.117	0.739	0.201	0.313	4.458	0.541	0.272	0.356	3.741	3.111
θ	0.119				7.920				0.189			
ν	6.846				3.097				2.139			

[a] This row has EMU values for the time period from 2001 to 2005.

of parameters reduces the implied value of ν to more realistic values especially in the time period from 2001 to 2005, which brings the implied estimate of the interest elasticity of money demand more in line with the values suggested by Hoffman and Rasche (1991). For the vast majority of countries Table 5.4 suggest realistic implied values of ω_{zz}, the weight of own traded goods in the consumption aggregate and h_z, the weight of nontraded goods. An interesting exception is Singapore, which has the highest net position in external assets relative to GDP from 1990 to 2000 (and second highest in the final period). It is the only country for which the model implies a bias towards consumption of foreign goods in the period from 1990 to 1995 and 2001 to 2005, but not in the period from 1996 to 2000, which is an outlier in the portfolio equity case.

Table 5.4 also suggests that both the elasticity of substitution between traded goods and nontraded goods λ_z and the elasticity of substitution between different traded goods γ_z are relatively high, the former parameter is generally larger than the latter and that overall they decline over time.

Table 5.5 shows the implied values for the heterogeneous specification of consumption preferences and total equity included in the data. The results in Table 5.5, when total equity is included in the data, share some features with the implied preferences in Table 5.5 in that for most countries a high value of home bias parameter and elasticities is implied. The most notable difference between preferences with portfolio equity and total equity is for the time

Table 5.5: Implied Heterogeneous Preferences, Total Equity

	1990 - 1995				1996 - 2000				2001 - 2005			
Country	ω_{zz}	h_z	λ_z	γ_z	ω_{zz}	h_z	λ_z	γ_z	ω_{zz}	h_z	λ_z	γ_z
Argentina	0.945	0.987	7.487	5.438	0.978	0.251	1.520	1.375	0.945	0.922	6.080	14.590
Australia	0.801	0.487	3.487	5.063	0.978	0.001	2.772	1.375	0.976	0.307	3.492	7.085
Brazil	0.964	0.097	5.550	7.125	0.978	0.008	1.001	1.373	0.932	0.968	11.502	4.990
Canada	0.852	0.112	6.487	4.344	0.978	0.947	10.112	1.310	0.836	0.809	4.459	6.174
China	0.945	0.112	13.487	5.563	0.978	0.517	1.020	1.029	0.922	0.956	8.181	2.741
France[a]	0.945	0.128	1.269	2.813	0.978	0.018	2.524	1.533	0.943	0.026	13.149	4.973
Germany	0.945	0.862	11.050	5.063	0.984	0.002	2.520	1.373				
Hong Kong	0.195	0.112	5.487	8.346	0.978	0.001	4.762	3.297	0.351	0.443	15.000	4.778
India	0.945	0.112	2.862	1.313	0.982	0.064	6.734	1.373	0.946	0.737	3.929	4.773
Italy	0.945	0.612	9.487	5.750	0.963	0.354	3.505	2.135				
Japan	0.945	0.987	5.769	4.063	0.988	0.014	4.522	1.374	0.979	0.350	11.178	4.279
Korea	0.945	0.722	9.534	5.563	0.925	0.126	7.393	1.373	0.953	0.009	2.668	6.850
Mexico	0.992	0.237	0.504	6.563	0.996	0.003	5.505	1.000	0.982	0.993	14.999	6.707
Portugal	0.945	0.362	0.987	5.375	0.999	0.001	6.407	2.248				
Singapore	0.070	0.112	13.487	14.797	0.523	0.776	4.941	1.248	0.817	0.038	11.482	4.731
Spain	0.945	0.112	8.487	5.563	0.982	0.011	2.521	1.373				
United Kingdom	0.960	0.987	13.487	4.563	0.979	0.975	10.366	1.037	0.948	0.229	2.306	6.463
United States	0.851	0.112	13.737	7.766	0.978	0.001	6.279	1.373	0.544	0.006	14.996	3.848
Rest of the World	0.945	0.175	13.987	3.063	0.979	0.001	4.520	1.373	0.944	0.624	14.999	5.100
Mean	0.846	0.391	7.718	5.691	0.954	0.214	4.680	1.504	0.868	0.495	4.997	5.411
Standard Deviation	0.257	0.352	4.649	2.771	0.106	0.338	2.745	0.534	0.181	0.385	5.689	4.856
θ	0.157				0.614				0.214			
ν	9.547				5.000				9.300			

[a] This row has EMU values for the time period from 2001 to 2005.

period between 1996 to 2000. The case with total equity, which has a much better overall FVU measure has uniformly higher values of the home bias parameter and generally lower elasticities of demand than the portfolio equity case.

It is worth to briefly study the robustness of results to varying the values of macroeconomic fundamentals by assuming that macroeconomic fundamentals are the same across countries. To construct homogeneous preferences, the autoregressive parameters ρ^X as well as the government spending weight g are set to equal the averages of the corresponding values in Tables A.13 through to A.16. Simple averages are used because macroeconomic fundamentals are estimated on a per capita basis and there are no points, where the confidence intervals for all of the 18 countries would overlap. In addition, for homogeneous macroeconomic fundamentals the covariance matrix of the shocks $\mathbf{\Sigma}$ is assumed to be diagonal and equal to $\sigma^2\mathbf{I}$, where σ^2 is set to be the mean of the diagonal elements of the original estimated covariance matrix of shocks. Solving the model with homogeneous fundamentals and heterogeneous preferences yields the total sum of squares underlying the penalty function that is about two orders of magnitude higher higher than those with homogeneous preferences and heterogeneous macroeconomic fundamentals. Naturally, it would perhaps be possible to find a set of consumption preferences that sustain observed asset allocation even with homogeneous macroeconomic fundamentals. However, given that differences in macroeconomic fundamentals are empirically relevant, this robustness check suggests that heterogeneity in macroeconomic fundamentals matters more for external asset allocation than heterogeneity in preferences.

The results outlined above should not be interpreted as detailed estimates of preference parameters for individual countries because the amount of uncertainty involved in the calibration procedure is considerable. One can, however, make several reasonably robust conclusions. First, considerable amounts of home bias help to improve the fit of the model to observed asset allocation patterns. Second, there is more substitutability between traded goods basket and non-traded goods, than between different components of the traded goods basket. This is consistent with the world, where countries are relatively specialised in their traded endowments. Third, a large elasticity of intertemporal substitution (larger than one) improves the fit of the model.

6 Conclusion

This paper studied the problem of determining optimal external asset allocation across asset classes as well as, for the first time, across countries in a multi-country framework with potentially heterogeneous consumption tastes. Empirically, this paper verified that the proposed model can to some extent replicate three key stylised facts on external asset allocation - that external asset holdings are positive, that there is heterogeneity in asset allocation across asset classes and that assets issued by some countries, for example the UK, have a larger share in portfolios of other regions than would be suggested by their share in the overall output.

In terms of methodology, this paper has generalised the solution method for the optimum portfolio allocation problem proposed by Devereux and Sutherland (2008) to the case of multiple countries and assets denominated in different currencies. The method relies on deriving the first order linearisation of the model, which is obtained by the method of Christiano (2002) and subsequently solving a matrix equation to which the general case of the optimal portfolio allocation problem can be reduced to. It was shown that for most plausible parameter specifications, there is likely to be only one solution to the asset allocation problem. The numerical solution allows relating optimum net foreign asset portfolios to many observable macroeconomic fundamentals (e.g. persistence and volatility of government spending processes) and structural preference parameters.

Optimum portfolio allocation can change significantly depending on the interactions between various preference parameters, most notably the elasticity of substitution between traded and nontraded goods as well the intertemporal elasticity of substitution. The proposed model is able to generate complicated patterns of asset allocation, including home bias in equity holdings, and different net positions in external debt and external equity. This paper has also shown that trade in goods and the terms of trade channel can play an important role in determining optimum asset allocation. In that context, it is important to acknowledge that an increase in financial integration has proceeded together with the rise in trade integration, which can conceivably lead to higher values of elasticity of substitution in traded goods.

A possible extension to the model is to obtain the solution for not just the steady state portfolios, but for their dynamics. Devereux and Sutherland (2007), in fact, propose another extension of the method applied in this paper, which relies on a third-order approximation of the portfolio choice condition and the second-order approximation of the non-portfolio equations. Such a second-order approximation could be solved, for example, using the method proposed by Lombardo and Sutherland (2007), but it would be considerably more complicated in a multi-country framework.

Another possible extension is to include a more explicit role for production and incorporate more New Keynesian features into the model, most notably price rigidity. Such an extension would likely help the model to match the high volatility of nominal exchange rate and would allow highlighting the role of other sources of risks, for example, the importance of labour income risk on the international asset allocation. With such modifications, the model could also be suitable for conducting simulations at business cycle frequencies, which allows comparing the effects of potentially suboptimal asset allocations on, for example, consumption or exchange rate volatility. The proposed model could easily nest the case with only a single riskless bond available (by setting all portfolio innovation terms $\xi_{z,t}$ to zero) or an arbitrary asset allocation (by varying the $\mathbf{H}$ matrix).

The empirical exercise in Section 5 shows that the model has some promise, but the results are sensitive to the way the penalty function is specified and there is potentially a lot of uncertainty over the measurement of macroeconomic fundamentals and the structure of the covariance matrix of shocks $\mathbf{\Sigma}$. Naturally, the empirical results are also obtained under a large amount of maintained hypotheses, perfect markets in goods and assets, the definition of equity and bonds as claims on the aggregate endowment and currency respectively, etc. Hence a more elaborate empirical exercise might focus on fewer, more similar countries and amend the model to achieve better empirical performance. The results presented here should be seen as obtained under fairly strict assumptions. The fact that the model still manages to reflect heterogeneity in net asset exposures and, to some extent, other stylised facts suggest that even as global financial markets become more integrated and barriers for trade are removed, heterogeneities in external asset allocation, including those that give rise to global imbalances, can continue.

In spite of the recent financial and economic crisis, it is likely that world trade and financial integration will continue and the composition of net foreign asset portfolios will acquire ever greater significance, not just from the viewpoint of global imbalances, but also for their role in the transmission of shocks. In particular, more detailed data on foreign asset portfolios is likely to be available in the future and hence the role of models, which can specifically consider bilateral and not just aggregate portfolio holdings and relate them to macroeconomic fundamentals is likely to increase. The model and methods outlined here could be considered a useful first step for such purposes.

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A Tables

Table A.1: Quandt-Andrews Breakpoint Test Results, p-values for the Null Hypothesis of No Structural Break

Country	Maximum		Exp		Average		Likeliest Breakpoint	
	Portfolio	Total	Portfolio	Total	Portfolio	Total	Portfolio	Total
Argentina	0.00	0.00	0.00	0.00	0.00	0.00	2001	2001
Australia	0.00	0.00	0.00	0.00	0.00	0.04	2001	2001
Brazil	0.00	0.00	0.00	0.00	0.00	0.04	1999	1999
Canada	0.61	0.60	0.26	0.26	0.19	0.18	1998	2001
China	0.00	0.00	0.00	0.00	0.00	0.00	1998	1998
France	0.28	0.00	0.27	0.00	0.39	0.00	2002	2000
Germany	0.00	0.00	0.00	0.00	0.00	0.00	2002	2001
Hong Kong	0.00	0.00	0.00	0.00	0.00	0.00	1999	1999
India	0.01	0.00	0.01	0.00	0.02	0.00	2002	2002
Italy	0.00	0.00	0.00	0.00	0.00	0.00	2001	2001
Japan	0.09	0.03	0.05	0.01	0.11	0.02	2002	2002
Korea	0.00	0.00	0.00	0.00	0.00	0.00	2002	2002
Mexico	0.00	0.23	0.00	0.12	0.00	0.10	2001	1995
Portugal	0.00	0.00	0.00	0.00	0.00	0.00	2000	2002
Singapore	0.00	0.00	0.00	0.00	0.00	0.00	1997	1997
Spain	0.00	0.01	0.00	0.00	0.00	0.01	1997	2002
United Kingdom	0.01	0.00	0.00	0.00	0.03	0.00	1993	1993
United States	0.41	0.49	0.14	0.24	0.01	0.20	1996	1995

Notes: Portfolio column refers to the net external assets with bonds and portfolio equity, whereas the Total column refers to the net external assets with bonds and total equity. The maximum statistic is the maximum of the individual Chow breakpoint F statistics, whereas the Average is the average of the individual statistics. Finally, Exp statistic is defined as $ExpF = \frac{1}{\tau_2 - \tau 1}\left(\sum_{\tau=\tau_1}^{\tau_2} \exp\left(\frac{1}{2}F(\tau)\right)\right)$, where τ_1 is the initial time period, τ_2 the final time period and $F(\tau)$ is the individual Chow statistic for breakpoint at time τ. The year of likeliest breakpoint is determined by the maximum value of the Chow statistic.

Table A.2: Average Private Sector Net External Asset Holdings (Aggregate), 1990 - 2005

Country	1990 - 1995		1996 - 2000		2001 - 2005	
	Portfolio	Total	Portfolio	Total	Portfolio	Total
Argentina	4.2	-2.9	-20.7	-35.2	0.0	-25.8
Australia	-36.3	-50.1	-16.7	-22.2	-53.4	-59.1
Brazil	-22.0	-26.8	-15.7	-26.9	-33.6	-46.4
Canada	-18.2	-20.1	-55.4	-61.7	-9.5	-4.3
China	-5.7	-10.9	-6.7	-16.4	1.0	-16.2
France	16.0	20.2	3.9	7.5		
Germany	-5.8	-6.3	2.2	23.7		
Hong Kong	138.7	52.9	63.6	75.5	180.9	152.4
India	30.8	-30.5	-5.9	-7.6	16.1	-22.5
Italy	-4.8	-3.0	0.0	0.2		
Japan	2.9	8.4	15.6	18.4	11.7	17.7
Korea	-12.3	-11.6	-16.1	-16.4	-28.9	-33.3
Mexico	-21.4	-30.6	-24.3	-34.7	-15.4	-37.6
Portugal	-8.2	-20.8	-25.7	-32.4		
Singapore	21.7	-29.1	-2.7	-307.9	137.2	82.7
Spain	-5.0	-16.1	-10.8	-13.8		
United Kingdom	-1.5	2.8	3.6	-5.5	-36.5	-11.3
United States	-0.3	1.9	3.2	3.2	-5.5	-1.0
EMU Total					13.4	22.2

Table A.3: Average Private Sector Net External Asset Holdings (18 Countries), 1990 - 2005

Country	Total Equity			Portfolio Equity			Debt			Aggregate		
	1990 - 1995	1996 - 2000	2001 - 2005	1990 - 1995	1996 - 2000	2001 - 2005	1990 - 1995	1996 - 2000	2001 - 2005	1990 - 1995	1996 - 2000	2001 - 2005
Argentina	-6.2	-16.2	-24.8	-0.2	-1.7	5.6	-14.9	-19.0	30.1	-21.1	-35.2	5.3
Australia	-10.5	-6.8	-7.7	1.1	-1.3	-2.2	-16.5	-15.4	-29.0	-27.1	-22.2	-36.7
Brazil	-9.6	-15.0	-20.8	-1.0	-3.8	-6.7	-11.0	-11.9	-10.7	-20.7	-26.9	-31.5
Canada	-15.8	-46.7	-0.4	-8.1	-40.3	9.3	-6.7	-15.1	-18.8	-22.5	-61.7	-19.2
China	-2.0	-10.1	-12.9	0.0	-0.4	-2.4	-9.4	-6.4	-3.0	-11.4	-16.4	-15.9
Germany	2.1	12.2	22.6	-2.8	-9.2	-7.1	10.7	11.5	15.8	12.9	23.7	38.3
France	-0.9	2.1	7.2	-1.3	-1.6	1.6	10.6	5.4	18.2	9.7	7.5	25.4
Hong Kong	-29.7	-4.8	18.4	-5.7	-16.7	-4.5	15.4	80.3	142.9	-14.3	75.5	161.3
India	-1.1	-3.5	-6.7	-0.2	-1.7	-4.4	-4.5	-4.1	-3.2	-5.6	-7.6	-9.9
Italy	-1.4	1.7	-2.8	-0.8	1.7	-1.5	-5.4	-1.6	-19.1	-6.7	0.2	-21.9
Japan	4.0	1.7	-2.4	-0.3	-1.1	-4.8	14.6	16.7	17.0	18.6	18.4	14.6
Korea	-1.4	-2.1	-13.7	0.0	-1.8	-12.8	-12.3	-14.3	-9.2	-13.7	-16.4	-22.9
Mexico	-9.2	-18.8	-19.4	-2.1	-8.5	-5.2	-17.8	-15.9	-7.0	-27.0	-34.6	-26.4
Portugal	-9.9	-13.1	-19.5	-1.5	-6.4	-4.2	-21.4	-19.3	-44.4	-31.3	-32.4	-63.9
Singapore	-35.9	-4.0	-34.4	4.4	32.1	30.6	-584.7	-303.9	-79.5	-620.6	-307.9	-114.0
Spain	-7.8	-11.8	-6.0	-2.2	-8.8	-7.7	1.2	-2.0	-12.3	-6.5	-13.8	-18.3
United Kingdom	3.2	-15.6	-21.7	7.5	-6.5	-9.2	0.4	10.1	12.6	3.6	-5.5	-9.0
United States	2.4	8.0	7.0	1.2	8.1	6.2	-3.2	-4.9	-7.7	-0.9	3.2	-0.7
EMU Total			6.7			-2.8			3.9			10.6

Notes: The data is in percent of GDP (in current prices). The aggregate column is the sum of total equity and debt columns. Table is based on the data from the External Wealth of Nations dataset by Lane and Milesi-Ferretti (2007) after excluding government assets and liabilities obtained from IMF (2009a).

Table A.4: Total Equity Composition, 1990 - 1995

	Issuing Country																	
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.7	14.2	1.1	0.1	2.1	2.6	0.3	0.0	1.2	2.2	0.0	4.8	0.2	0.2	6.0	4.8	59.4	100
Australia	0.4		0.8	1.4	0.1	0.6	1.1	7.9	0.2	0.6	4.0	0.1	0.0	0.1	3.3	0.4	38.2	40.8	100
Brazil	12.9	1.6		2.3	0.3	2.3	3.4	0.7	0.1	1.5	1.8	0.0	1.9	2.0	0.6	2.1	6.9	59.7	100
Canada	0.3	1.6	1.2		0.1	3.4	1.3	0.9	0.1	0.6	1.8	0.0	0.4	0.1	1.3	0.3	12.2	74.3	100
China	0.9	3.4	2.4	1.0		1.9	3.2	34.1	0.3	1.3	10.9	0.0	0.7	0.3	11.3	1.6	5.5	21.1	100
France	0.9	1.2	2.4	3.0	0.3		12.2	0.7	0.1	8.5	1.2	0.3	0.6	1.6	1.1	10.9	19.7	35.2	100
Germany	0.8	1.4	4.0	3.2	0.3	16.5		0.8	0.2	6.3	3.7	0.3	1.3	1.1	1.0	6.1	15.8	37.2	100
Hong Kong	0.4	8.5	1.0	1.9	8.5	1.0	1.5		0.5	0.6	6.3	0.5	0.3	0.1	7.2	0.7	12.5	48.5	100
India	0.3	3.9	0.7	1.5	0.5	1.8	2.3	5.0		1.0	5.4	0.0	0.2	0.1	2.8	0.9	17.0	56.7	100
Italy	3.1	1.0	6.7	1.4	0.2	21.6	12.7	0.5	0.1		3.7	0.0	0.3	0.8	0.4	9.8	15.1	22.6	100
Japan	0.4	7.2	2.8	2.4	2.2	2.5	3.1	4.6	0.1	0.9		2.2	0.8	0.1	2.9	1.2	10.0	56.5	100
Korea	0.7	3.2	0.1	11.3	16.2	2.0	3.4	3.1	0.6	0.4	4.8		0.6	0.6	0.8	0.5	3.7	48.0	100
Mexico	2.8	0.3	1.5	1.0	0.1	1.0	1.2	0.2	0.0	0.5	1.0	0.0		0.1	0.1	2.6	2.2	85.5	100
Portugal	0.4	0.3	6.9	0.7	0.5	9.9	5.7	0.3	0.0	2.6	2.5	0.0	1.9		0.2	26.5	13.4	28.3	100
Singapore	0.4	6.5	1.1	0.9	7.0	1.0	1.2	27.6	0.5	0.8	5.9	0.4	0.2	0.1		0.8	12.3	33.4	100
Spain	9.0	1.0	3.3	1.2	0.4	10.1	8.6	0.7	0.1	3.9	1.9	0.0	5.6	2.5	0.5		15.8	35.4	100
United Kingdom	0.1	4.7	1.0	3.3	0.1	11.2	9.2	1.7	0.2	3.1	3.4	0.1	0.3	0.7	1.7	3.0		56.2	100
United States	0.8	3.7	3.6	26.4	0.2	6.1	8.0	2.2	0.2	3.3	6.9	0.6	3.4	0.3	1.6	2.1	30.5		100
Average	2.0	2.9	3.2	3.8	2.2	5.6	4.7	5.4	0.2	2.2	4.0	0.3	1.4	0.6	2.2	4.4	13.8	47.0	
Output Share	1.4	1.5	4.5	2.6	10.5	5.0	7.5	0.6	6.4	4.9	12.2	2.2	2.8	0.6	0.3	2.8	4.6	29.5	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.5: Total Equity Composition, 1996 - 2000

	Issuing Country																	
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.7	17.4	1.0	0.2	1.9	2.1	0.3	0.0	1.0	1.6	0.2	4.9	0.2	0.2	6.2	4.7	57.4	100
Australia	0.6		0.6	0.9	0.3	1.0	1.1	5.1	0.3	0.6	4.2	0.4	0.2	0.1	1.7	0.5	29.4	53.2	100
Brazil	10.3	1.3		2.0	0.5	2.3	3.1	0.5	0.1	1.4	1.7	0.3	1.6	1.9	0.5	1.9	6.9	63.6	100
Canada	0.7	1.1	1.3		0.1	4.5	1.7	1.2	0.1	0.7	1.9	0.2	0.9	0.1	0.8	0.4	13.4	70.9	100
China	0.9	3.4	2.2	0.9		1.8	3.0	30.9	0.4	1.2	10.2	3.4	0.6	0.3	12.2	1.6	5.6	21.4	100
France	1.5	1.3	2.9	4.4	0.5		12.7	0.6	0.2	6.4	2.2	0.4	0.5	1.0	1.1	6.9	20.2	37.1	100
Germany	0.6	1.1	2.4	1.6	1.1	17.1		0.7	0.3	5.6	3.2	0.5	1.0	1.0	0.9	4.1	18.5	40.3	100
Hong Kong	0.2	2.2	0.5	0.8	62.2	0.5	1.3		0.4	0.3	3.3	0.8	0.1	0.1	3.3	0.4	6.7	17.1	100
India	0.4	5.2	1.0	1.9	1.4	1.6	2.1	5.4		1.0	3.9	0.6	0.2	0.2	4.5	1.0	19.3	50.6	100
Italy	1.4	0.7	2.8	0.8	0.4	18.9	11.1	0.6	0.2		4.2	0.4	0.4	0.7	0.4	6.6	17.5	32.8	100
Japan	1.0	4.0	1.8	1.8	4.7	2.7	3.5	3.8	0.5	0.9		2.3	0.6	0.3	3.8	1.0	11.6	55.9	100
Korea	0.6	2.4	1.0	2.3	26.5	1.6	3.4	5.0	2.7	0.5	3.9		1.2	0.3	2.0	0.6	5.0	40.9	100
Mexico	2.6	0.3	1.4	1.0	0.1	0.9	1.1	0.2	0.0	0.5	0.8	0.1		0.1	0.1	2.7	2.4	85.8	100
Portugal	0.4	0.2	38.0	0.4	0.2	5.9	3.6	0.1	0.0	1.8	1.0	0.1	1.3		0.0	25.5	8.3	13.3	100
Singapore	0.2	4.8	0.6	0.6	12.9	1.1	1.4	14.1	0.9	0.7	5.9	1.5	0.2	0.1		0.7	17.4	37.1	100
Spain	7.8	0.9	2.8	1.1	0.7	11.7	8.5	0.6	0.1	3.7	2.0	0.4	4.8	2.5	0.6		16.8	35.1	100
United Kingdom	0.3	2.8	0.9	2.0	0.2	11.4	9.8	1.9	0.3	3.3	4.3	0.3	0.4	0.6	1.3	3.5		56.6	100
United States	1.0	3.0	3.1	26.9	0.5	5.7	5.9	2.3	0.4	2.5	6.0	0.6	3.5	0.4	1.6	2.3	34.2		100
Average	1.8	2.1	4.7	3.0	6.6	5.3	4.4	4.3	0.4	1.9	3.5	0.7	1.3	0.6	2.1	3.9	14.0	45.2	
Output Share	1.4	1.6	4.1	2.6	14.1	4.5	6.7	0.6	7.0	4.3	10.4	2.4	2.7	0.5	0.4	2.6	4.4	29.7	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.6: Total Equity Composition, 2001 - 2005

	Issuing Country													
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	EMU	HK	India	Japan	Korea	Mexico	Sing.	UK	USA	Total
Argentina		0.1	3.2	0.1	0.0	7.4	0.0	0.0	0.3	0.0	1.2	0.0	1.2	86.5	100
Australia	0.2		0.2	1.3	0.3	4.0	2.3	0.3	3.8	0.8	0.2	1.3	20.2	65.2	100
Brazil	15.6	0.1		1.5	0.3	40.3	0.0	0.0	0.7	0.1	1.0	0.1	4.8	35.4	100
Canada	0.8	2.1	1.3		0.2	9.0	1.3	0.1	5.4	0.7	0.9	0.9	13.9	63.4	100
China	0.1	1.6	0.3	0.2		1.4	88.0	0.0	1.0	1.9	0.4	1.3	0.6	3.2	100
EMU	1.8	1.4	3.5	4.0	1.3		0.9	0.5	6.8	0.9	1.8	1.1	28.8	47.3	100
Hong Kong	0.1	3.3	0.2	0.7	65.3	1.3		0.3	3.0	1.2	0.1	2.5	16.5	5.6	100
India	0.4	6.3	1.1	2.3	3.1	5.2	7.1		2.8	1.0	0.3	6.9	16.4	47.1	100
Japan	0.4	3.4	1.0	2.2	3.9	9.5	2.5	0.4		1.4	0.6	2.5	12.6	59.6	100
Korea	0.4	2.1	1.1	1.6	27.2	5.5	6.5	2.3	4.4		1.0	1.9	4.7	41.3	100
Mexico	1.0	0.2	0.6	0.5	0.1	2.6	0.1	0.0	0.4	0.1		0.1	1.3	93.0	100
Singapore	0.2	6.5	0.6	0.7	14.0	4.0	18.0	2.0	8.1	6.7	0.2		14.1	24.9	100
United Kingdom	0.3	5.3	1.0	2.0	0.9	26.4	4.4	0.6	8.9	1.8	0.9	2.0		45.5	100
United States	0.6	4.8	2.7	14.3	1.0	21.3	2.9	0.9	13.5	2.8	4.0	3.1	28.1		100
Average	1.7	2.9	1.3	2.4	9.0	10.6	10.3	0.6	4.6	1.5	1.0	1.8	12.6	47.5	
Output Share	1.2	1.6	3.9	2.6	17.5	17.4	0.6	7.8	9.0	2.4	2.6	0.4	4.3	28.7	100

[a] Aus. denotes Australia. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.7: Portfolio Equity Composition, 1990 - 1995

	Issuing Country																		
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.2	4.9	0.6	0.0	2.6	2.7	0.2	0.0	1.3	2.9	0.0	2.6	0.1	0.1	5.2	4.8	71.7	100
Australia	0.0		0.2	0.7	0.0	1.7	1.8	4.7	0.3	0.8	14.3	0.1	0.1	0.1	1.5	0.6	13.2	59.8	100
Brazil	0.6	0.2		0.7	0.0	3.1	3.3	0.2	0.0	1.5	3.2	0.0	0.5	1.0	0.1	1.1	5.8	78.4	100
Canada	0.0	0.3	0.3		0.0	5.4	1.0	0.4	0.0	0.5	1.1	0.0	0.2	0.1	0.1	0.3	9.4	80.9	100
China	0.0	0.8	0.3	0.2		3.2	3.5	27.9	0.2	1.6	29.9	0.2	0.1	0.1	2.6	1.0	4.7	23.5	100
France	0.0	0.2	0.4	0.9	0.0		33.0	0.4	0.0	10.9	5.4	0.0	0.2	0.4	0.1	7.8	22.9	17.3	100
Germany	0.0	0.2	0.4	0.3	0.0	30.6		0.4	0.0	4.6	5.2	0.0	0.2	0.3	0.1	2.8	19.2	35.5	100
Hong Kong	0.0	2.8	0.1	0.6	0.8	1.6	1.8		0.3	0.8	13.9	0.1	0.1	0.1	1.7	0.5	13.2	61.6	100
India	0.0	1.8	0.2	0.7	0.1	2.2	2.4	3.7		1.1	7.6	0.0	0.1	0.1	1.1	0.7	17.1	61.2	100
Italy	0.1	0.2	0.5	0.4	0.0	24.2	11.6	0.4	0.0		5.9	0.0	0.2	0.4	0.1	3.5	14.2	38.1	100
Japan	0.1	2.8	0.6	0.5	0.3	7.6	8.3	4.6	0.2	3.7		0.2	0.4	0.3	1.1	2.5	11.1	55.7	100
Korea	0.0	1.2	0.3	0.2	0.2	3.5	3.9	2.4	0.1	1.7	52.5		0.1	0.1	0.5	1.1	5.3	26.6	100
Mexico	0.2	0.1	0.3	0.4	0.0	1.0	1.0	0.1	0.0	0.4	1.0	0.0		0.1	0.0	1.8	1.7	91.9	100
Portugal	0.1	0.2	1.5	0.4	0.0	8.7	8.2	0.4	0.0	3.9	4.9	0.0	0.2		0.1	10.6	15.2	45.5	100
Singapore	0.0	3.4	0.2	0.6	0.3	1.7	1.9	6.7	0.4	0.9	11.6	0.1	0.1	0.1		0.6	14.0	57.6	100
Spain	0.2	0.3	0.6	0.5	0.0	29.4	12.3	0.5	0.1	5.9	6.7	0.0	0.7	1.8	0.1		17.9	23.2	100
United Kingdom	0.0	0.5	0.4	1.0	0.0	13.3	12.1	0.8	0.1	3.7	4.6	0.0	0.2	0.4	0.2	2.8		59.8	100
United States	0.2	1.2	1.4	36.1	0.0	4.6	4.8	1.8	0.2	2.1	4.8	0.0	2.5	0.3	0.5	1.6	38.0		100
Average	0.1	1.0	0.7	2.6	0.1	8.5	6.7	3.3	0.1	2.7	10.3	0.0	0.5	0.3	0.6	2.6	13.4	52.3	
Output Share	1.4	1.5	4.5	2.6	10.5	5.0	7.5	0.6	6.4	4.9	12.2	2.2	2.8	0.6	0.3	2.8	4.6	29.5	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.8: Portfolio Equity Composition, 1996 - 2000

	Issuing Country																	
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.2	12.3	0.6	0.0	2.1	2.1	0.1	0.0	1.0	1.9	0.1	3.3	0.2	0.1	5.5	4.6	65.9	100
Australia	0.1		0.3	1.0	0.2	2.3	2.3	4.2	0.4	1.2	11.5	0.8	0.2	0.1	2.1	0.8	15.8	56.9	100
Brazil	0.6	0.2		0.8	0.0	3.0	2.9	0.2	0.0	1.4	2.5	0.2	0.5	1.2	0.1	1.3	6.4	78.5	100
Canada	0.1	0.6	0.5		0.0	6.4	1.5	0.7	0.1	0.7	1.8	0.1	0.5	0.1	0.3	0.7	13.6	72.2	100
China	0.0	1.0	0.3	0.2		3.2	3.3	26.3	0.4	1.5	23.2	2.7	0.1	0.2	3.9	1.3	5.6	26.9	100
France	0.1	0.3	0.5	1.2	0.1		26.5	0.5	0.1	9.7	5.1	0.3	0.6	0.5	0.2	8.8	24.0	21.5	100
Germany	0.0	0.2	0.4	0.4	0.1	28.0		0.3	0.1	4.2	4.0	0.3	0.2	0.4	0.1	3.3	21.1	36.8	100
Hong Kong	0.0	3.1	0.1	0.6	2.8	1.5	1.5		0.5	0.7	10.2	0.8	0.1	0.1	2.4	0.6	14.2	60.7	100
India	0.0	2.2	0.2	0.8	0.3	2.2	2.3	3.6		1.1	6.3	0.4	0.1	0.1	1.6	0.9	19.3	58.6	100
Italy	0.1	0.4	1.2	0.5	0.1	17.7	9.6	0.5	0.1		5.9	0.5	0.3	0.4	0.2	3.8	17.7	40.9	100
Japan	0.1	2.6	0.4	1.0	0.7	6.2	6.7	3.5	0.5	2.5		2.1	0.2	0.3	1.2	2.2	12.5	57.3	100
Korea	0.1	1.2	0.2	0.2	1.1	3.5	4.8	7.5	0.2	1.5	31.7		0.3	0.2	1.9	1.2	10.1	34.1	100
Mexico	0.3	0.1	0.4	0.4	0.0	0.9	0.9	0.1	0.0	0.4	0.8	0.0		0.1	0.0	2.0	2.0	91.6	100
Portugal	0.1	0.3	1.7	0.5	0.1	7.7	7.2	0.3	0.1	3.1	3.8	0.3	1.1		0.2	11.7	16.5	45.4	100
Singapore	0.0	4.2	0.1	0.6	1.7	1.5	2.1	9.2	0.6	0.7	7.8	0.7	0.1	0.1		0.6	19.1	51.0	100
Spain	1.4	0.3	0.6	0.5	0.1	24.8	11.4	0.4	0.1	4.5	4.1	0.4	1.1	2.0	0.2		19.4	28.7	100
United Kingdom	0.1	1.1	0.6	1.2	0.1	13.7	11.7	1.4	0.2	4.2	5.9	0.3	0.4	0.6	0.5	4.2		53.6	100
United States	0.3	1.8	1.7	32.4	0.1	5.3	4.9	1.9	0.4	2.4	5.4	0.3	2.8	0.4	0.8	2.1	37.0		100
Average	0.2	1.1	1.2	2.4	0.4	7.2	5.7	3.4	0.2	2.3	7.3	0.6	0.7	0.4	0.9	2.8	14.4	48.9	
Output Share	1.4	1.6	4.1	2.6	14.1	4.5	6.7	0.6	7.0	4.3	10.4	2.4	2.7	0.5	0.4	2.6	4.4	29.7	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.9: Portfolio Equity Composition, 2001 - 2005

	Issuing Country													
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	EMU	HK	India	Japan	Korea	Mexico	Sing.	UK	USA	Total
Argentina		0.0	2.4	0.0	0.0	7.2	0.0	0.0	0.2	0.0	0.9	0.0	1.1	88.1	100
Australia	0.0		0.2	1.4	0.2	7.8	2.2	0.4	9.2	1.2	0.1	1.0	9.9	66.3	100
Brazil	4.7	0.1		0.6	0.1	33.6	0.0	0.0	0.2	0.0	0.3	0.0	4.6	55.8	100
Canada	0.0	1.6	0.5		0.1	10.5	1.5	0.2	7.7	1.1	0.7	0.5	13.7	61.8	100
China	0.0	1.0	0.2	0.2		8.7	29.8	0.6	19.7	4.6	0.1	4.0	6.0	25.1	100
EMU	0.1	1.1	1.0	0.7	0.4		1.1	0.7	12.6	1.1	0.3	0.4	32.0	48.5	100
Hong Kong	0.0	1.4	0.1	1.4	23.4	2.1		0.4	5.8	2.5	0.0	2.7	46.5	13.7	100
India	0.0	2.2	0.1	0.9	0.7	6.4	6.5		7.4	0.9	0.1	9.2	11.8	53.9	100
Japan	0.0	2.6	0.1	2.5	1.0	12.6	2.6	0.3		0.4	0.1	0.6	14.6	62.8	100
Korea	0.0	1.4	0.0	0.9	1.2	3.3	8.3	1.9	16.7		0.0	0.7	8.2	57.3	100
Mexico	0.1	0.0	0.1	0.2	0.0	1.8	0.0	0.0	0.3	0.1		0.0	0.9	96.4	100
Singapore	0.0	4.9	0.3	0.7	5.8	4.9	18.3	2.3	11.2	8.5	0.1		15.4	27.6	100
United Kingdom	0.1	4.2	0.9	0.5	0.9	29.7	3.6	0.6	16.4	3.0	1.1	1.4		37.6	100
United States	0.1	3.6	2.5	10.4	0.8	23.4	2.3	1.2	19.4	4.0	2.4	1.6	28.5		100
Average	0.4	1.8	0.6	1.6	2.7	11.7	5.9	0.7	9.7	2.1	0.5	1.7	14.9	53.5	
Output Share	1.2	1.6	3.9	2.6	17.5	17.4	0.6	7.8	9.0	2.4	2.6	0.4	4.3	28.7	100

[a] Aus. denotes Australia. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.10: Debt Security Composition, 1990 - 1995

	Issuing Country																		
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.3	7.4	1.5	0.1	4.0	3.3	0.9	0.1	2.3	3.4	0.2	5.4	0.3	1.0	2.7	14.7	52.4	100
Australia	0.3		0.7	0.8	0.6	2.5	2.1	10.1	0.3	1.5	20.2	1.3	0.4	0.2	11.9	0.6	20.3	26.1	100
Brazil	2.9	0.3		1.8	0.2	4.6	3.9	1.0	0.1	2.7	3.9	0.3	2.2	1.0	1.2	1.1	16.9	55.9	100
Canada	1.1	1.6	3.8		0.1	2.8	0.9	1.5	0.2	0.9	2.5	0.3	4.7	0.0	3.5	0.2	11.9	64.1	100
China	0.2	1.0	0.6	0.4		3.4	2.9	22.8	0.2	2.0	26.1	2.3	0.4	0.2	14.0	0.8	10.2	12.5	100
France	0.6	0.4	1.9	1.4	1.0		7.0	3.1	0.3	7.7	20.8	0.4	1.1	1.1	3.3	3.5	30.2	16.2	100
Germany	1.3	0.7	1.7	1.9	0.7	11.2		3.2	0.6	11.4	8.9	1.0	1.0	1.1	2.4	2.8	40.5	9.8	100
Hong Kong	0.2	2.9	0.6	0.9	4.1	3.1	2.6		0.5	1.9	19.0	1.7	0.4	0.2	15.5	0.7	21.9	23.8	100
India	0.3	1.7	0.7	1.0	0.6	3.7	3.3	8.7		2.3	10.3	0.8	0.4	0.2	9.2	0.9	27.9	28.2	100
Italy	1.2	0.1	0.4	0.2	0.1	14.9	6.6	1.9	0.0		7.7	0.1	0.8	0.4	3.0	3.6	44.0	15.0	100
Japan	0.3	1.9	0.7	1.1	1.2	3.9	3.2	37.6	0.2	1.1		1.3	0.4	0.1	13.6	0.4	12.5	20.6	100
Korea	0.3	1.6	0.7	0.4	1.8	3.9	3.3	7.6	0.2	2.3	43.1		0.4	0.2	6.2	0.9	11.7	15.3	100
Mexico	1.9	0.1	1.8	1.5	0.1	2.1	1.7	0.5	0.0	1.2	1.6	0.1		0.2	0.5	1.4	8.0	77.3	100
Portugal	0.6	0.3	3.5	0.8	0.2	10.3	7.9	1.2	0.1	5.2	4.5	0.3	0.8		1.3	3.9	34.6	24.4	100
Singapore	0.3	3.3	0.7	0.9	2.4	3.2	2.7	15.6	0.5	1.9	17.3	1.3	0.4	0.2		0.8	22.6	26.0	100
Spain	1.0	0.1	0.5	0.5	0.5	14.1	3.6	1.2	0.0	11.0	5.1	0.0	1.6	4.8	1.1		39.2	15.7	100
United Kingdom	0.5	0.9	0.9	3.1	0.3	9.6	14.1	4.2	0.4	12.0	22.4	0.8	1.5	0.6	4.5	2.6		21.5	100
United States	2.0	0.8	4.3	6.2	0.4	5.5	3.1	4.3	0.3	3.4	32.4	3.0	5.8	0.3	2.6	1.0	24.6		100
Average	0.9	1.1	1.8	1.4	0.8	6.1	4.2	7.4	0.2	4.2	14.7	0.9	1.6	0.7	5.6	1.6	23.0	29.7	
Output Share	1.3	1.5	4.5	2.6	10.5	5.0	7.5	0.6	6.4	4.9	12.2	2.2	2.8	0.6	0.3	2.8	4.6	29.5	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.11: Debt Security Portfolio Composition, 1996 - 2000

	Issuing Country																		
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	France	Ger.	HK	India	Italy	Japan	Korea	Mexico	Portugal	Sing.	Spain	UK	USA	Total
Argentina		0.3	7.6	1.5	0.2	3.8	3.1	0.9	0.1	2.2	3.1	0.3	5.1	0.3	1.1	2.7	15.0	52.6	100
Australia	0.1		0.1	0.3	1.1	0.5	1.8	5.7	0.1	0.2	6.5	1.3	0.0	0.0	12.8	0.1	32.0	37.3	100
Brazil	3.1	0.3		1.8	0.2	4.4	3.7	1.0	0.1	2.6	3.5	0.3	2.1	1.1	1.2	1.2	17.0	56.5	100
Canada	1.0	0.5	1.4		0.2	1.5	1.2	2.1	0.1	0.4	2.7	0.6	2.9	0.0	1.8	0.1	18.2	65.2	100
China	0.2	1.1	0.6	0.4		3.4	2.8	20.7	0.2	1.9	24.8	2.6	0.4	0.2	15.7	0.8	10.7	13.4	100
France	0.7	0.5	1.1	1.6	1.3		12.1	2.8	0.2	11.9	11.4	0.9	0.9	1.6	3.5	5.6	26.7	17.2	100
Germany	1.1	0.5	1.3	1.9	0.9	11.9		2.1	0.8	12.5	7.2	1.0	0.7	1.9	2.2	5.4	34.0	14.7	100
Hong Kong	0.2	2.9	0.6	0.9	5.4	2.9	2.4		0.5	1.8	17.5	1.8	0.4	0.2	16.5	0.7	22.0	23.3	100
India	0.3	1.7	0.6	1.0	0.8	3.6	3.0	8.5		2.2	9.3	0.9	0.4	0.2	9.9	0.9	28.2	28.5	100
Italy	1.5	0.1	0.9	0.2	0.2	16.5	12.1	0.8	0.0		2.7	0.1	0.6	1.0	1.7	5.1	46.0	10.6	100
Japan	0.1	1.4	0.3	1.0	1.1	3.9	5.6	22.2	0.2	1.2		1.8	0.2	0.0	11.4	0.6	15.7	33.2	100
Korea	0.3	1.7	0.7	0.5	2.6	3.9	3.3	7.6	0.2	2.3	40.8		0.4	0.2	6.9	0.9	12.2	15.5	100
Mexico	1.9	0.1	1.6	1.4	0.1	2.0	1.6	0.5	0.0	1.1	1.5	0.1		0.2	0.6	1.3	7.9	78.2	100
Portugal	0.2	0.1	4.3	0.5	0.3	10.7	11.2	0.1	0.0	22.1	0.6	0.1	0.5		0.1	20.7	17.0	11.5	100
Singapore	0.3	3.4	0.6	0.9	3.2	3.1	2.6	15.3	0.6	1.8	15.9	1.5	0.4	0.2		0.8	23.1	26.5	100
Spain	2.4	0.1	4.6	0.3	1.0	16.8	11.4	0.4	0.0	10.3	1.8	0.1	2.4	7.1	0.4		28.3	12.6	100
United Kingdom	0.4	1.3	0.8	2.9	0.4	10.0	16.9	2.6	0.3	10.9	16.1	0.9	0.7	1.0	3.2	3.3		28.2	100
United States	2.6	1.6	4.9	10.5	0.4	6.9	7.5	2.8	0.5	2.6	14.3	3.3	5.1	0.3	1.6	1.2	34.1		100
Average	1.0	1.0	1.9	1.6	1.1	6.2	6.0	5.7	0.2	5.2	10.6	1.0	1.4	0.9	5.3	3.0	22.8	30.9	
Output Share	1.4	1.6	4.1	2.6	14.1	4.5	6.7	0.6	7.0	4.3	10.4	2.4	2.7	0.5	0.4	2.6	4.4	29.7	100

[a] Aus. denotes Australia. Ger. denotes Germany. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.12: DEBT SECURITY PORTFOLIO COMPOSITION, 2001 - 2005

	Issuing Country													
Portfolio owner	Argentina	Aus.[a]	Brazil	Canada	China	EMU	HK	India	Japan	Korea	Mexico	Sing.	UK	USA	Total
Argentina		0.0	3.3	0.0	0.0	2.3	0.0	0.0	0.1	0.0	0.1	0.0	0.2	93.8	100
Australia	0.0		0.1	2.4	0.5	9.1	5.5	0.4	5.1	0.6	0.0	9.8	39.4	27.1	100
Brazil	1.0	0.1		1.6	0.1	11.8	0.2	0.0	2.6	0.3	0.0	0.5	11.6	70.2	100
Canada	0.3	0.5	1.4		0.2	2.7	1.4	0.2	1.2	0.2	1.5	0.9	20.5	68.9	100
China	0.2	1.1	0.6	0.4		9.1	20.4	0.2	23.9	3.0	0.4	16.3	11.1	13.4	100
EMU	0.8	1.0	1.5	1.0	0.9		1.6	0.5	6.7	0.6	1.3	3.1	56.3	24.8	100
Hong Kong	0.0	20.1	0.4	4.9	3.5	12.7		0.4	5.7	5.9	0.1	2.5	12.9	31.0	100
India	0.0	0.4	0.1	0.7	0.5	9.1	5.1		3.0	0.3	0.1	7.2	18.7	54.6	100
Japan	0.0	1.6	0.3	1.5	0.9	19.2	4.9	0.2		1.1	0.2	7.3	17.7	45.1	100
Korea	0.1	0.8	0.1	0.3	0.8	7.7	3.6	0.5	1.9		0.5	2.2	9.3	72.2	100
Mexico	0.9	0.0	0.7	1.5	0.1	9.5	0.2	0.0	0.4	0.1		0.1	4.2	82.4	100
Singapore	0.0	13.1	0.1	2.0	0.7	17.9	2.6	1.2	5.8	4.8	0.1		29.0	22.5	100
United Kingdom	0.1	1.9	0.3	2.5	0.3	43.7	1.4	0.3	11.9	0.7	0.3	2.0		34.6	100
United States	0.8	1.2	2.4	9.9	0.9	20.5	1.2	0.3	10.6	2.0	2.2	0.8	47.2		100
Average	0.3	3.2	0.9	2.2	0.7	13.5	3.7	0.3	6.1	1.5	0.5	4.0	21.4	49.3	
Output Share	1.2	1.6	3.9	2.6	17.5	17.4	0.6	7.8	9.0	2.4	2.6	0.4	4.3	28.7	100

[a] Aus. denotes Australia. HK denotes Hong Kong. Sing. denotes Singapore.

Table A.13: TRADED ENDOWMENT PARAMETER ESTIMATES

	1990 - 1995		1996 - 2000		2001 - 2005	
Country	ρ_z^T	$(\sigma_z^T)^2$	ρ_z^T	$(\sigma_z^T)^2$	ρ_z^T	$(\sigma_z^T)^2$
Argentina	0.626 (0.237)	3.34×10^{-5} (1.43×10^{-5})	0	2.07×10^{-4} (6.56×10^{-5})	0	6.51×10^{-4} (2.06×10^{-4})
Australia	0.125 (0.212)	3.11×10^{-5} (8.99×10^{-6})	*	3.81×10^{-5} (1.2×10^{-5})	0	2.07×10^{-5} (6.54×10^{-6})
Brazil	0	2.18×10^{-3} (9.30×10^{-4})	0	2.88×10^{-4} (9.1×10^{-5})	*	8.55×10^{-4} (2.70×10^{-4})
Canada	0.625 (0.150)	2.20×10^{-5} (6.35×10^{-6})	0.039 (0.240)	1.39×10^{-5} (4.38×10^{-6})	0.080 (0.292)	2.33×10^{-5} (7.36×10^{-6})
China	0.683 (0.169)	7.81×10^{-6} (2.26×10^{-6})	0	8.38×10^{-6} (2.65×10^{-6})	0	1.35×10^{-5} (4.26×10^{-6})
France[a]	0.262 (0.223)	2.39×10^{-5} (6.91×10^{-6})	0.000 (0.223)	8.06×10^{-6} (2.55×10^{-6})	0.380 (0.222)	8.13×10^{-6} (2.64×10^{-6})
Germany	0.806 (0.112)	7.09×10^{-6} (2.05×10^{-6})	0.852 (0.123)	2.79×10^{-6} (8.89×10^{-7})		
Hong Kong	0.115 (0.199)	4.44×10^{-5} (1.28×10^{-5})	0.813 (0.123)	1.01×10^{-5} (3.20×10^{-6})	0.942 (0.071)	1.14×10^{-5} (3.67×10^{-6})
India	0	4.41×10^{-5} (1.47×10^{-5})	0	4.41×10^{-5} (1.47×10^{-5})	0.013 (0.224)	6.26×10^{-5} (1.98×10^{-5})
Italy	0.167 (0.205)	3.81×10^{-5} (1.10×10^{-5})	0	2.55×10^{-5} (8.08×10^{-6})		
Japan	0.424 (0.187)	3.62×10^{-5} (1.05×10^{-5})	0.530 (0.183)	3.60×10^{-5} (1.14×10^{-5})	0.654 (0.174)	4.70×10^{-5} (1.49×10^{-5})
Korea	0	4.54×10^{-5} (1.31×10^{-5})	0.413 (0.210)	1.90×10^{-4} (6.00×10^{-5})	0.178 (0.215)	3.64×10^{-5} (1.15×10^{-5})
Mexico	0.536 (0.230)	2.56×10^{-5} (1.10×10^{-5})	0.527 (0.481)	3.56×10^{-5} (1.13×10^{-5})	0.333 (0.214)	1.77×10^{-5} (5.60×10^{-6})
Portugal	0	3.28×10^{-5} (9.68×10^{-6})	0	3.28×10^{-5} (9.68×10^{-6})		
Singapore	0.290 (0.195)	2.90×10^{-5} (8.55×10^{-6})	0.041 (0.233)	2.02×10^{-4} (6.75×10^{-5})	0.457 (0.217)	4.27×10^{-5} (1.39×10^{-5})
Spain	0	1.59×10^{-5} (4.79×10^{-6})	0	1.59×10^{-5} (4.79×10^{-6})		
United Kingdom	0.293 (0.196)	1.39×10^{-5} (4.01×10^{-6})	0	5.04×10^{-6} (1.59×10^{-6})	0	8.52×10^{-6} (2.69×10^{-6})
United States	0.869 (0.114)	2.35×10^{-6} (6.99×10^{-7})	0.422 (0.487)	2.98×10^{-5} (9.70×10^{-6})	0.015 (0.229)	5.32×10^{-5} (1.77×10^{-5})
Indonesia	0.672 (0.168)	8.54×10^{-7} (2.52×10^{-7})	0.492 (0.241)	6.20×10^{-5} (1.96×10^{-5})	0	7.37×10^{-6} (2.33×10^{-6})
Russia[c]	0.845 (0.128)	3.31×10^{-5} (1.08×10^{-5})	0.845 (0.128)	3.31×10^{-5} (1.08×10^{-5})	0.511 (0.212)	7.78×10^{-5} (2.46×10^{-5})
South Africa	0.381 (0.185)	3.47×10^{-5} (1.00×10^{-5})	0.734 (0.148)	5.35×10^{-6} (1.70×10^{-6})	0.314 (0.209)	1.60×10^{-5} (5.05×10^{-6})
Rest of the World	0.633	2.29×10^{-5}	0.690	3.35×10^{-5}	0.275	3.37×10^{-5}
Mean[b]	0.340	1.40×10^{-4}	0.228	6.46×10^{-5}	0.222	1.26×10^{-4}
Standard Deviation	0.300	4.94×10^{-4}	0.315	8.63×10^{-5}	0.287	2.58×10^{-4}

Notes: Standard error is given in brackets. If estimated ρ_z^T is less than zero, the assumption of it being zero is tested by likelihood ratio test. If the assumption is not rejected at 5% zero is entered, * denotes rejection at 5%. In either case, the estimates for the variance $(\sigma_z^T)^2$ are then taken from restricted model.

[a] Values for the EMU region are reported in this row for the 2001 - 2005 period.

[b] The mean is calculated over the Rest of the World and the 18 main countries in the dataset.

[c] Data is not available for the period from 1990 to 1995, therefore the next period data is used.

Table A.14: Nontraded Endowment Parameter Estimates

Country	1990 - 1995		1996 - 2000		2001 - 2005	
	ρ_z^N	$(\sigma_z^N)^2$	ρ_z^N	$(\sigma_z^N)^2$	ρ_z^N	$(\sigma_z^N)^2$
Argentina	0.535 (0.267)	5.24×10^{-5} (2.24×10^{-5})	0.859 (0.100)	9.28×10^{-6} (2.95×10^{-6})	0.821 (0.112)	4.13×10^{-5} (1.31×10^{-5})
Australia	0.605 (0.156)	5.27×10^{-6} (1.52×10^{-6})	0.122 (0.230)	7.92×10^{-6} (2.51×10^{-6})	0	4.20×10^{-6} (1.33×10^{-6})
Brazil	0.307 (0.280)	3.63×10^{-4} (1.55×10^{-4})	0	9.36×10^{-5} (2.96×10^{-5})	$*$	6.23×10^{-4} (1.97×10^{-4})
Canada	0.281 (0.193)	1×10^{-5} (2.89×10^{-6})	0.387 (0.203)	7.31×10^{-6} (2.31×10^{-6})	0.103 (0.217)	1.37×10^{-5} (4.34×10^{-6})
China	0.126 (0.228)	1.64×10^{-5} (4.73×10^{-6})	0	8.23×10^{-6} (2.60×10^{-6})	0	9.35×10^{-6} (2.96×10^{-6})
France[a]	0.197 (0.197)	1.97×10^{-6} (5.69×10^{-7})	0.516 (0.197)	1.13×10^{-6} (3.36×10^{-7})	0.104 (0.227)	6.06×10^{-7} (1.97×10^{-7})
Germany	0.855 (0.093)	1.08×10^{-6} (3.13×10^{-7})	0.544 (0.179)	8.01×10^{-7} (2.54×10^{-7})		
Hong Kong	0.706 (0.154)	1.96×10^{-6} (5.67×10^{-6})	$*$	6.66×10^{-4} (2.10×10^{-4})	$*$	9.14×10^{-5} (2.89×10^{-5})
India	0	3.01×10^{-5} (1.00×10^{-5})	0	3.01×10^{-5} (1.00×10^{-5})	0	1.68×10^{-5} (5.30×10^{-6})
Italy	0	3.46×10^{-6} (9.98×10^{-7})	0.326 (0.220)	3.58×10^{-6} (1.13×10^{-6})		
Japan	0	1.69×10^{-5} (4.88×10^{-6})	0	2.31×10^{-6} (7.32×10^{-6})	0.001 (0.231)	3.67×10^{-6} (1.16×10^{-6})
Korea	0.257 (0.204)	6.24×10^{-6} (1.80×10^{-6})	0.265 (0.210)	5.29×10^{-5} (1.67×10^{-5})	0.267 (0.211)	9.28×10^{-6} (2.94×10^{-6})
Mexico	0.291 (0.285)	1.84×10^{-4} (7.85×10^{-5})	0.040 (0.250)	1.69×10^{-5} (5.36×10^{-6})	0	2.83×10^{-5} (8.94×10^{-6})
Portugal[c]	0.323 (0.192)	2.48×10^{-5} (7.32×10^{-6})	0.323 (0.192)	2.48×10^{-5} (7.32×10^{-6})		
Singapore	0	2.01×10^{-4} (5.94×10^{-5})	0.141 (0.231)	1.26×10^{-4} (4.21×10^{-5})	0	8.98×10^{-5} (2.91×10^{-5})
Spain[c]	0.806 (0.193)	8.06×10^{-6} (2.45×10^{-6})	0.806 (0.193)	8.06×10^{-6} (2.45×10^{-6})		
United Kingdom	0	3.06×10^{-5} (8.83×10^{-6})	0	2.37×10^{-6} (7.50×10^{-7})	0	1.28×10^{-6} (4.04×10^{-7})
United States	0	2.00×10^{-4} (5.91×10^{-5})	0	2.84×10^{-6} (9.48×10^{-7})	0	5.42×10^{-6} (1.75×10^{-6})
Indonesia	0.756 (0.148)	8.11×10^{-7} (2.40×10^{-7})	0.803 (0.115)	6.92×10^{-5} (2.20×10^{-5})	0.493 (0.357)	6.61×10^{-6} (2.10×10^{-6})
Russia[c]	0.856 (0.123)	2.60×10^{-4} (8.50×10^{-5})	0.856 (0.123)	2.60×10^{-4} (8.50×10^{-5})	0.299 (0.228)	6.77×10^{-5} (2.14×10^{-5})
South Africa	0.822 (0.104)	1.36×10^{-6} (3.95×10^{-7})	0.476 (0.188)	2.45×10^{-6} (7.76×10^{-7})	0.306 (0.209)	1.45×10^{-6} (4.58×10^{-7})
Rest of the World	0.811	8.74×10^{-5}	0.712	1.11×10^{-4}	0.366	2.35×10^{-5}
Mean[b]	0.321	6.55×10^{-5}	0.265	6.18×10^{-5}	0.111	6.42×10^{-5}
Standard Deviation	0.309	9.95×10^{-5}	0.297	1.51×10^{-4}	0.226	1.57×10^{-4}

Notes: Standard error is given in brackets. If estimated ρ_z^N is less than zero, the assumption of it being zero is tested by likelihood ratio test. If the assumption is not rejected at 5% zero is entered, $*$ denotes rejection at 5%. In either case, the estimates for the variance $(\sigma_z^N)^2$ are then taken from restricted model.

[a] Values for the EMU region are reported in this row for the 2001 - 2005 period.

[b] The mean is calculated over the Rest of the World and the 18 main countries in the dataset.

[c] Data is not available for the period from 1990 to 1995, therefore the next period data is used.

Table A.15: Government Spending Parameter Estimates

	1990 - 1995		1996 - 2000		2001 - 2005	
Country	ρ_z^G	$(\sigma_z^G)^2$	ρ_z^G	$(\sigma_z^G)^2$	ρ_z^G	$(\sigma_z^G)^2$
Argentina	0	2.44×10^{-4} (1.04×10^{-4})	$*$	7.26×10^{-5} (2.29×10^{-5})	0.221 (0.213)	6.64×10^{-4} (2.10×10^{-4})
Australia	$*$	4.83×10^{-5} (1.39×10^{-5})	0.037 (0.231)	2.28×10^{-5} (7.21×10^{-6})	0	5.10×10^{-6} (1.61×10^{-6})
Brazil	0	3.00×10^{-3} (1.28×10^{-3})	$*$	4.65×10^{-4} (1.47×10^{-4})	0	8.90×10^{-4} (2.81×10^{-4})
Canada	0.314 (0.212)	1.77×10^{-5} (5.10×10^{-6})	0.179 (0.217)	1.97×10^{-5} (6.24×10^{-6})	0.276 (0.214)	2.07×10^{-5} (6.54×10^{-6})
China	0	3.33×10^{-4} (9.60×10^{-5})	0.005 (0.267)	1.33×10^{-5} (4.20×10^{-6})	0.642 (0.198)	5.84×10^{-6} (1.85×10^{-6})
France[a]	0.374 (0.201)	4.20×10^{-6} (1.21×10^{-6})	0.605 (0.170)	1.23×10^{-6} (3.89×10^{-7})	$*$	2.32×10^{-6} (7.53×10^{-7})
Germany	0	4.49×10^{-5} (1.30×10^{-5})	0	2.22×10^{-5} (7.02×10^{-6})		
Hong Kong	0	9.23×10^{-5} (2.66×10^{-5})	0	9.83×10^{-5} (3.11×10^{-5})	0.249 (0.228)	8.70×10^{-5} (2.75×10^{-5})
India	0	8.54×10^{-4} (2.70×10^{-4})	0	8.54×10^{-4} (2.70×10^{-4})	$*$	1.21×10^{-3} (3.82×10^{-4})
Italy	0.721 (0.135)	3.22×10^{-6} (9.31×10^{-7})	0.051 (0.222)	3.84×10^{-6} (1.21×10^{-6})		
Japan	$*$	2.66×10^{-5} (7.69×10^{-6})	0.074 (0.222)	1.20×10^{-5} (3.79×10^{-6})	$*$	1.96×10^{-5} (6.20×10^{-6})
Korea	0	4.46×10^{-5} (1.29×10^{-5})	0	5.44×10^{-5} (1.72×10^{-5})	$*$	3.38×10^{-5} (1.07×10^{-5})
Mexico	0.387 (0.273)	5.17×10^{-4} (2.21×10^{-4})	0	1.72×10^{-3} (5.45×10^{-4})	0.279 (0.267)	1.37×10^{-4} (4.34×10^{-5})
Portugal[c]	0.776 (0.120)	1.32×10^{-6} (3.91×10^{-7})	0.776 (0.120)	1.32×10^{-6} (3.91×10^{-7})		
Singapore	0.009 (0.209)	6.53×10^{-4} (1.97×10^{-4})	0	4.83×10^{-4} (1.61×10^{-4})	0	4.40×10^{-4} (1.39×10^{-4})
Spain	0	8.15×10^{-6} (2.40×10^{-6})	0	8.15×10^{-6} (2.40×10^{-6})		
United Kingdom	0	1.16×10^{-5} (3.36×10^{-6})	0.068 (0.219)	1.08×10^{-5} (3.41×10^{-6})	0.246 (0.221)	4.65×10^{-6} (1.47×10^{-6})
United States	0.128 (0.215)	7.32×10^{-6} (2.16×10^{-6})	$*$	1.05×10^{-5} (3.33×10^{-6})	0	8.81×10^{-6} (2.79×10^{-6})
Indonesia	0	1.12×10^{-3} (3.30×10^{-4})	0	3.17×10^{-3} (1.00×10^{-3})	0	3.51×10^{-3} (1.11×10^{-3})
Russia	0	4.82×10^{-5} (1.56×10^{-5})	0	4.82×10^{-5} (1.56×10^{-5})	0.137 (0.252)	6.82×10^{-6} (2.16×10^{-6})
South Africa	0	5.55×10^{-5} (1.60×10^{-5})	0.729 (0.162)	9.37×10^{-6} (2.97×10^{-6})	0	1.16×10^{-5} (3.67×10^{-6})
Rest of the World	0	4.08×10^{-4}	0.243	1.08×10^{-3}	0.046	1.18×10^{-3}
Mean[b]	0.143	3.33×10^{-4}	0.107	2.61×10^{-4}	0.131	3.14×10^{-4}
Standard Deviation	0.252	6.94×10^{-4}	0.218	4.73×10^{-4}	0.186	4.50×10^{-4}

Notes: Standard error is given in brackets. If estimated ρ_z^G is less than zero, the assumption of it being zero is tested by likelihood ratio test. If the assumption is not rejected at 5% zero is entered, $*$ denotes rejection at 5%. In either case, the estimates for the variance $(\sigma_z^G)^2$ are then taken from restricted model.

[a] Values for the EMU region are reported in this row for the 2001 - 2005 period.

[b] The mean is calculated over the Rest of the World and the 18 main countries in the dataset.

[c] Data is not available for the period from 1990 to 1995, therefore the next period data is used.

Table A.16: Money Supply Parameter Estimates

	1990 - 1995		1996 - 2000		2001 - 2005	
Country	ρ_z^M	$(\sigma_z^M)^2$	ρ_z^M	$(\sigma_z^M)^2$	ρ_z^M	$(\sigma_z^M)^2$
Argentina	0.963 (0.045)	2.68×10^{-3} (7.85×10^{-4})	0.345 (0.227)	1.12×10^{-4} (3.54×10^{-5})	0.166 (0.222)	5.31×10^{-4} (1.68×10^{-4})
Australia	0.250 (0.213)	1.76×10^{-5} (5.08×10^{-6})	0	1.20×10^{-5} (3.79×10^{-6})	0	4.26×10^{-5} (1.35×10^{-5})
Brazil	0.446 (0.252)	3.51×10^{-2} (1.50×10^{-2})	0	1.66×10^{-3} (5.23×10^{-4})	0.171 (0.218)	2.63×10^{-4} (8.31×10^{-5})
Canada	0.630 (0.176)	6.69×10^{-6} (1.93×10^{-6})	0.835 (0.104)	3.00×10^{-6} (9.51×10^{-7})	0.216 (0.216)	4.56×10^{-6} (1.44×10^{-6})
China	0	2.08×10^{-4} (5.99×10^{-5})	0.301 (0.223)	3.04×10^{-5} (9.62×10^{-6})	0.159 (0.216)	1.76×10^{-5} (5.58×10^{-6})
France[a]	0.097 (0.270)	5.11×10^{-5} (1.48×10^{-5})	0.351 (0.271)	1.85×10^{-5} (5.84×10^{-6})	0.495 (0.312)	8.61×10^{-6} (2.73×10^{-6})
Germany	*	7.14×10^{-4} (2.06×10^{-4})	0	3.38×10^{-4} (1.07×10^{-4})		
Hong Kong	0	4.13×10^{-5} (1.38×10^{-5})	0	1.48×10^{-4} (4.67×10^{-5})	0	6.51×10^{-5} (2.06×10^{-5})
India	0.023 (0.203)	5.93×10^{-5} (1.71×10^{-5})	0	3.90×10^{-5} (1.23×10^{-5})	0	5.52×10^{-5} (1.75×10^{-5})
Italy	0.132 (0.202)	4.41×10^{-5} (1.27×10^{-5})	0	9.09×10^{-5} (2.87×10^{-5})		
Japan	0	4.69×10^{-5} (1.35×10^{-5})	0.385 (0.216)	1.47×10^{-5} (4.64×10^{-6})	0.645 (0.159)	6.77×10^{-5} (2.14×10^{-5})
Korea	0	3.16×10^{-5} (9.33×10^{-6})	0.709 (0.147)	3.76×10^{-5} (1.19×10^{-5})	0.230 (0.213)	2.08×10^{-5} (6.56×10^{-6})
Mexico	0.442 (0.185)	8.16×10^{-5} (2.36×10^{-5})	0.639 (0.238)	3.43×10^{-5} (1.09×10^{-5})	0.059 (0.222)	1.78×10^{-5} (5.63×10^{-6})
Portugal	0.045 (0.215)	7.20×10^{-5} (2.08×10^{-5})	0	5.65×10^{-5} (1.79×10^{-5})		
Singapore	0.084 (0.204)	4.20×10^{-5} (1.21×10^{-5})	0.046 (0.218)	3.18×10^{-4} (1.01×10^{-4})	0.103 (0.221)	6.57×10^{-5} (2.08×10^{-5})
Spain	*	5.83×10^{-5} (1.68×10^{-5})	0.005 (0.206)	6.81×10^{-5} (2.01×10^{-5})		
United Kingdom	0	3.36×10^{-5} (9.70×10^{-6})	0	1.39×10^{-5} (4.41×10^{-6})	0	3.11×10^{-6} (9.84×10^{-7})
United States	0.472 (0.190)	3.77×10^{-6} (1.09×10^{-6})	0.510 (0.185)	1.86×10^{-6} (5.32×10^{-7})	0.362 (0.215)	8.58×10^{-6} (2.72×10^{-6})
Indonesia	0	4.13×10^{-3} (1.46×10^{-3})	0	4.13×10^{-3} (1.46×10^{-3})	0	4.13×10^{-3} (1.46×10^{-3})
Russia	0.522 (0.185)	3.44×10^{-4} (1.12×10^{-4})	0.522 (0.185)	3.44×10^{-4} (1.12×10^{-4})	0.159 (0.217)	2.81×10^{-5} (8.89×10^{-6})
South Africa	0.294 (0.196)	4.11×10^{-5} (1.19×10^{-5})	0.472 (0.205)	3.80×10^{-5} (1.20×10^{-5})	0	4.65×10^{-5} (1.47×10^{-5})
Rest of the World	0.272	1.51×10^{-3}	0.331	1.50×10^{-3}	0.053	1.40×10^{-3}
Mean[b]	0.203	2.10×10^{-3}	0.235	2.37×10^{-4}	0.177	1.71×10^{-4}
Standard Deviation	0.272	8.00×10^{-3}	0.280	4.84×10^{-4}	0.193	3.67×10^{-4}

Notes: Standard error is given in brackets. If estimated ρ_z^M is less than zero, the assumption of it being zero is tested by likelihood ratio test. If the assumption is not rejected at 5% zero is entered, $*$ denotes rejection at 5%. In either case, the estimates for the variance $(\sigma_z^M)^2$ are then taken from restricted model.

[a] Values for the EMU region are reported in this row for the 2001 - 2005 period.

[b] The mean is calculated over the Rest of the World and the 18 main countries in the dataset.

Table A.17: STEADY STATE PARAMETERS

	1990 - 1995				1996 - 2000				2001 - 2005			
Country	$\frac{\bar{Y}^N}{\bar{Y}^T}$	Output	$\bar{g}_z$	Pop.	$\frac{\bar{Y}^N}{\bar{Y}^T}$	Output	$\bar{g}_z$	Pop.	$\frac{\bar{Y}^N}{\bar{Y}^T}$	Output	$\bar{g}_z$	Pop.
Argentina	1.94	0.011	0.133	0.006	2.05	0.012	0.129	0.006	1.98	0.005	0.122	0.006
Australia	2.64	0.016	0.184	0.003	2.88	0.016	0.176	0.003	3.18	0.018	0.172	0.003
Brazil	2.26	0.027	0.196	0.029	2.14	0.030	0.200	0.029	1.88	0.021	0.197	0.029
Canada	2.41	0.029	0.229	0.005	2.33	0.026	0.194	0.005	2.54	0.022	0.193	0.005
China	0.60	0.026	0.220	0.218	0.62	0.041	0.210	0.240	0.67	0.057	0.220	0.207
France[a]	3.97	0.067	0.248	0.011	3.89	0.058	0.010	0.213	3.18	0.220	0.201	0.041
Germany	1.86	0.100	0.193	0.015	2.16	0.087	0.193	0.014				
Hong Kong	7.54	0.006	0.078	0.001	9.25	0.007	0.089	0.001	13.06	0.006	0.099	0.001
India	1.17	0.015	0.127	0.162	1.17	0.017	0.127	0.165	1.43	0.018	0.115	0.169
Italy	2.80	0.056	0.208	0.011	2.78	0.048	0.188	0.010				
Japan	0.91	0.203	0.141	0.023	0.97	0.176	0.159	0.022	1.02	0.172	0.179	0.020
Korea	2.57	0.019	0.116	0.008	2.49	0.020	0.119	0.008	2.14	0.026	0.131	0.008
Mexico	2.40	0.018	0.116	0.016	2.20	0.019	0.109	0.017	2.35	0.030	0.117	0.017
Portugal	3.05	0.005	0.194	0.002	3.06	0.005	0.194	0.002				
Singapore	1.84	0.003	0.055	0.001	1.86	0.004	0.059	0.001	2.03	0.004	0.070	0.001
Spain	2.95	0.027	0.179	0.007	2.95	0.024	0.174	0.007				
UK	3.38	0.052	0.239	0.011	3.36	0.056	0.217	0.010	4.29	0.076	0.213	0.010
USA	3.57	0.323	0.162	0.048	3.44	0.354	0.146	0.047	3.56	0.448	0.155	0.046
Indonesia	0.86		0.088		0.82		0.067		0.84		0.077	
Russia	2.03		0.211		2.03		0.211		2.03		0.176	
South Africa	2.02		0.214		2.21		0.192		2.39		0.191	
Rest of the W.	1.63		0.171	0.423	1.69		0.157	0.430	1.75		0.148	0.437

[a] Values for the EMU region are reported in this row for the 2001 - 2005 period.

B Steady State with Homogeneous Consumption Tastes

To derive the steady state with homogeneous consumption tastes, it is assumed that $\lambda_z = 1$, $\gamma_z = 1$, $h_z = \frac{1}{2}$ and $\omega_{zz} = \kappa$ and $\omega_{zj} = \frac{1-\kappa}{X-1}$ as well as $\bar{W}_z = 0$ for all z.

The asset choice first-order conditions in (3.31), and (3.32) and the fact that from (3.20) it follows that the interest risk premium is zero in the steady state, ensure that gross nominal returns on all assets are equal in their respective currencies, thus for all z:

$$\bar{d}_z = \bar{i}_z = \frac{1}{\beta}$$

Therefore steady state excess returns are zero. There are no changes in nominal exchange rates in a non-stochastic steady state, so excess returns are also zero when adjusted for currency appreciation. Therefore, non-zero gross asset positions can be ignored for this derivation.

The aggregate steady state budget constraint in nominal per capita terms is given by:

$$\bar{P}^T_z \bar{Y}^T_z + \bar{P}^N_z \bar{Y}^N_z = \bar{P}_z(\bar{C}_z + \bar{G}_z) \tag{B.1}$$

By solving the expenditure minimisation problem and taking the limit of the resulting price index, with the assumption of homogeneous consumption tastes imposed, it is standard to show that the price indices for the consumption aggregators given in (3.2),$\bar{P}_z$, and (3.4), $\bar{P}^T_{z\Omega}$, in the steady state are given by:[32]

$$\bar{P}_z = (\bar{P}^T_{z\Omega})^{\frac{1}{2}}(\bar{P}^N_z)^{\frac{1}{2}} \qquad \bar{P}^T_{z\Omega} = \prod_{j\neq z}\left[\bar{P}^T_{zj}\right]^{\frac{1-\kappa}{X-1}}(\bar{P}^T_z)^{\kappa} \tag{B.2}$$

[32]See e.g. Obstfeld and Rogoff (1996).

and the consumption demand for own traded, foreign traded and nontraded goods ($\bar{C}_z^T$, $\bar{C}_{zj}^T$ and $\bar{C}_z^N$ respectively) in the steady state is given by:

$$\bar{C}_z^T = \frac{\kappa}{2}\left(\frac{\bar{P}_{z\Omega}^T}{\bar{P}_z^T}\right)\left(\frac{\bar{P}_z}{\bar{P}_{z\Omega}^T}\right)\bar{C}_z \quad \bar{C}_{zj}^T = \frac{1-\kappa}{2(X-1)}\left(\frac{\bar{P}_{z\Omega}^T}{\bar{P}_{zj}^T}\right)\left(\frac{\bar{P}_z}{\bar{P}_{z\Omega}^T}\right)\bar{C}_z \quad \bar{C}_z^N = \frac{1}{2}\left(\frac{\bar{P}_z}{\bar{P}_z^N}\right)\bar{C}_z \tag{B.3}$$

with government consumption demands for own traded, foreign traded and nontraded goods ($\bar{G}_z^T$, $\bar{G}_{zj}^T$ and $\bar{G}_z^N$ respectively) having the same structure.

It is possible to characterise the steady state by finding equilibrium terms of trade and nominal exchange rates, which combine to obtain equilibrium real exchange rates. Consider first, the market clearing condition for the nontraded good of region z:

$$\bar{P}_z^N\bar{Y}_z^N = \bar{P}_z^N(\bar{C}_z^N + \bar{G}_z^N)$$

which after substituting (B.1) and using (B.3) yields the relative price of nontraded endowment with respect to traded endowment:

$$\frac{\bar{P}_z^N}{\bar{P}_z^T} = \frac{\bar{Y}_z^T}{\bar{Y}_z^N} \tag{B.4}$$

The market clearing condition for the traded good of region z in the currency of region z is:

$$L_z\bar{P}_z^T\bar{Y}_z^T = \sum_{j\neq z}^{X} L_j\left(\bar{P}_{jz}^T(\bar{C}_{jz}^T + \bar{G}_{jz}^T)\right)\times\frac{\bar{S}_z}{\bar{S}_j} + L_z\bar{P}_z^T(\bar{C}_z^T + \bar{G}_z^T)$$

Using the law of one price (3.15), demand equations (B.3), steady state budget constraint (B.1) and, finally, equilibrium in nontraded goods market (B.4) yields after rearranging:

$$L_z\bar{P}_z^T\bar{Y}_z^T = \sum_{j\neq z}^{X}\left[\left(\frac{1}{X-1}\right)\left(\frac{\bar{P}_j^T\bar{Y}_j^T L_j\bar{S}_z}{\bar{S}_j}\right)\right]$$

Define ϑ_{jz} and τ_{jz} such that $\vartheta_{jz} = \frac{L_j\bar{Y}_j^T}{L_z\bar{Y}_z^T}$ and $\tau_{jz} = \frac{\bar{P}_j^T\bar{S}_z}{\bar{S}_j\bar{P}_z^T}$. ϑ_{jz} can be interpreted as the relative abundance of traded goods in region j to traded goods in region z and τ_{jz} is the ratio of export prices of region j to export prices of region z, that is the terms of trade of region j with region z. The market clearing condition for the endowment of region z can be rewritten by dividing by $\bar{P}_z^T\bar{Y}_z^T$:

$$\sum_{j\neq z}^{X}\left[\left(\frac{1}{X-1}\right)\vartheta_{jz}\tau_{jz}\right] = 1 \tag{B.5}$$

(B.5) intuitively shows that a rise in the abundance of the traded good from region j *ceteris paribus* must necessarily be accompanied by a worsening of the terms of trade of region j with region z.

Applying (B.5) to write the market clearing condition for the traded good of each country yields a system of X equations and $\frac{X(X-1)}{2}$ terms of trade.[33] By inspection, for any X, $\vartheta_{jz}\tau_{jz} = 1$ for any j and z is always a unique symmetric solution. Thus the steady state terms of trade:

$$\frac{\bar{P}_j^T\bar{S}_z}{\bar{S}_j\bar{P}_z^T} = \frac{L_z\bar{Y}_z^T}{L_j\bar{Y}_j^T}. \tag{B.6}$$

[33]Note that to construct $\frac{X(X-1)}{2}$ terms of trade, it is sufficient to define terms of trade with respect to the numeraire region, that is only $X-1$ relative prices are required, as usual.

Using (B.2) and (B.4), one can express the real exchange rate as a function of $\frac{\bar{P}^T_{j\Omega}\bar{S}_z}{\bar{S}_j\bar{P}^T_{z\Omega}}$ and relative nontraded endowments. Using (B.2) again as well as the steady state version of (3.15), substituting (B.6) and rearranging yields the real exchange rate given in (3.29):

$$\frac{\bar{P}_z\bar{S}_j}{\bar{P}_j\bar{S}_z} = \left(\frac{L_z\bar{Y}^T_z}{L_j\bar{Y}^T_j}\right)^{\frac{1-\kappa X}{2(X-1)}} \left(\frac{L_j\bar{Y}^N_j}{L_z\bar{Y}^N_z}\right)^{\frac{1}{2}}. \tag{B.7}$$

Substituting (B.4) into (B.1) and using (B.6) together with (B.7), it follows that the ratio of aggregate consumption in region z to consumption in region j is equal to the real exchange rate of region j with region z:

$$\frac{(\bar{C}_z+\bar{G}_z)L_z}{(\bar{C}_j+\bar{G}_j)L_j} = \frac{\bar{P}_j\bar{S}_z}{\bar{P}_z\bar{S}_j} = \left(\frac{L_j\bar{Y}^T_j}{L_z\bar{Y}^T_z}\right)^{\frac{1-\kappa X}{2(X-1)}} \left(\frac{L_z\bar{Y}^N_z}{L_j\bar{Y}^N_j}\right)^{\frac{1}{2}}. \tag{B.8}$$

Intuitively, (B.8) means that aggregate nominal GDP is equal in all countries, since prices of all endowments adjust to reflect their relative scarcity. One can substitute (B.1) into the definition $\bar{g}_z$ from (3.10):

$$\bar{g}_z = \frac{\bar{P}_z\bar{G}_z}{\bar{P}^T_z\bar{Y}^T_z + \bar{P}^N_z\bar{Y}^N_z} \rightarrow \bar{g}_z = \frac{\bar{P}_z\bar{G}_z}{\bar{P}_z(\bar{C}_z+\bar{G}_z)} \rightarrow \bar{G}_z = \frac{\bar{g}_z}{1-\bar{g}_z}\bar{C}_z$$

Hence the ratio of aggregate private sector consumption can be obtained from (B.8):

$$\frac{L_z\bar{C}_z}{L_j\bar{C}_j} = \left(\frac{1-\bar{g}_z}{1-\bar{g}_j}\right)\left(\frac{L_j\bar{Y}^T_j}{L_z\bar{Y}^T_z}\right)^{\frac{1-\kappa X}{2(X-1)}} \left(\frac{L_z\bar{Y}^N_z}{L_j\bar{Y}^N_j}\right)^{\frac{1}{2}} \tag{B.9}$$

Combining (3.27) for regions z and j one can write:

$$\frac{\bar{P}_z}{\bar{P}_j} = \left(\frac{\bar{M}_z}{\bar{M}_j}\right)\left(\frac{\bar{C}_j}{\bar{C}_z}\right)^{\frac{\theta}{\nu}}$$

Applying (B.6), (B.7), and (B.9) the combination of the money market equilibria can be written as:

$$\frac{\bar{P}_z}{\bar{P}_j} = \left(\frac{(1-\bar{g}_j)L_z}{(1-\bar{g}_z)L_j}\right)^{\frac{\theta}{\nu}}\left(\frac{\bar{M}_z}{\bar{M}_j}\right)\left(\frac{L_z\bar{Y}^T_z}{L_j\bar{Y}^T_j}\right)^{\frac{(1-\kappa X)\theta}{2\nu(X-1)}} \left(\frac{L_j\bar{Y}^N_j}{L_z\bar{Y}^N_z}\right)^{\frac{\theta}{2\nu}}$$

which can be plugged in (B.7) to obtain the nominal exchange rate of region j to region z in the steady state given in (3.30):

$$\frac{\bar{S}_z}{\bar{S}_j} = \left(\frac{(1-\bar{g}_j)L_z}{(1-\bar{g}_z)L_j}\right)^{\frac{\theta}{\nu}}\left(\frac{\bar{M}_z}{\bar{M}_j}\right)\left(\frac{L_z\bar{Y}^T_z}{L_j\bar{Y}^T_j}\right)^{\frac{(\theta-\nu)(1-\kappa X)}{2\nu(X-1)}} \left(\frac{L_j\bar{Y}^N_j}{L_z\bar{Y}^N_z}\right)^{\frac{\theta-\nu}{2\nu}} \tag{B.10}$$

C First-Order Approximate Solution

Subtracting (3.38) for region 1 from (3.38) for region z yields the equation for net foreign asset differential (C.1):

$$
\begin{aligned}
\hat{w}_{z,t} - \hat{w}_{1,t} = &\left(\frac{\bar{w}_z}{\beta}\right)\left(\hat{i}_{1,t} - \beta\varsigma\hat{w}_{1,t-1}\right) + \frac{1}{\beta}\hat{w}_{z,t-1} + \bar{y}_z^T(\hat{p}_{z,t}^T + \hat{y}_{z,t}^T - \hat{s}_{z,t}) \\
&+ \bar{y}_z^N(\hat{p}_{z,t}^N + \hat{y}_{z,t}^N - \hat{s}_{z,t}) - \bar{c}_z(\hat{c}_{z,t} + \hat{p}_{z,t} - \hat{s}_{z,t}) - \bar{g}_z(\hat{g}_{z,t} + \hat{p}_{z,t} - \hat{s}_{z,t}) \\
&- \bar{y}_1^T(\hat{p}_{1,t}^T + \hat{y}_{1,t}^T) - \bar{y}_1^N(\hat{p}_{1,t}^N + \hat{y}_{1,t}^N) + \bar{c}_1(\hat{p}_{1,t} + \hat{c}_{1,t}) + \bar{g}_1(\hat{g}_{1,t} + \hat{p}_{1,t}) \\
&- \frac{1}{\beta}\hat{w}_{1,t-1} - \left(\frac{\bar{w}_1}{\beta}\right)\left(\hat{i}_{1,t} - \beta\varsigma\hat{w}_{1,t-1}\right) + \xi_{z,t} - \xi_{1,t} + O(\epsilon^2)
\end{aligned}
\tag{C.1}
$$

To derive the risk sharing condition, note that linearised (3.18) is:

$$
E_t\Big[(\hat{p}_{j,t+1} - \hat{s}_{j,t+1} - \hat{p}_{1,t+1}) - (\hat{q}_{jB,t}^* + \hat{p}_{j,t} - \hat{s}_{j,t} - \hat{p}_{1,t})\Big] = E_t[-\hat{q}_{1B,t}^*]
$$

and the linearised first-order condition for asset returns (3.31) is:

$$
\begin{aligned}
&-\theta\hat{c}_{z,t} + \hat{s}_{z,t} - \hat{s}_{j,t} - \hat{p}_{z,t} \\
&\qquad = E_t[\hat{i}_{j,t+1} - \beta\varsigma\hat{w}_{j,t} - \theta\hat{c}_{z,t+1} + \hat{s}_{z,t+1} - \hat{s}_{j,t+1} - \hat{p}_{z,t+1}]
\end{aligned}
\tag{C.2}
$$

where $\hat{i}_{j,t} - \beta\varsigma\hat{w}_{j,t}$ can be replaced by equity denominated in the currency of region j. Given that expected returns on riskless nominal bonds should be equal to the expected returns on riskless real bonds, one can obtain uncovered interest parity, written in terms of nominal riskless rates:

$$
E_t[\hat{i}_{j,t+1}] = E_t[\hat{i}_{1,t+1}] + E_t[\hat{s}_{j,t+1} - \hat{s}_{j,t}] \tag{C.3}
$$

Note that $\hat{s}_{z,t}$ is the price of a unit of currency 1 in terms of currency z, that is a positive $\hat{s}_{z,t}$ represents depreciation of currency of region z or an appreciation of currency 1. Applying (C.3) and subtracting (C.2) written for region z from (C.2) written for region 1 (and their corresponding bonds), yields the risk sharing condition (3.43).

To ensure the stationarity of the model it is necessary to ensure that there are no bubbles in prices or exchange rates, that is:

$$
\lim_{T\to\infty} E_t[\hat{s}_{t+T}] = 0 \qquad \lim_{T\to\infty} E_t\left[\sum_{j=1}^{X}\hat{p}_{j,t+T}\right] = 0. \tag{C.4}
$$

To justify (C.4), first note that if bubbles in individual price levels do not arise, then no bubble can arise in nominal exchange rate, since the real exchange rate is driven by real variables. Obstfeld and Rogoff (1986) show that implosive price bubbles can be ruled out in models of the type considered here, by relying on the transversality condition $\lim_{t\to\infty}\beta^t\frac{u'(\bar{C})}{P_t} = 0$ and an additional assumption that the utility function for real money balances is bounded from above, which it is in (3.1), under the assumption that $\nu > 1$. This rules out a case when $\lim_{T\to\infty}\hat{p}_{z,t+T} = -\infty$.

It is harder to rule out hyperinflationary bubbles, when $\lim_{T\to\infty}\hat{p}_{z,t+T} = \infty$. A suitable argument for our purposes is given by Obstfeld and Rogoff (1983), who show, under mild assumptions, that if the government provides a minimum level of fractional backing for the currency, that is it ensures that a unit of currency will always be able to buy some consumption good, then hyperinflations can be ruled out.

A simplified version[34] of the canonical form of the linear rational expectations model used in Christiano (2002) is given by:

$$E_t\left[\sum_{i=0}^{r}\mathbf{\Theta}_i\mathbf{x}_{t+r-1-i}+\sum_{i=0}^{r-1}\mathbf{\Xi}_i\mathbf{v}_{t+r-1-i}\right]=0 \tag{C.5}$$

In (C.5) $r=2$, $\mathbf{x}_t$ is a $(5X-1)\times 1$ vector of endogenous variables:

$$\mathbf{x}_t' = \begin{bmatrix}\hat{c}_{1,t} & \hat{p}^T_{1,t} & \hat{p}^N_{1,t} & \hat{i}_{1,t} & \hat{c}_{2,t} & \hat{p}^T_{2,t} & \hat{p}^N_{2,t} & \hat{s}_{2,t} & \hat{w}_{2,t} & \ldots & \hat{c}_{X,t} & \hat{p}^T_{X,t} & \hat{p}^N_{X,t} & \hat{s}_{X,t} & \hat{w}_{X,t}\end{bmatrix}$$

In addition, $\mathbf{v}_t$ in (C.5) is a $5X-1\times 1$ vector containing exogenous variables, which are relevant for determining endogenous variables:

$$\mathbf{v}_t' = \begin{bmatrix}\hat{y}^T_{1,t} & \hat{y}^N_{1,t} & \hat{g}_{1,t} & \hat{m}_{1,t} & \ldots & \hat{y}^T_{X,t} & \hat{y}^N_{X,t} & \hat{g}_{X,t} & \hat{m}_{X,t} & \xi_{X,t}\end{bmatrix}$$

In (C.5), $\mathbf{\Theta}_i$ are $5X-1\times 5X-1$ matrices, whose rows consist of coefficients from (3.41), (3.42), (3.40), (3.43) and (C.1). The matrices $\mathbf{\Xi}_0$ and $\mathbf{\Xi}_1$ have size $5X-1\times 5X-1$. The solution for the behaviour of endogenous variables is of the form:

$$\mathbf{x}_t=\mathbf{C}_1\mathbf{x}_{t-1}+\mathbf{C}_2\mathbf{v}_t \tag{C.6}$$

Following Christiano (2002), one can first set the exogenous variables in v_t to zero and write (C.5) as:

$$\mathbf{A}\mathbf{z}_{t+1}+\mathbf{B}\mathbf{z}_t=0$$

where $\mathbf{z}_t$ is a $2(5X-1)\times 1$ vector given by:

$$\mathbf{z}_t=\begin{bmatrix}\mathbf{x}_t\\ \mathbf{x}_{t-1}\end{bmatrix}$$

and matrices $\mathbf{A}$ and $\mathbf{B}$ are $2(5X-1)\times 2(5X-1)$ matrices given by:

$$\mathbf{A}=\begin{bmatrix}\mathbf{\Theta_0} & \mathbf{0}_{5X-1\times 5X-1}\\ \mathbf{0}_{5X-1\times 5X-1} & \mathbf{I}_{5X-1}\end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix}\mathbf{\Theta}_1 & \mathbf{\Theta}_2\\ -\mathbf{I}_{5X-1} & \mathbf{0}_{5X-1\times 5X-1}\end{bmatrix}$$

The $\mathbf{\Theta_0}$ matrix is singular and hence $\mathbf{A}$ is a singular matrix as well. Therefore, following Christiano (2002), it is necessary to apply the QZ decomposition to find orthonormal matrices $\mathbf{Q}$ and $\mathbf{Z}$ and upper triangular matrices $\mathbf{T_0}$ and $\mathbf{T_1}$, such that:

$$\mathbf{QAZ}=\mathbf{T}_0 \qquad \mathbf{QBZ}=\mathbf{T}_1$$

and the l zeros on the diagonal of $\mathbf{T_0}$, which arise due to the singularity of $\mathbf{\Theta_0}$ matrix, are located in the lower right part of $\mathbf{T_0}$[35]. It is assumed that the upper $2(5X-1)-l\times 2(5X-1)-l$ block of $\mathbf{T_0}$, which, as in Christiano (2002), is denoted by $\mathbf{G}_0$ is non-singular. Let $\mathbf{G_1}$ denote the corresponding upper $2(5X-1)-l\times 2(5X-1)-l$ block of $\mathbf{T_1}$, and assume that the lower right $l\times l$ block of $\mathbf{T}_1$ is nonsingular. One can also partition $\mathbf{Z}'$:

$$\mathbf{Z}'=\begin{bmatrix}\mathbf{L}_1\\ \mathbf{L}_2\end{bmatrix}$$

[34] The simplification is in assuming that all countries use the same information set.

[35] The Matlab routines `qzswitch.m` and `qzdiv.m` developed by Sims (2002) are used to find the right decomposition.

such that $\mathbf{L}_1$ is a $2(5X-1)-l \times 2(5X-1)$ matrix and $\mathbf{L}_2$ is a $l \times 2(5X-1)$ matrix. Now, one can use the eigenvector, eigenvalue decomposition to find matrices Λ and $\mathbf{P}$ such that:

$$\mathbf{P}\Lambda\mathbf{P^{-1}} = -\mathbf{G}_0^{-1}\mathbf{G}_1$$

and find the matrix $\tilde{\mathbf{P}}$, which is composed of the rows of $\mathbf{P}^{-1}$ corresponding to diagonal terms in Λ that exceed unity in absolute value. The $\tilde{\mathbf{P}}$ matrix has to have $5X-2-l$ rows for the unique solution to exist. Then, one can define $\mathbf{D}$:

$$\mathbf{D} = \begin{bmatrix} \tilde{\mathbf{P}}\mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix}$$

One can then partition $\mathbf{D}$ into two matrices $\mathbf{D}_1$ and $\mathbf{D}_2$, such that $\mathbf{D}_1$ consists of the first $5X-1$ columns of $\mathbf{D}$ and $\mathbf{D}_2$ consists of remaining columns. The matrix $\mathbf{C}_1$ in (C.6) is composed of the bottom $5X-1$ rows of $-(\mathbf{D}_1)^{-1}\mathbf{D}_2$.

To compute the impact of exogenous variables on consumption and net foreign assets, one can define the law of motion for exogenous variables (and preference constants) such that:

$$\mathbf{v}_t = \mathbf{\Phi}\mathbf{v}_{t-1} + \hat{\boldsymbol{\varepsilon}}_t, \tag{C.7}$$

where $\boldsymbol{\varepsilon}_t$ is a $5X-1 \times 1$ vector of shocks defined as:

$$\boldsymbol{\varepsilon}'_t = \begin{bmatrix} \hat{\varepsilon}^T_{1,t} & \hat{\varepsilon}^N_{1,t} & \hat{\varepsilon}^G_{1,t} & \hat{\varepsilon}^M_{1,t} & \dots & \hat{\varepsilon}^T_{X,t} & \hat{\varepsilon}^N_{X,t} & \hat{\varepsilon}^G_{X,t} & \hat{\varepsilon}^M_{X,t} & \xi_{X,t} \end{bmatrix}$$

The equations in (C.7) are obtained by linearising the laws of motion for exogenous variables, e.g. the linearisation of (3.6) is:

$$\bar{Y}^T_z + \bar{Y}^T_z \hat{y}^T_{z,t} = \bar{Y}^T_z + \hat{a}^T_{z,t}\bar{Y}^T_z + O(\epsilon^2)$$

Thus $\hat{y}^T_z = \hat{a}^T_{z,t}$ and substituting for (3.7) one obtains the linearisations of traded and nontraded endowments:

$$\hat{y}^T_{z,t} = \rho^T_z \hat{y}^T_{z,t-1} + \hat{\varepsilon}^T_{z,t} + O(\epsilon^2) \qquad \hat{y}^N_{z,t} = \rho^N_z \hat{y}^N_{z,t-1} + \hat{\varepsilon}^N_{z,t} + O(\epsilon^2)$$

The laws of motion for government spending (3.8) and money supply (3.12) are linearised in a similar manner:

$$\hat{g}_{z,t} = \rho^G_z \hat{g}_{z,t-1} + \hat{\varepsilon}^G_{z,t} + O(\epsilon^2) \qquad \hat{m}_{z,t} = \rho^M_z \hat{m}_{z,t-1} + \hat{\varepsilon}^M_{z,t} + O(\epsilon^2)$$

In (C.7) is a $5X-1 \times 5X-1$ matrix with nondiagonal elements equal to zero and the diagonal containing autoregressive parameters and zeros:

$$diag(\mathbf{\Phi}) = \begin{bmatrix} \rho^T_1 & \rho^N_1 & \rho^G_1 & \rho^M_1 & \rho^T_2 & \rho^N_2 & \rho^G_2 & \rho^M_2 & 0 & \dots & \rho^T_X & \rho^N_X & \rho^G_X & \rho^M_X & 0 \end{bmatrix}$$

One can also define matrices $\mathbf{K}_i$, $\mathbf{n}$ and $\mathbf{m}$ such that:

$$\mathbf{K}_i = \mathbf{\Theta}_0\mathbf{C}^i_1 + \mathbf{\Theta}_1\mathbf{C}^{i-1}_1 + \ldots + \mathbf{\Theta}_i \qquad \mathbf{n} = \sum_{j=0}^{r-1}\Big[\mathbf{K}_j \otimes (\mathbf{\Phi}^{r-1-j})'\Big] \qquad \mathbf{m} = vec\Big[\sum_{j=0}^{r-1}\Big((\mathbf{\Phi}')^{r-1-j}\Xi'_j\Big)\Big]$$

Following Christiano (2002) the matrix $\mathbf{C}_2$ is then given by:

$$vec(\mathbf{C}'_2) = -\mathbf{n}^{-1}\mathbf{m}$$

Having obtained (C.6), one can derive the excess returns on various assets. First note that from (3.19) for country j, it follows that $\hat{i}_{j,t} - \beta\varsigma\hat{w}_{j,t-1} = -\hat{q}_{jB,t-1}$. From (3.36) expected excess returns, adjusted for currency appreciation are zero, the realized excess return on bonds, adjusted for appreciation is:

$$\begin{aligned} \hat{i}_{j,t} - \beta\varsigma\hat{w}_{j,t-1} - (\hat{i}_{1,t} - \beta\varsigma\hat{w}_{1,t-1}) - (\hat{s}_{j,t} - \hat{s}_{j,t-1}) \\ = -E_{t-1}[\hat{s}_{j,t} - \hat{s}_{j,t-1}] + O(\epsilon^2) \end{aligned} \tag{C.8}$$

that is the realised excess return on bonds from country j depends only on the unexpected appreciation of the currency of region j.

The linearised return on equity from region j, can be obtained by linearising (3.21):

$$\hat{d}_{j,t} = \bar{y}_z^T(\hat{y}_{j,t}^T + \hat{p}_{j,t}^T) + \bar{y}_z^N(\hat{y}_j^N + \hat{p}_{j,t}^N) - \hat{q}_{jK,t-1}$$

Given that expected excess return on equity is zero, when adjusted for currency appreciation, one can write the realised excess return on equity adjusted for appreciation as:

$$\begin{aligned} \hat{d}_{j,t} - (\hat{i}_{1,t} - \beta\varsigma\hat{w}_{1,t-1}) - (\hat{s}_{j,t} - \hat{s}_{j,t-1}) = \bar{y}_z^T\Big(\hat{p}_{j,t}^T + \hat{y}_{j,t}^T - E_{t-1}(\hat{p}_{j,t}^T + \hat{y}_{j,t}^T)\Big) + \\ + \bar{y}_z^N\Big(\hat{p}_{j,t}^N + \hat{y}_{j,t}^N - E_{t-1}(\hat{p}_{j,t}^N + \hat{y}_{j,t}^N)\Big) - (\hat{s}_{j,t} - E_{t-1}\hat{s}_{j,t-1}) \end{aligned} \tag{C.9}$$

that is the realised excess return on equity is given by unexpected revenue in the traded and nontraded sectors minus the unexpected currency depreciation.