Discussion of:

Observing the Crisis: Characterising the spectrum of financial markets with high frequency data, 2004-2008

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Sixth Cambridge-Princeton Conference, September 10-11, 2010

1. Summary of the Theory: Spectrum Statistics

- Basic framework: X, often the log of an asset price, is assumed to follow an Itô semimartingale.

- A semimartingale can be decomposed into the sum of a drift, a continuous Brownian-driven part and a discontinuous, or jump, part.
  - The jump part can in turn be decomposed into a sum of small jumps and big jumps.
  - Such a process will always generate a finite number of big jumps.
  - But it may give rise to either a finite or infinite number of small jumps.

- The model is

$$X_t = X_0 + \underbrace{\int_0^t b_s ds}_{\text{drift}} + \underbrace{\int_0^t \sigma_s dW_s}_{\text{continuous part}} + \text{JUMPS}$$

$$\text{JUMPS} = \underbrace{\int_0^t \int_{\{|x| \leq \varepsilon\}} x(\mu - \nu)(ds, dx)}_{\text{small jumps}} + \underbrace{\int_0^t \int_{\{|x| > \varepsilon\}} x\mu(ds, dx)}_{\text{big jumps}}$$

- μ is the jump measure of X, and its predictable compensator is the Lévy measure ν.

- The distinction between small and big jumps (ε) is arbitrary. What is important is that $\varepsilon > 0$ is fixed.

- The paper uses statistics that focus on specific parts of the distribution of high frequency returns in order to learn something about the different components of the semimartingale that produced those returns
  - decide which component(s) need to be included in the model (jumps, finite or infinite activity, continuous component, etc.)
  - determine their relative magnitude
  - magnify specific components of the model if they are present, so we can analyze their finer characteristics (such as the degree of activity of jumps)

- Based on power variations of the increments, suitably truncated and/or sampled at different frequencies.

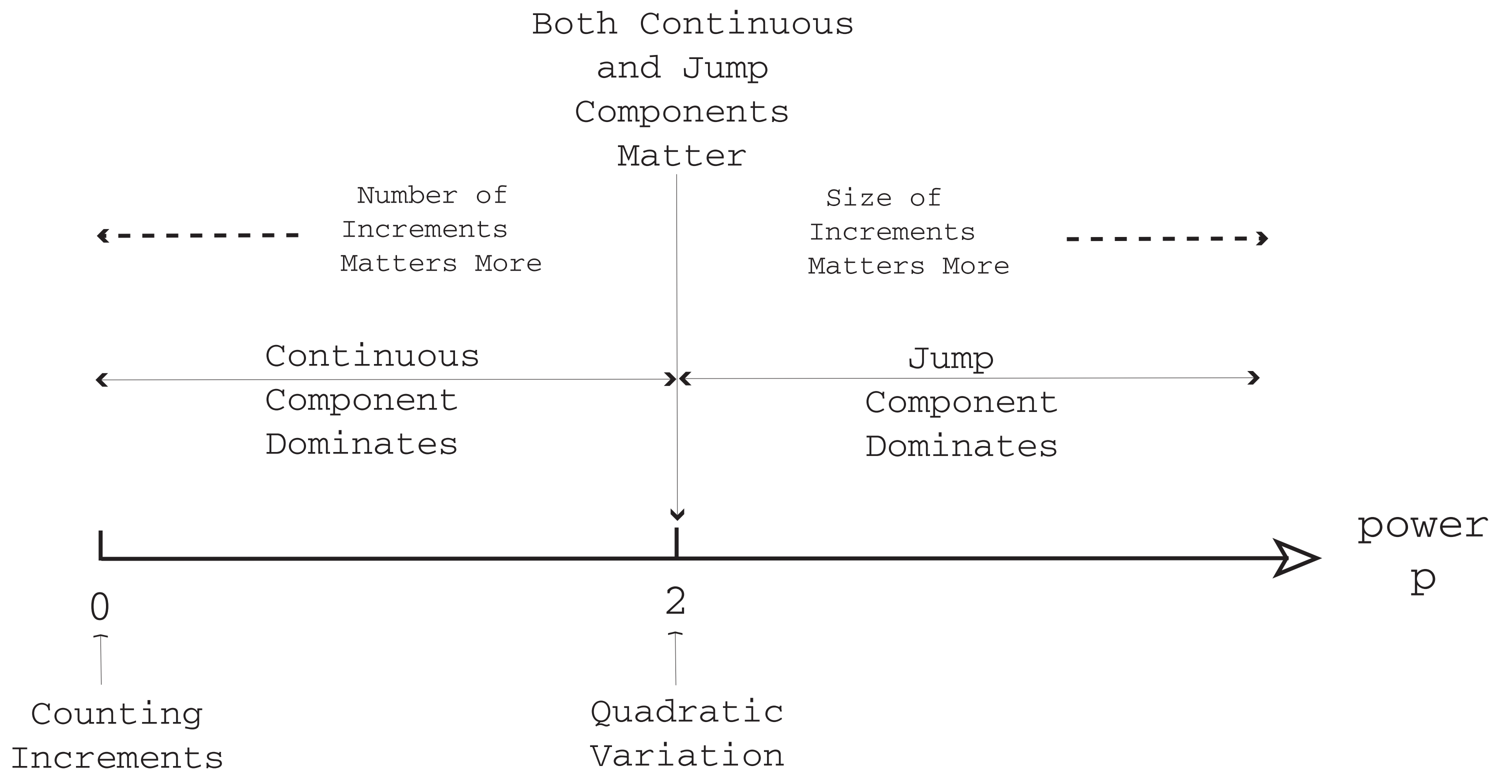
- Exploit the different asymptotic behavior of the variations as we vary:

  – the power p

  – the truncation level u

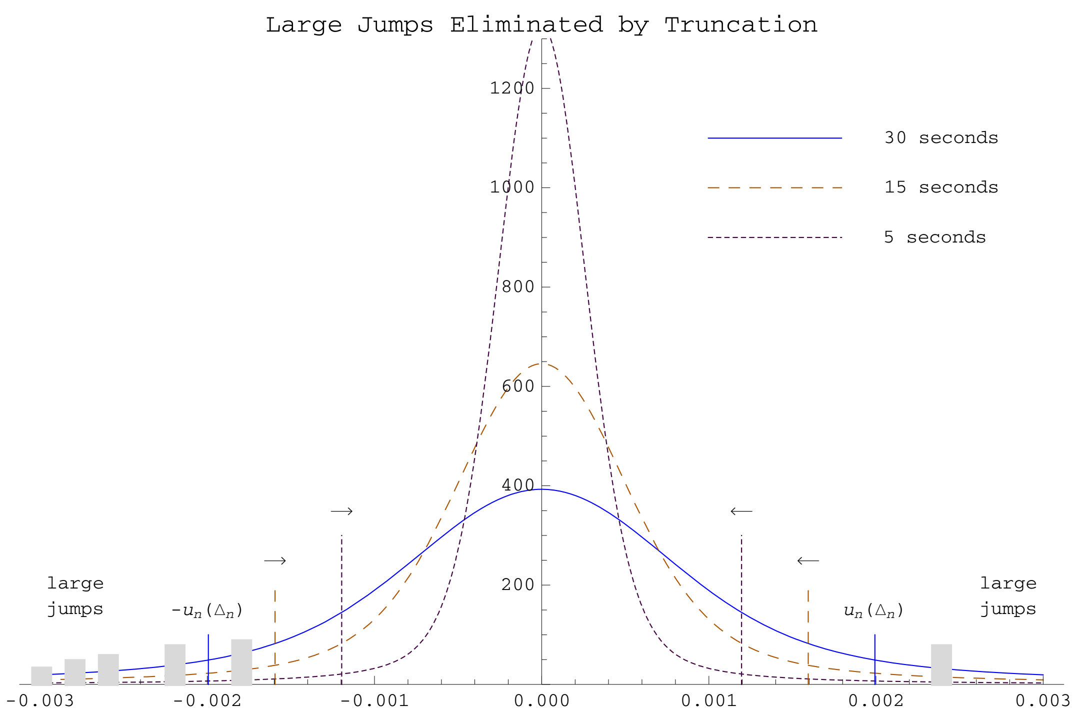
  – the sampling frequency Δ

- Varying the power
  - Powers $p < 2$ will emphasize the continuous component of the underlying sampled process.
  - Powers $p > 2$ will conversely accentuate its jump component.
  - The power $p = 2$ puts them on an equal footing.

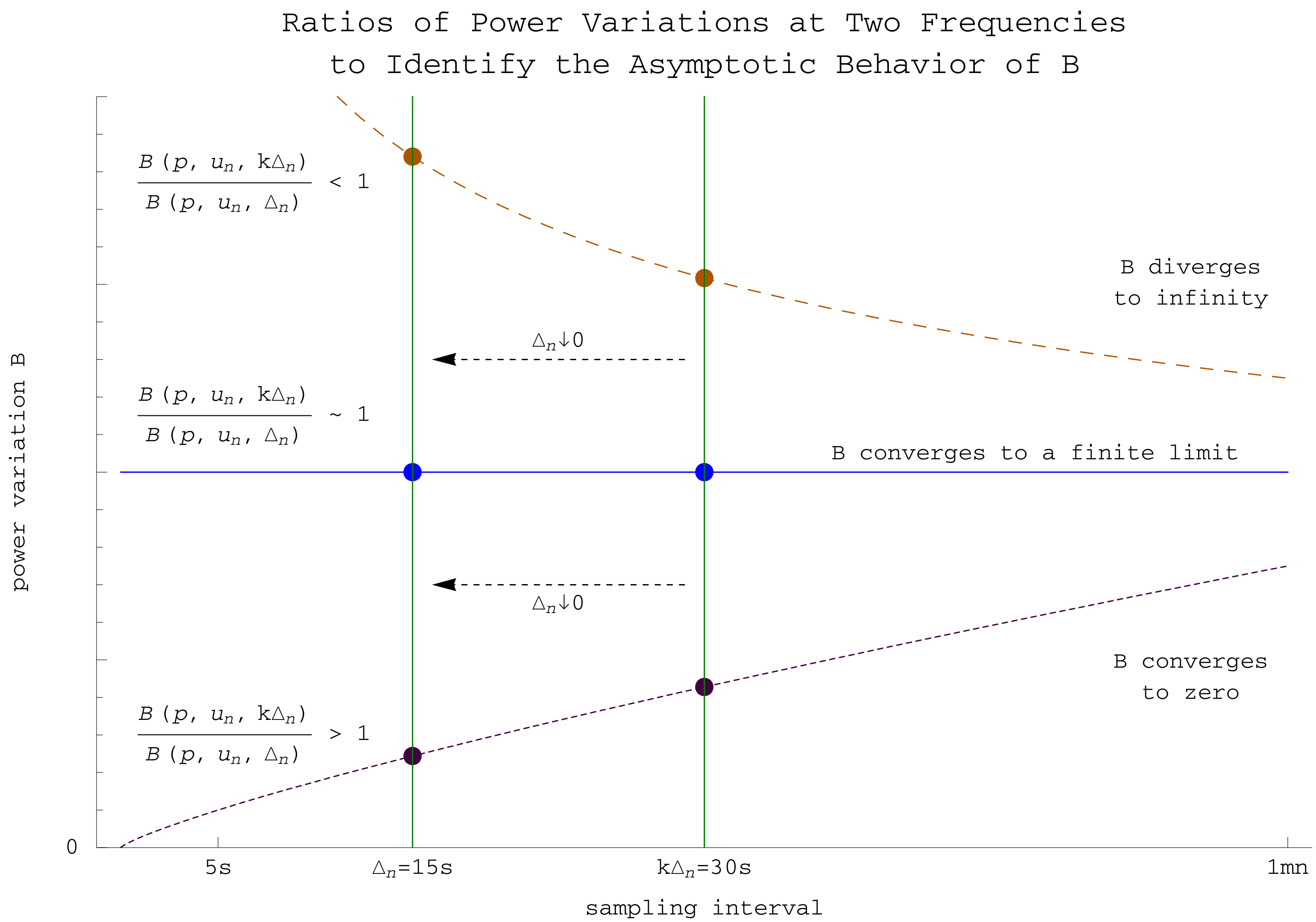

Both Continuous
and Jump
Components
Matter
Number of
Increments
Matters More
Size of
Increments
Matters More
Continuous
Component
Dominates
Jump
Component
Dominates
power
p
0
2
Counting
Increments
Quadratic
Variation

- Truncating the large increments at a suitably selected cutoff level can eliminate the big jumps when needed

- Early use of this device: Mancini (2001)



- Sampling at different frequencies can let us distinguish between situations where the variations:
  - converge to a finite limit;
  - converge to zero;
  - diverge to infinity.



Ratios of Power Variations at Two Frequencies
to Identify the Asymptotic Behavior of B
$\frac{B(p,\ u_n,\ k\Delta_n)}{B(p,\ u_n,\ \Delta_n)} < 1$
B diverges
to infinity
$\Delta_n \downarrow 0$
$\frac{B(p,\ u_n,\ k\Delta_n)}{B(p,\ u_n,\ \Delta_n)} \sim 1$
B converges to a finite limit
$\Delta_n \downarrow 0$
B converges
to zero
$\frac{B(p,\ u_n,\ k\Delta_n)}{B(p,\ u_n,\ \Delta_n)} > 1$
power variation B
0
5s
Δ_n=15s
kΔ_n=30s
1mn
sampling interval

- These various limiting behaviors of the variations are indicative of which component of the model dominates at a particular power and in a certain range of returns (by truncation)

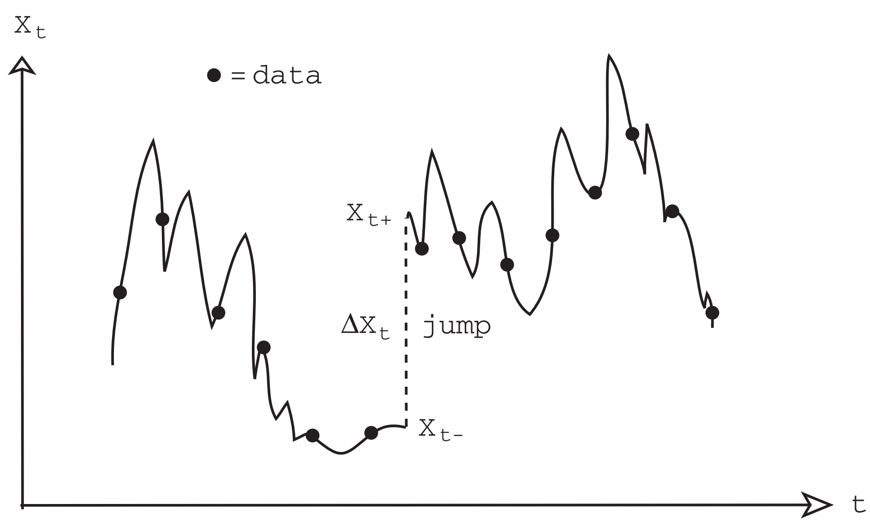
- So they effectively allow one to distinguish between all manners of null and alternative hypotheses.

- There are n observed increments of X on $[0, T]$, which are

$$\Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n},$$

to be contrasted with the actual (unobservable) jumps of X :

$$\Delta X_s = X_s - X_{s-}$$



- For any real $p \geq 0$, the basic instruments are the sum of the p^{th}power of the increments of X, sampled at time interval Δ_n, and truncated at level u_n :

$$B(p, u_n, \Delta_n) \; = \; \sum_{i=1}^{[T/\Delta_n]} |\Delta_i^n X|^p \, 1_{\{|\Delta_i^n X| \leq u_n\}}$$

- The entire methodology relies only on the computation of B for various values of (p, u_n, Δ_n) :

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B(p,u,del)=sum((abs(dX(del)).^p).*(abs(dX(del))<=u(del)))
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- T is fixed, asymptotics are all with respect to $\Delta_n \to 0$.

- u_n is the cutoff level for truncating the increments

- $u_n \to 0$ when $n \to \infty$: in the form $u_n = \alpha\Delta_n^{\varpi}$ for some $\varpi \in (0, 1/2)$.

- $\varpi < 1/2$ to keep all the increments which contain a Brownian contribution.

- There will be further restrictions on the rate at which $u_n \to 0$, expressed in the form of restrictions on the choice of ϖ.

- If we don't want to truncate, we write $B(p, \infty, \Delta_n)$.

- Sometimes we will truncate in the other direction, that is retain only the increments larger than u :

$$U(p, u_n, \Delta_n) = \sum_{i=1}^{[T/\Delta_n]} |\Delta_i^n X|^p \, 1_{\{|\Delta_i^n X| > u_n\}}.$$

- With $u_n = \alpha \Delta_n^{\varpi}$ and $\varpi < 1/2$, that can allow us to eliminate all the increments from the continuous part of he model.

- In terms of the power variations B :

$$U(p, u_n, \Delta_n) = B(p, \infty, \Delta_n) - B(p, u_n, \Delta_n).$$

- Sometimes, we will simply count the number of increments of X, that is, take the power $p = 0$

$$U(0, u_n, \Delta_n) = \sum_{i=1}^{[T/\Delta_n]} 1_{\{|\Delta_i^n X| > u_n\}}.$$

2. Combinations of (p, u, Δ)

H_1 \ H_0	Jumps: Present or Not Ω_T^c	Ω_T^j
Ω_T^c	$\ddots$	S_J : $\begin{pmatrix} p > 2 \\ \infty \\ \Delta_n,\ k\Delta_n \end{pmatrix}$
Ω_T^j	S_J : $\begin{pmatrix} p > 2 \\ \infty \\ \Delta_n,\ k\Delta_n \end{pmatrix}$	$\ddots$

H_1 \ H_0	Jumps: Finite or Infinite Activity Ω_T^f	Ω_T^i
Ω_T^f	$\ddots$	S_{IA} : $\begin{pmatrix} p > 2,\ p' > 2 \\ u_n,\ \gamma u_n \\ \Delta_n \end{pmatrix}$
Ω_T^i	S_{FA} : $\begin{pmatrix} p > 2 \\ u_n \\ \Delta_n,\ k\Delta_n \end{pmatrix}$	$\ddots$

H_0 / H_1	Brownian Motion: Present or Not	
	Ω_T^{W}	$\Omega_T^{\text{no}W}$
Ω_T^{W}	$\ddots$	$S_{\text{no}W}$: $\begin{pmatrix} p = 0,\ p' = 2 \\ u_n,\ \gamma u_n \\ \Delta_n \end{pmatrix}$
$\Omega_T^{\text{no}W}$	S_W : $\begin{pmatrix} p < 2 \\ u_n \\ \Delta_n,\ k\Delta_n \end{pmatrix}$	$\ddots$

Relative Magnitude of the Components
$\begin{pmatrix} p = 2 \\ u_n \\ \Delta_n \end{pmatrix}$

Estimating the Degree of Jump Activity β	
$\hat{\beta}$	$\begin{pmatrix} p = 0 \\ u_n,\ \gamma u_n \\ \Delta_n \end{pmatrix}$
$\hat{\beta}'$	$\begin{pmatrix} p = 0 \\ u_n \\ \Delta_n,\ k\Delta_n \end{pmatrix}$

3. The Paper

3.1. Contribution

- Uses high frequency spectrum statistics to compare US Treasury markets before and after the crisis. Provides a view of the crisis as it unfolded under the microscope.

- Explores the dependence of the statistics on p, as a third dimension of variation.

- Proposes a statistics focused on asymmetric tail behavior between the two sample periods.

3.2. The Results

- High frequency data for US Treasuries during the global financial crisis period from July 2007 until December 2008 and a pre-crisis period from July 2004 to July 2007.

- 2, 5, 10 and 30 year maturities.

- Provide interesting insights since US Treasuries are the main recipients of funds flying to quality in periods of crisis (2007-08 but also Asian currencies, LTCM, etc.), so the differences between before and after should be more visible there than in equities.

- The spectrum statistics provide a much finer characterization of the environment compared to using only realized variance, which would only show an increase in total risk ($=$ variance) during the crisis: 90% implied vol at the height of the crisis in equities

- S_J : Pre-crisis the distribution is less distinguishable from noise whereas during the crisis period, almost all of the distribution supports the null of jump activity.

- S_{FA} : Both the pre-crisis and crisis distributions are centered around 1, consistent with infinite activity jumps. The distribution has a lower variance in the crisis period, centering its mass tightly on 1.

- S_W : The distribution is barely changed across the pre-crisis and crisis samples, and clearly supports the presence of Brownian motion in the data generating process (on top of jumps of infinite activity).

- Relative magnitudes of the components
  - The Brownian component accounts for 40% or less of quadratic variation in the pre-crisis period, and this drops to less than 20%, and as low as 15% for the 2 and 5 year bonds, in the crisis period.
  - The proportion of small jumps does not change very much from the pre-crisis to crisis periods, but the proportion of large jumps increases for all maturities in the crisis period.
  - The shorter maturities show the greatest proportion of large jumps during the crisis.

- Conclusion:
  - fairly stable underlying process (with continuous component and jumps)
  - but greater certainty in distinguishing the presence of jumps from microstructure noise
  - and reveals the increased presence of large jumps during the crisis period.

3.3. Comments/Questions

- Concatenating the spectrum statistics across different power (two dimensional analysis) suppresses potentially useful information in the three-dimensional analysis: I agree
  - Usefulness depends on your prior: is the null limit dependent on p (as in $k^{p/2-1}$) or not (as in 1)
  - If it is, then one should either not concatenate, or report a range of limits
- Interaction between size of tails and powers: isn't this what higher powers (p) should pick up?

- Distinction between a regime switch and a (large) jump?

- Before vs. after: Could construct formal Chow-type tests for the spectrum statistics?

3.4. Asymmetric Tail Statistics

- Nice idea
  - construct a statistic describing each tail of the distribution: S_E^+ and S_E^-
  - instead of using absolute results
  - minor point: should be defined including p

- Compare before and after crisis:
  - Right tail: greater mass during the crisis period, consistent with existing results about increasing right skewness during crisis periods
  - Left tail: Long maturities increases, short maturities decreases (rates dropped to near 0)
  - Consistent with flight to cash and quality in US Treasury bonds. During the crisis there was high demand for these assets and they experienced relatively little of the large falls in return experienced in other asset markets.