Understanding Asset Returns

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What’s the story?
What’s the story?

When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth.

— ”The Adventure of the Blanched Soldier”
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Stylized facts of asset returns
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- Log returns are not Gaussian;
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- Volatility clustering;

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- Log returns are not Gaussian;
- Log returns are not autocorrelated;
- Absolute log returns are serially correlated;
- Aggregational gaussianity;
- Volatility clustering;
- Gain/loss asymmetry;

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Previous models

- Fitting the unconditional distribution of log returns;
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  - being able to fit 'the fat tail';
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- GARCH...
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Two-regime Markovian Model
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An unobserved ergodic Markov chain \((\xi_n)_{n \in \mathbb{Z}}\). Independently of \(\xi\), we have two sequences \((X^i_n)_{n \in \mathbb{Z}}\) of independent random variables, \(i = 1, 2\), with \(X^i_n \sim F_i\) for all \(n\) and \(i\), in terms of which the return \(r_n\) on day \(n\) is \(\sum_{i=1}^{2} I\{\xi_n = i\} X^i_n\). Let \(\mu_i\) denote the mean of regime \(i\).
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Proposition

Suppose that \(\mu_1 = \mu_2 = \mu\). Then \(\mathbb{E}[r_n r_{n+k}] = \mu^2\) for any \(k > 0\) and \(n \in \mathbb{Z}\).
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Proposition

**Suppose that** \(\mu_1 = \mu_2 = \mu\). **Then** \(\mathbb{E}[r_nr_{n+k}] = \mu^2\) **for any** \(k > 0\) **and** \(n \in \mathbb{Z}\).

**Proof.** Fix \(k > 0\) and let \(\mathcal{X} \equiv \sigma(\xi_m, m \in \mathbb{Z})\). We see that

\[
\mathbb{E}[r_nr_{n+k}] = \mathbb{E}[\mathbb{E}[r_nr_{n+k} | \mathcal{X}]]
\]

\[
= \sum_{i=1}^{2} \sum_{j=1}^{2} \mathbb{E}[\mathbb{E}[X_n^iX_{n+k}^j | \mathcal{X}] ; \xi_n = i, \xi_{n+k} = j]
\]

\[
= \sum_{i=1}^{2} \sum_{j=1}^{2} \mathbb{E}[\mu_i\mu_j ; \xi_n = i, \xi_{n+k} = j]
\]

\[
= \mu^2,
\]

using the fact that the \(X\)'s are independent of \(\mathcal{X}\) and of each other, and then using the hypothesis that \(\mu_1 = \mu_2\).
Autocovariance of absolute returns

The autocorrelation of absolute returns has been found to decay quite slowly with lag (Granger et al. 2000). If we set $\pi$ as the invariant law of $\xi$ and

$$
\nu_i = \int |x - \mu| F_i(dx)
$$

for the (centered) absolute first moment in regime $i$, we find that

$$
\begin{align*}
\mathbb{E}|r_n - \mu| &= \pi_1 \nu_1 + \pi_2 \nu_2 \\
\mathbb{E}(|r_n - \mu)(r_{n+k} - \mu)| &= (\pi_1 \nu_1 \pi_2 \nu_2) P^k \binom{\nu_1}{\nu_2} .
\end{align*}
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\begin{cases}
E|r_n - \mu| = \pi_1 \nu_1 + \pi_2 \nu_2 \\
E|(r_n - \mu)(r_{n+k} - \mu)| = \left( \begin{array}{cc}
\pi_1 & \pi_2
\end{array} \right) P^k \left( \begin{array}{c}
\nu_1 \\
\nu_2
\end{array} \right)
\end{cases}
$$

It now follows that the covariance of the centred absolute returns is given by (for $k > 0$)

$$
\text{cov}(|r_n - \mu|, |r_{n+k} - \mu|) = \left( \begin{array}{cc}
\pi_1 & \pi_2
\end{array} \right) \left( P^k - \left( \begin{array}{c}
1 \\
1
\end{array} \right) \left( \begin{array}{cc}
\pi_1 & \pi_2
\end{array} \right) \right) \left( \begin{array}{c}
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$$

$$
= \left( \begin{array}{cc}
\pi_1 & \pi_2
\end{array} \right) v \lambda^k u^T \left( \begin{array}{c}
\nu_1 \\
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\end{array} \right),
$$

where $\lambda$ is the eigenvalue of $P$ different from 1, and $v$ (respectively, $u$) is the right (respectively, left) eigenvector of $\lambda$. 

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Distributions of Regimes
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We consider conditional distributions $F_i$ of returns given the regime of the Markov chain $\xi$ which are members of the generalized hyperbolic class of distributions, or of some subclass.
Distributions of Regimes

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$$x \mapsto \frac{(\gamma/\delta)^\lambda}{\sqrt{2\pi} K_\lambda(\delta \gamma)} \frac{K_{\lambda-1/2} \left( \alpha \sqrt{\delta^2 + (x - \mu)^2} \right)}{\left( \sqrt{\delta^2 + (x - \mu)^2}/\alpha \right)^{1/2-\lambda}} e^{\beta(x-\mu)}$$

where $\gamma \equiv \sqrt{\alpha^2 - \beta^2}$, and the moment-generating function (MGF) is

$$z \mapsto \frac{e^{\mu z} \gamma^\lambda}{(\alpha^2 - (\beta + z)^2)^{\lambda/2}} \frac{K_\lambda(\delta \sqrt{\alpha^2 - (\beta + z)^2})}{K_\lambda(\delta \gamma)}.$$
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Various subfamilies of the GH class are of interest in their own right:
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- taking $\delta = 0$ and $\beta = 0$ gives the symmetric variance-gamma class;
- taking $\alpha = \beta = 0$ and $\lambda = -\nu/2$ gives a Student-$t_\nu$ distribution.
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Data Set

Figure: 1990-2009 daily Stock indices (S&P500, FTSE, DAX, NIKKEI, CAC40) adjusted by US currency

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Maximum likelihood estimation for HMM

The log-likelihood function of an observed sequence $r_1, r_2, \ldots, r_m$ of returns is

$$
\mathcal{L}(\theta_1, \theta_2; r_1, \ldots, r_m) = \log (\pi F(r_1; \theta_1, \theta_2) P F(r_2; \theta_1, \theta_2) P \cdots P F(r_m; \theta_1, \theta_2) 1)
$$

where

$$
\pi = (\pi_1 \pi_2), \quad F(r; \theta_1, \theta_2) = \left( \begin{array}{c} f(r; \theta_1) \\ f(r; \theta_2) \end{array} \right), \quad 1 = \left( \begin{array}{c} 1 \\ 1 \end{array} \right).
$$

Then we can calculate maximum-likelihood estimators for the parameters by assuming that the returns are symmetric hyperbolic.
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**Figure:** autocovariances of absolute return with 50 lags (1990-2009 daily S&P500)
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Then we can calculate maximum-likelihood estimators for the parameters by assuming that the returns are symmetric hyperbolic.

We therefore introduce a penalty function to improve the fit:

\[
\mathcal{P}(\theta_1, \theta_2) = A \sum_{k=0}^{w} (\hat{\rho}_k - \rho_k)^2
\]

where \( w \) is the total lag number for summation, \( A \) is the scalar of the penalty function, and \( \hat{\rho}_k \) and \( \rho_k \) are theoretical and empirical autocovariances of absolute returns with \( k \) lags. Explicitly, we maximize

\[
\mathcal{L}(\theta_1, \theta_2; r_1, \ldots, r_m) - \mathcal{P}(\theta_1, \theta_2).
\]
Maximum likelihood estimation for HMM

Figure: autocovariances of absolute return with penalty function (1990-2009 daily S&P500)
Figure: autocovariances of absolute return with common Markov chain (1990-2009 daily S&P500)

Maximum likelihood estimation for HMM
Kolmogorov-Smirnov (KS) test of unconditional log return distribution
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- Individual Markov chain for each index, all of four distributions can pass the test;
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- For common Markov chain, the highest significance level
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  symmetric variance-gamma 91.46%
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  - symmetric variance-gamma: 91.46%
  - symmetric hyperbolic: 89.07%
Kolmogorov-Smirnov (KS) test of unconditional log return distribution

- Individual Markov chain for each index, all of four distributions can pass the test;

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  - symmetric variance-gamma: 91.46%
  - symmetric hyperbolic: 89.07%
  - hyperbolic: 78.28%
  - symmetric generalized hyperbolic: 90.70%
- Regime distribution: symmetric hyperbolic / hyperbolic.

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Figure: 2008-2009 daily posterior probability of being in 'good mood' (10-day moving average)
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- Applications: Optimal investment, Option pricing;
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- Stochastic Volatility Model? Maybe...
Heteroskedasticity

Realized variance of 29 SP500 stocks, 200 day moving average

Realized quadratic variation of 29 stocks from S&P500
(taking 200-day moving averages, 2000.07 - 2010.07)

Rogers and Zhang (2010) Understanding Asset Returns