Crashes and Collateralized Lending

Jakub W. Jurek and Erik Stafford

Princeton University – Bendheim Center for Finance

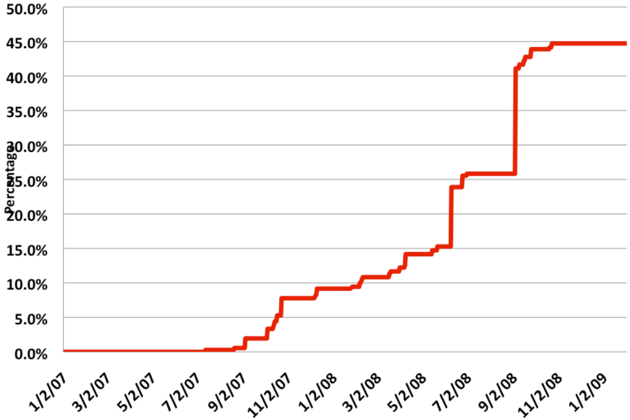
September 2010

Collateral and Financing of Securities

- Investors can buy securities on margin.
- Investor (borrower) contributes some share, $\mathcal{H}$, called the **haircut** or margin.
  - Reg T for stocks says $\mathcal{H} = 50\%$.
  - Portfolio margin for stocks says $\mathcal{H} = 25\%$.
  - Portfolio margin for a broad stock index says $\mathcal{H} \approx 10\%$.
- Lender finances the remainder.
  - Loan of $1 - \mathcal{H}$ against the security, which is used as collateral for the loan.
  - Loan is very short term (one-day) $\longrightarrow$ **repo market** fits this well.

Collateral and Financing of Securities

Gorton and Metrick (2010) report the average haircut for nine asset classes (corporate bonds, ABS, RMBS, CLO, CDO, etc.)



Collateral and Financing of Securities

How should financing terms be set?

The borrowing rate in excess of the riskfree rate (financing spread) should reflect the risk to the lender.

1. How big is the haircut?
   - This determines the borrower's share and the lender's share (leverage).
   - A schedule of spreads/haircuts (Geanakoplos (1997,2003)).

2. What can happen to the collateral over the life of the loan (1 day)?
   - Fundamental shocks (small and big)
   - Liquidity/mispricing shocks
   - Information asymmetry and agency concerns about collateral

Collateral and Financing of Securities

Basic idea

- Crash risk exists and commands a risk premium.
- Crash exposure varies across securities and through time.
- Investors using leverage (margin purchase or repo market financing) share this exposure with the lender $\longrightarrow$ Modigliani-Miller (1958).
- Produce a schedule of haircuts/financing rates for a variety of securities appropriate for bearing crash risk.

Collateral and Financing of Securities

- Crash risk exists and commands a risk premium.
- Crash exposure varies across securities and through time.
- Investors using leverage (margin purchase or repo market financing) share this exposure with the lender $\longrightarrow$ Modigliani-Miller (1958).
- Produce a schedule of haircuts/financing rates for a variety of securities appropriate for bearing crash risk.

Ingredients:

1. Utility function and market structure, $U(W_t)$.
2. Crash distribution, $f^{\mathbb{P}}(x)$.
3. Crash consequence function (security-level crash exposure), $\mathcal{B}(x)$.

Valuing Crash Risk

The investor can purchase an insurance claim, $I(x)$, for the price, ξ:

$$E_t^{\mathbb{P}}\left[U\left(\tilde{W}_t\right)\right] = (1-\pi^{\mathbb{P}})\cdot U\left(W_{t-} - q\cdot\xi\right) + \pi^{\mathbb{P}}\cdot\int_0^1 U\left(W_t(x) + q\cdot I(x) - q\cdot\xi\right)\cdot f^{\mathbb{P}}(x)\cdot dx$$

- An indifference argument indicates the **price of insurance**, ξ, will be given by:

$$\begin{aligned}\xi &= \pi^{\mathbb{P}}\cdot\frac{\int_0^1 U'\left(W_t(x)\right)\cdot I(x)\cdot f^{\mathbb{P}}(x)\cdot dx}{(1-\pi^{\mathbb{P}})\cdot U'\left(W_{t-}\right) + \pi^{\mathbb{P}}\cdot\int_0^1 U'\left(W_t(x)\right)\cdot f^{\mathbb{P}}(x)\cdot dx}\\ &\equiv \pi^{\mathbb{Q}}\cdot\int_0^1 f^{\mathbb{Q}}(x)\cdot I(x)\cdot dx\end{aligned}$$

- The **risk-neutral** quantities are:

$$\begin{aligned}\pi^{\mathbb{Q}} &= \pi^{\mathbb{P}}\cdot\frac{\int_0^1 U'\left(W_t(x)\right)\cdot f^{\mathbb{P}}(x)\cdot dx}{(1-\pi^{\mathbb{P}})\cdot U'\left(W_{t-}\right) + \pi^{\mathbb{P}}\cdot\int_0^1 U'\left(W_t(x)\right)\cdot f^{\mathbb{P}}(x)\cdot dx}\\ f^{\mathbb{Q}}(x) &= \frac{U'\left(W_t(x)\right)\cdot f^{\mathbb{P}}(x)}{\int_0^1 U'\left(W_t(x)\right)\cdot f^{\mathbb{P}}(x)\cdot dx}\end{aligned}$$

Valuing Crash Risk

The loss function for a borrower posting margin, $\mathcal{H}$, and financing the remainder is:

$$l_b(x) \quad = \quad \min\left(\mathcal{B}(x), \mathcal{H}\right) = \begin{cases} \mathcal{B}(x) & x < \hat{x} \\ \mathcal{H} & x \geq \hat{x} \end{cases} \quad \text{where} \quad \hat{x} = \mathcal{B}^{-1}(\mathcal{H})$$

Valuing Crash Risk

The loss function for a borrower posting margin, $\mathcal{H}$, and financing the remainder is:

$$l_b(x) \quad = \quad \min\left(\mathcal{B}(x), \mathcal{H}\right) = \begin{cases} \mathcal{B}(x) & x < \hat{x} \\ \mathcal{H} & x \geq \hat{x} \end{cases} \qquad \text{where} \quad \hat{x} = \mathcal{B}^{-1}(\mathcal{H})$$

The cost of insuring the strategy can be split between the borrow and the lender:

- Borrower's *dollar* cost of capital.

$$\begin{aligned} \xi_b &= \pi^{\mathbb{Q}} \cdot \int_0^1 f^{\mathbb{Q}}(x) \cdot l_b(x) \cdot dx \\ &= \pi^{\mathbb{Q}} \cdot \left(\mathcal{H} \cdot \left(1 - F^{\mathbb{Q}}(\hat{x})\right) + E^{\mathbb{Q}}\left[\mathcal{B}(x) \mid x < \hat{x}\right] \cdot F^{\mathbb{Q}}(\hat{x})\right) \end{aligned}$$

Valuing Crash Risk

The loss function for a borrower posting margin, $\mathcal{H}$, and financing the remainder is:

$$l_b(x) = \min\left(\mathcal{B}(x), \mathcal{H}\right) = \begin{cases} \mathcal{B}(x) & x < \hat{x} \\ \mathcal{H} & x \geq \hat{x} \end{cases} \quad \text{where} \quad \hat{x} = \mathcal{B}^{-1}(\mathcal{H})$$

The cost of insuring the strategy can be split between the borrow and the lender:

- Borrower's *dollar* cost of capital.

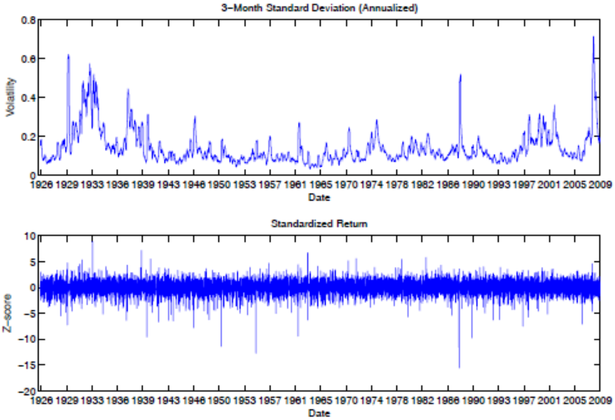
$$\begin{aligned} \xi_b &= \pi^{\mathbb{Q}} \cdot \int_0^1 f^{\mathbb{Q}}(x) \cdot l_b(x) \cdot dx \\ &= \pi^{\mathbb{Q}} \cdot \left(\mathcal{H} \cdot \left(1 - F^{\mathbb{Q}}(\hat{x})\right) + E^{\mathbb{Q}}\left[\mathcal{B}(x) \mid x < \hat{x}\right] \cdot F^{\mathbb{Q}}(\hat{x})\right) \end{aligned}$$

- Lender's *dollar* cost of capital:

$$\begin{aligned} \xi_l = \xi - \xi_b &= \pi^{\mathbb{Q}} \cdot \left(E^{\mathbb{Q}}\left[\mathcal{B}(x) \mid x > \hat{x}\right] - \mathcal{H}\right) \cdot \left(1 - F^{\mathbb{Q}}(\hat{x})\right) \\ &\approx \pi^{\mathbb{Q}} \cdot \left(1 - F^{\mathbb{Q}}(\hat{x})\right) \cdot \left.\frac{\partial B}{\partial x}\right|_{x=\hat{x}} \cdot E^{\mathbb{Q}}\left[(x - \hat{x}) \mid x > \hat{x}\right] \end{aligned}$$

Characterizing the Crash Distribution

- Examine returns that are large relative to recent volatility (e.g. $Z < -6$).
- Daily returns from CRSP (1929-2009) measured relative to standard deviation of returns over past 3-months.
- 19 observations (once every 5 years) $\rightarrow$ 6 in the last third of the sample.



Characterizing the Crash Distribution

1. Annualized arrival rate ≈ 0.2 (0.09% per trading day).
2. Assume Z-scores have a Beta(a,b) (or Gamma(a, b)) distribution.
   - Scaled by the prevailing volatility to obtain *returns* (Bates (2000), Pan (2002))
3. Assume the largest Z-score corresponds to the 95th percentile of the crash distribution → extreme value theory.
   - The most severe event corresponds to a Z-score of -15.5.
   - The estimated 95% confidence interval for corresponding percentile ranges from 68% to 100%.
4. Choose parameters to minimize fitting errors between the empirical and theoretical CDFs at the median and 95^{th} percentiles.
   - Mitigates "peso problems" relative to moment-matching methods.

Percentile	$0.8 \cdot$ VIX	$\hat{a}$	$\hat{b}$	95th-tile Crash Size	Jump Variance	Jump Share in Total Variance	Jump Risk Premium
1	8.18%	3.59	84.73	8.00%	0.0004	5.88%	0.12%
5	8.79%	3.58	78.24	8.59%	0.0005	5.88%	0.14%
25	10.43%	3.53	64.57	10.20%	0.0007	5.88%	0.20%
50	13.30%	3.45	48.78	13.00%	0.0011	5.88%	0.34%
75	18.51%	3.30	32.68	18.09%	0.0021	5.87%	0.71%
95	36.09%	2.81	12.95	35.28%	0.0081	5.85%	3.85%
99	53.23%	2.31	6.54	52.03%	0.0175	5.83%	14.50%

Crash Consequence Function, $\mathcal{B}(x)$

- Change in securitys value as a consequence of the crash, x:

$$\mathcal{B}(x) \quad = \quad V\left(t, x, \sigma_m(x), \gamma\right) - V\left(t^-, x = 0, \sigma_m(x = 0), \gamma\right)$$

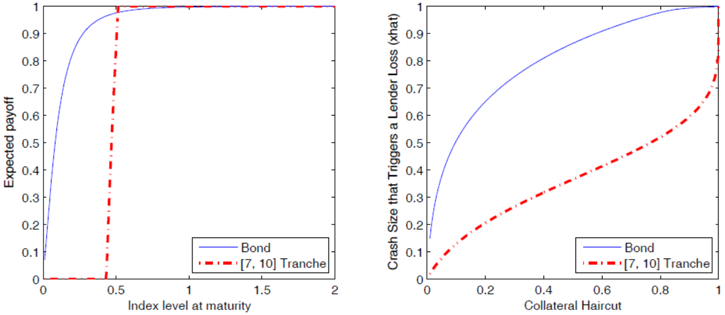
  - For **corporate bonds** and **structured securities**, use the structural valuation model developed in Coval, Jurek, and Stafford (AER 2009) $\rightarrow$ structural credit risk model.
  - State-contingent valuation of bonds, bond portfolios, CDO tranches.

- Crash insurance covers the crash loss, $I(x) = -\mathcal{B}(x)$, and costs ξ.

- The borrow and lender split the cost of insurance: $\xi = \xi_b + \xi_l$.
  - Split is a function of the posted haircut, $\mathcal{H} \rightarrow$ schedule of haircuts and financing spreads.

Crash Consequence Function, $\mathcal{B}(x)$

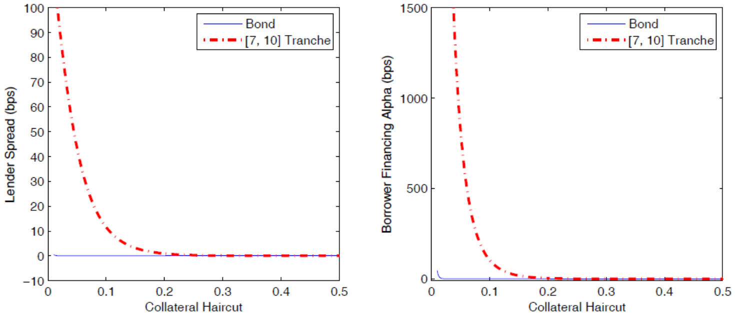
Corporate bonds vs. structured securities



- Both are highly rated (Bond is AA; [7,10] tranche of the Dow Jones CDX.NA.IG was considered AAA).
- Recovery, conditional on a market crash, is very different.
  - Bond expected to drop in value, but not by very much
  - Tranche expected to lose much more value

Crash Consequence Function, $\mathcal{B}(x)$

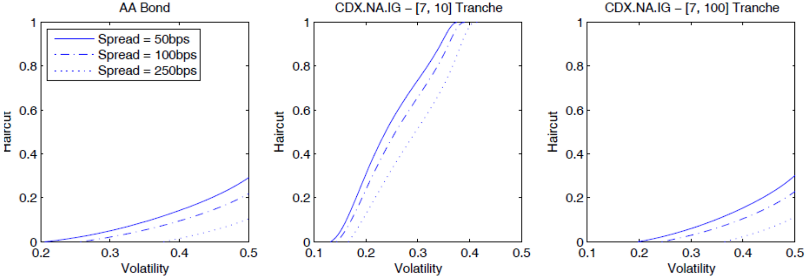
Corporate bonds vs. structured securities



- Financing spread on structured securities is extremely sensitive to the chosen haircut level, $\mathcal{H}$.
- A borrower who can finance structured securities at rates applicable to identically-rated bonds gains significant **financing alpha**.

Security Financing Stress Tests

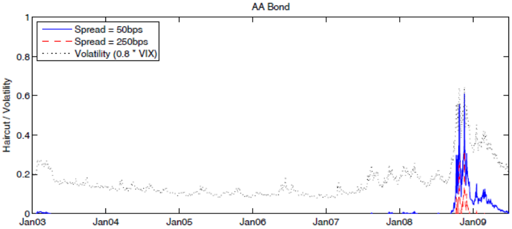
Volatility

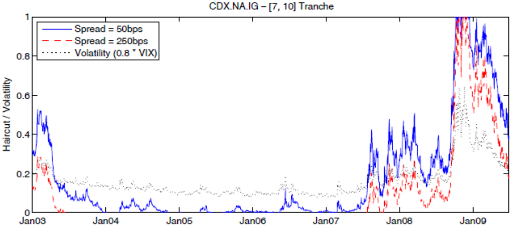


- As market volatility increases, thin mezzanine tranches become un-repoable ($\mathcal{H} \rightarrow 1$) at conventional spreads.
- In 2008, the repo market was estimated to be around $12 trillion.
  - Repo market relies on "high quality" collateral.
  - One function of CDOs was to manufacture high quality collateral.
- Prior to July 2007, no haircut applied to AAA-rated CDO tranches in the repo market. Haircut rises to 3-7% by Q2 2007, and further to 15-20% by Q1 2008.

Security Financing Stress Tests

Model-predicted haircuts





Security Financing Stress Tests

Good vs. poor collateral

Participants seem to want stable haircuts and low financing spreads.

- Haircut needs to be set such that the collateralized loan has a high recovery value in the event of a crash.

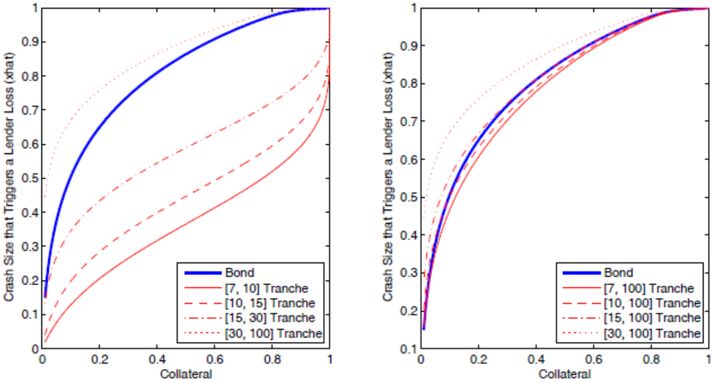
Good collateral: Investment grade bonds and thick senior tranches.

Poor collateral: Highly rated mezzanine tranches (of even high quality asset pools) are expected to have low recovery values in a crash.

- Re-securitizing mezzanine tranches does not change this.
- Most of the securities that had large "runs on repo" were mezz-like, e.g. thin mezzanine tranches, mortgage-backed CDOs (really CDO^2) and sub-prime CDOs.

Are All Securitizations Poor Collateral?

Increasing thickness allows tranches to withstand crashes of greater magnitude.



Conclusions

- A simple framework for understanding the **contribution of systematic crash risk to the cost of financing security purchases**.
  - The sensitivity of the borrower's and the lender's cost of capital to the underlying crash distribution, underlying securities, and leverage choices (borrower haircut).
  - How the borrower's cost of capital is altered when the lender's financing rule is imperfect $\longrightarrow$ *financing alpha*.

- Identifies **mezzanine-like collateral** - assets whose value is quickly exhausted as the consequence of a crash - as being expected to have volatile haircuts/spreads in the time series.