Understanding Asset Returns

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September 2010

What's the story?

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When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth.

– "The Adventure of the Blanched Soldier"

Outline

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Stylized facts of asset returns

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- Gain/loss asymmetry;

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An unobserved ergodic Markov chain $(\xi_n)_{n\in\mathbb{Z}}$. Independently of ξ, we have two sequences $(X_n^i)_{n\in\mathbb{Z}}$ of independent random variables, $i = 1, 2$, with $X_n^i \sim F_i$ for all n and i, in terms of which the return r_n on day n is $\sum_{i=1}^2 \mathbf{I}_{\{\xi_n=i\}} X_n^i$. Let μ_i denote the mean of regime i.

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Suppose that $\mu_1 = \mu_2 = \mu$. Then $\mathbb{E}[r_n r_{n+k}] = \mu^2$ for any $k > 0$ and $n \in \mathbb{Z}$.

Proof. Fix $k > 0$ and let $\mathcal{X} \equiv \sigma(\xi_m, m \in \mathbb{Z})$. We see that

$$\begin{aligned}
\mathbb{E}[r_n r_{n+k}] &= \mathbb{E}[\mathbb{E}[r_n r_{n+k} \,|\, \mathcal{X}]] \\
&= \sum_{i=1}^{2}\sum_{j=1}^{2} \mathbb{E}[\mathbb{E}[X_n^i X_{n+k}^j \,|\, \mathcal{X}]; \xi_n = i, \xi_{n+k} = j] \\
&= \sum_{i=1}^{2}\sum_{j=1}^{2} \mathbb{E}[\mu_i \mu_j; \xi_n = i, \xi_{n+k} = j] \\
&= \mu^2,
\end{aligned}$$

using the fact that the X's are independent of $\mathcal{X}$ and of each other, and then using the hypothesis that $\mu_1 = \mu_2$.

Autocovariance of absolute returns

The autocorrelation of absolute returns has been found to decay quite slowly with lag (Granger *et al.* 2000). If we set π as the invariant law of ξ and

$$\nu_i = \int |x - \mu| \; F_i(dx)$$

for the (centered) absolute first moment in regime i, we find that

$$\begin{cases} \mathbb{E}|r_n - \mu| & = & \pi_1\nu_1 + \pi_2\nu_2 \\ \mathbb{E}|(r_n - \mu)(r_{n+k} - \mu)| & = & \begin{pmatrix} \pi_1\nu_1 & \pi_2\nu_2 \end{pmatrix} P^k \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \end{cases} .$$

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It now follows that the covariance of the centred absolute returns is given by (for $k>0$)

$$\begin{aligned} \operatorname{cov}(|r_n-\mu|, |r_{n+k}-\mu|) &= \begin{pmatrix}\pi_1\nu_1 & \pi_2\nu_2\end{pmatrix} \left(P^k - \begin{pmatrix}1 \\ 1\end{pmatrix} \begin{pmatrix}\pi_1 & \pi_2\end{pmatrix} \right) \begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix} \\ &= \begin{pmatrix}\pi_1\nu_1 & \pi_2\nu_2\end{pmatrix} v\lambda^k u^T \begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix}, \end{aligned}$$

where λ is the eigenvalue of P different from 1, and v (respectively, u) is the right (respectively, left) eigenvector of λ.

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$$x \mapsto \frac{(\gamma/\delta)^\lambda}{\sqrt{2\pi} K_\lambda(\delta\gamma)} \frac{K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\left(\sqrt{\delta^2 + (x-\mu)^2}/\alpha\right)^{1/2-\lambda}} \; e^{\beta(x-\mu)}$$

where $\gamma \equiv \sqrt{\alpha^2 - \beta^2}$, and the moment-generating function (MGF) is

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Data Set

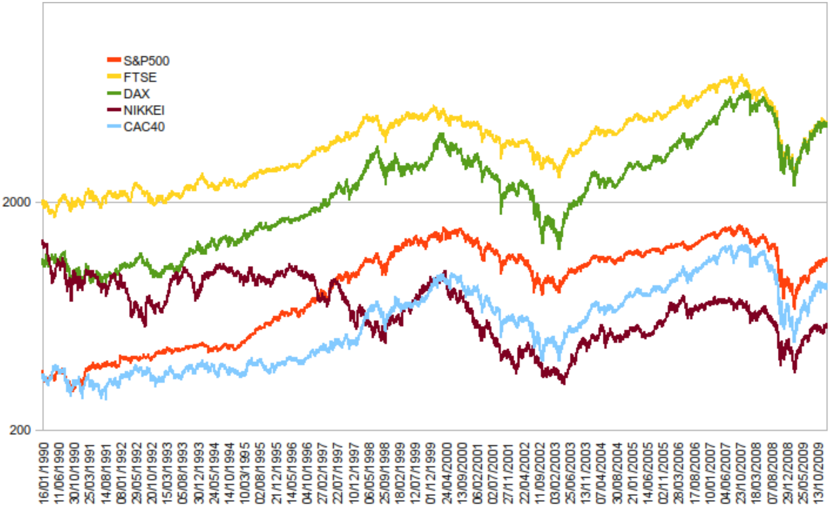

Figure: 1990-2009 daily Stock indices (S&P500, FTSE, DAX, NIKKEI, CAC40) adjusted by US currency

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Maximum likelihood estimation for HMM

The log-likelihood function of an observed sequence $r_1, r_2, \ldots, r_m$ of returns is

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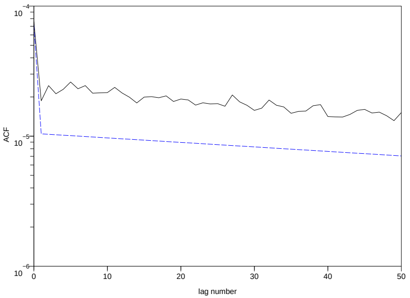

Figure: autocovariances of absolute return with 50 lags (1990-2009 daily S&P500)

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We therefore introduce a penalty function to improve the fit:

$$\mathcal{P}(\theta_1, \theta_2) = A \sum_{k=0}^{w} (\hat{\rho}_k - \rho_k)^2$$

where w is the total lag number for summation, A is the scalar of the penalty function, and $\hat{\rho}_k$ and ρ_k are theoretical and empirical autocovariances of absolute returns with k lags. Explicitly, we maximize

$$\mathcal{L}(\theta_1, \theta_2; r_1, \ldots, r_m) - \mathcal{P}(\theta_1, \theta_2).$$

Maximum likelihood estimation for HMM

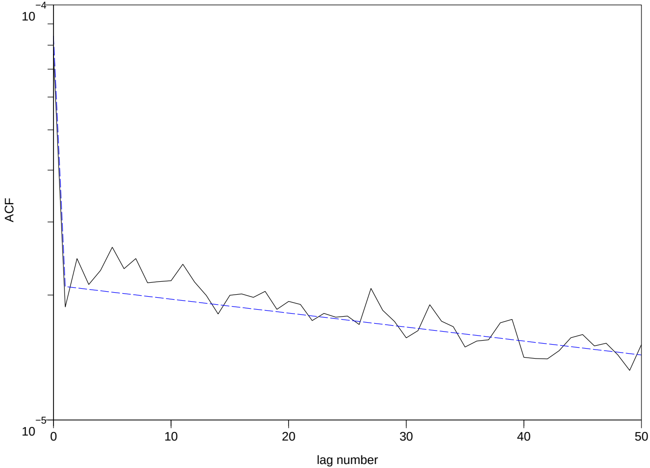

Figure: autocovariances of absolute return with penalty function (1990-2009 daily S&P500)

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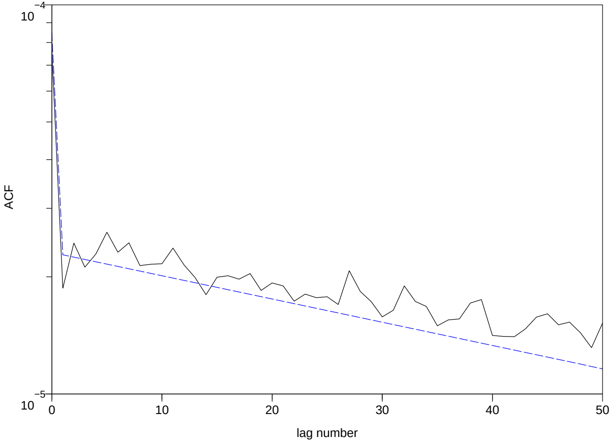

Figure: autocovariances of absolute return with common Markov chain (1990-2009 daily S&P500)

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Posterior probability

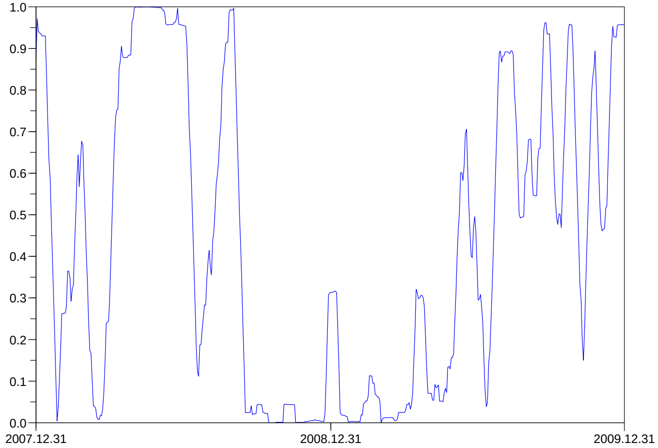

Figure: 2008-2009 daily posterior probability of being in 'good mood' (10-day moving average)

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- A common regime model (based on symmetric hyperbolic / hyperbolic)
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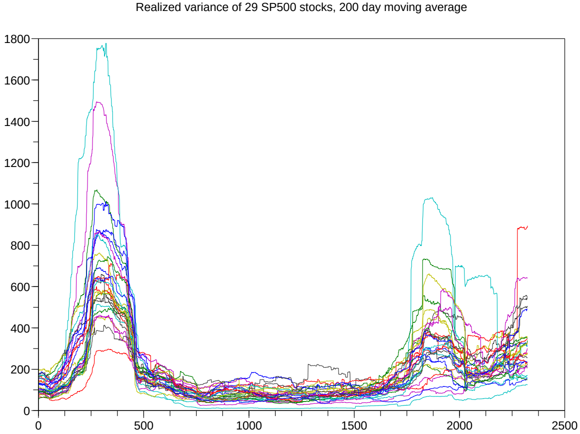
  - reveals an economic significance to the regime of HMM, corresponding 'good time' and 'bad time'.
- Applications: Optimal investment, Option pricing;
- Why not log-Lévy ?
- Why not GARCH? Heteroskedasticity
- Stochastic Volatility Model? Maybe...

Heteroskedasticity



Realized quadratic variation of 29 stocks from S&P500 (taking 200-day moving averages, 2000.07 - 2010.07)

go back to GARCH
go back to conclusion