Observing the crisis: Characterising the spectrum of markets with high frequency data, 2004-2008

Mardi Dungey, Jet Holloway and Abdullah Yalama

University of Tasmania (UTAS)

September 2010

Price discontinuities and crises

- Basic idea: What distinguishes financial market data in crises from non-crisis periods?
- Existing models based around:
  - Increased volatility
  - Underlying data generating processes unchanged, but augmented
  - Transmission of tail events across markets

Price discontinuities and crises

- Descriptive statistics by Ait-Sahalia and Jacod (2009) distinguish
  - Presence of Jumps (S_J)
  - Intensity of Jumps (infinite or finite) (S_{FA})
  - Presence of Brownian Motion (S_W)
- These work by comparing discretely sampled data and changing the following sampling characteristics:
  - k the sampling interval (eg 5 minutes versus 10 minutes)
  - p the power function of returns: we can look at $\sum \Delta X^p$, $p = 2, 3, 4.....$
  - u truncation: look at power functions of returns which are smaller/larger than u, eg $\sum \Delta X^p|_{\Delta X<u}$

The test statistics

- Jumps present:

$$S_J(p,k,\Delta_n)_t = \frac{\sum_{s\le T}|k\Delta X_s|^p}{\sum_{s\le T}|\Delta X_s|^p} \to \begin{cases} 1 & \text{jumps no noise} \\ k^{p/2-1} & \text{no jumps no noise} \\ 1/k & \text{additive noise dominant} \\ 1/k^{1/2} & \text{rounding error dominant} \end{cases}$$

- Finite Jump Activity:

$$S_{FA}(p,u_n,k,\Delta_n)_t \to \begin{cases} 1 & \text{infinite activity jumps no noise} \\ k^{p/2-1} & \text{finite activity jumps no noise} \\ 1/k & \text{additive noise dominant} \\ 1/k^{1/2} & \text{rounding error dominant} \end{cases}$$

- Brownian Motion

$$S_W(p,u_n,k,\Delta_n)_t = \frac{1}{S_{FAt}} \to \begin{cases} 1 & \text{no Brownian motion no noise} \\ k^{1-p/2} & \text{Brownian motion no noise} \\ k & \text{additive noise dominant} \\ k^{1/2} & \text{rounding error dominant} \end{cases}$$

Application

- July 2004 to December 2008: crisis from July 16 2007
  - Tick by tick observations, sampled at 5 minute intervals

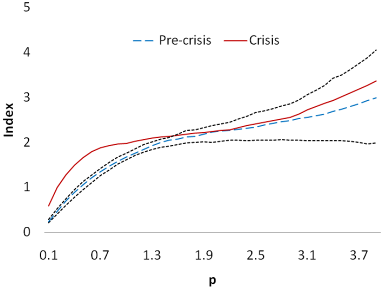
- **US Treasuries data:**
  - 2,5,10 and 30 year maturities in the secondary spot market
  - Trading time 8:00EST to 17:30EST
  - 115 observations per trading day

- **Equity Futures: S&P500, Nasdaq100**
  - CME exchange
  - overnight data: trades from 15:30CST to 8:15CST
  - 201 observations per overnight trading session

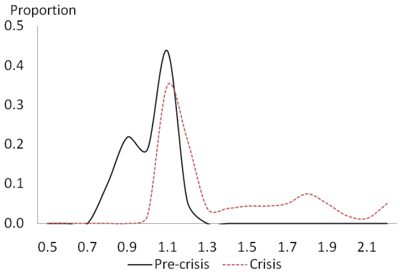
Basics: 2 year bond

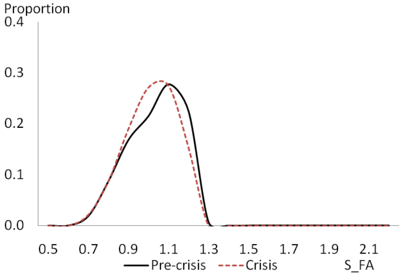
	pre-crisis	crisis
Blumenthal-Getoor index, $\widehat{\beta}$, $k = 2$,	0.6951	0.7696
proportion of quadratic variation in continuous	0.3730	0.1593
large jumps	0.4256	0.6557
small jumps	0.2014	0.1850

Activity Signature Plot (Tauchen and Todorov, 2009)



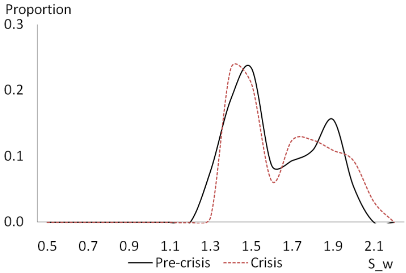
Distributions of statistics for US Treasuries





S_j : presence of jumps

S_{FA} : finite jump activity



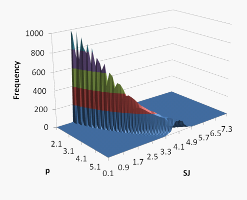
Big picture result

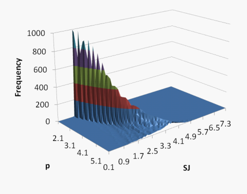
- Brownian motion and infinite jumps are confirmed in both non-crisis and crisis periods
  - Little difference in the distributions of these statistics
- The test for the presence of jumps shows something interestingly different
  - in crisis periods we are more sure that we are observing price discontinuities not noise
  - The right skew in the crisis distribution is worthy of exploration
- These 2D representations concatenate results over p and $k = 2, 3$
- Consider 3D representations.

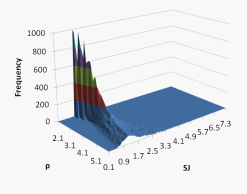
Monte Carlo Experiments

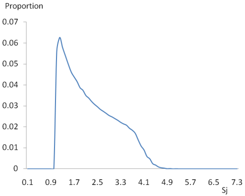
- Three possibilities for underlying distribution:
  1. Brownian motion
  2. Heston stochastic volatility model or
  3. skewed normal (Azzalini,1985)
  4. multiple regimes
- Examine these with no jumps, small jumps and large jumps
  - Calibrated to match Ait-Sahalia and Jacod (2009) representing a liquid equity stock
  - skewness of 0.78, consistent with shape parameter $\alpha = 4$
- Simulate 6000 trading days, with 201 observations per day, generates 100 months of observations on each statistic
- 10,000 jumps randomly distributed across observations

Brownian Motion:

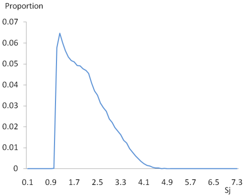




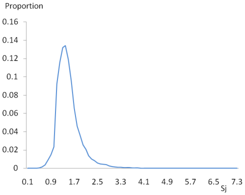




No Jumps

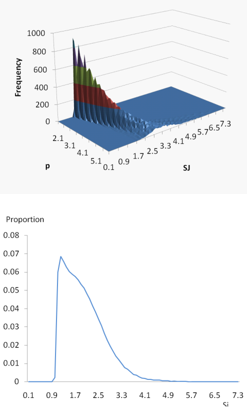


Small Jumps

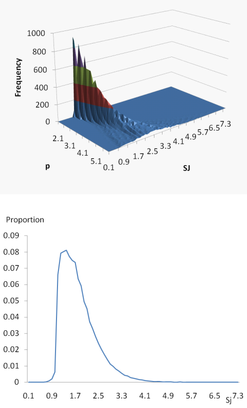


Large jumps

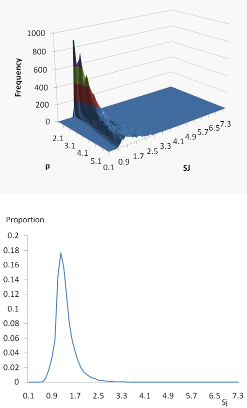
Skewness:



No Jumps

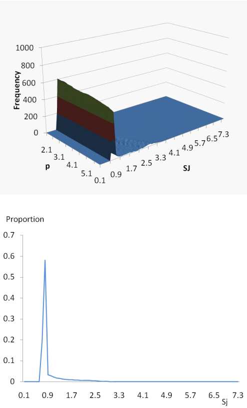


Small Jumps

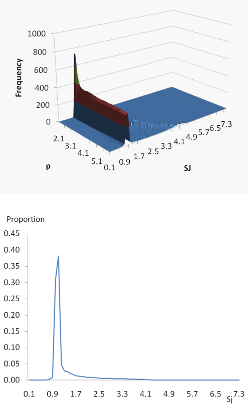


Large jumps

Some others:

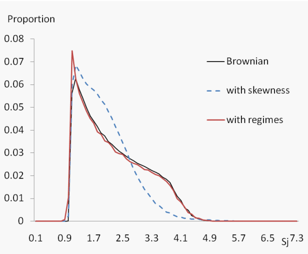


Rounding
No Jumps

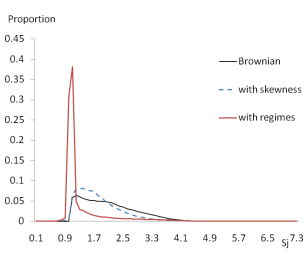


Regimes
Small Jumps

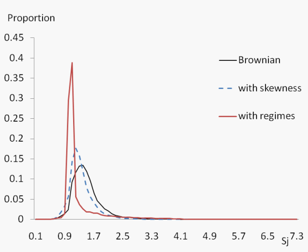
Comparisons of Monte Carlo outcomes: 2D



S_j : with no jumps

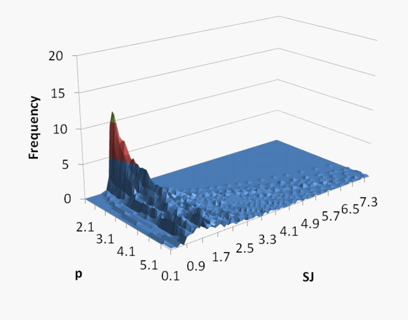


S_j : with small jumps

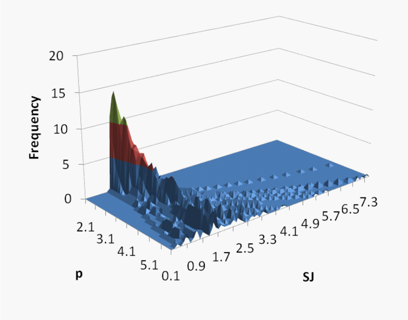


S_j : with large jumps

Treasuries data

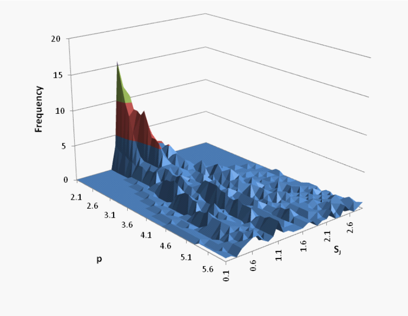


S_j : pre-crisis

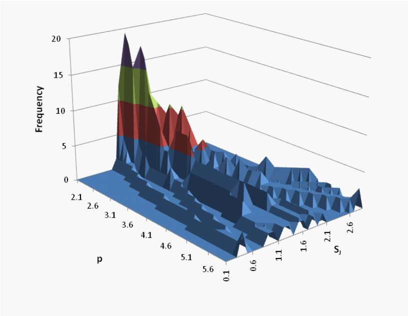


S_j : crisis

S&P futures data



S_j : pre-crisis



S_j : crisis

Findings thus far

- The 2 D representation hides some additional richness
  - During the move from non-crisis to crisis
    - no change in evidence for Brownian motion
    - no change in evidence that jumps are finite
    - Increase in our ability to identify jumps from noise
- Tail behaviour is distinguished as p increases
- Leads to right skews in the 2D representations
- Change between non-crisis and crisis identifiable at individual p
- This is consistent with many existing theories of crisis transmission
- Choosing p is important to distinguishing across non-crisis and crisis data

Where to next?

- Consider the role of tail behaviour more fully
- Use this to compare across non-crisis and crisis data
- Positive tail measure:

$$S_E^+ = \frac{\sum_{i=1}^{T/\Delta_n} 1_{\{k\Delta_i^n X > u_n\}}}{\sum_{i=1}^{T/\Delta_n} 1_{\{\Delta_i^n X > u_n\}}}$$

- Negative tail measure:

$$S_E^- = \frac{\sum_{i=1}^{T/\Delta_n} 1_{\{k\Delta_i^n X < -u_n\}}}{\sum_{i=1}^{T/\Delta_n} 1_{\{\Delta_i^n X < -u_n\}}}$$

- These have same sorts of properties as the S_J statistic, as they are simply the tails of that distribution

- Construct statistics which allow comparisons across different data sets:
    - take the value 1 under the null of no change between the two periods
- Compare positive tails

$$S^{+} = \frac{S^{+}_{E,crisis}}{S^{+}_{E,noncrisis}}$$

- Compare negative tails

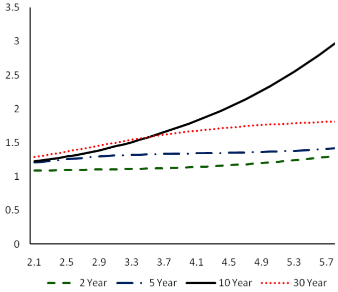
$$S^{-} = \frac{S^{-}_{E,crisis}}{S^{-}_{E,noncrisis}}$$

- Compare total tails

$$S^{abs} = \frac{\left\{\left[\sum_{i=1}^{T/\Delta_n} 1_{\{|k\Delta_i^n X|>u_n\}}\right] / \left[\sum_{i=1}^{T/\Delta_n} 1_{\{|\Delta_i^n X|>u_n\}}\right]\right\}_{crisis}}{\left\{\left[\sum_{i=1}^{T/\Delta_n} 1_{\{|k\Delta_i^n X|>u_n\}}\right] / \left[\sum_{i=1}^{T/\Delta_n} 1_{\{|\Delta_i^n X|>u_n\}}\right]\right\}_{noncrisis}}.$$

Application to the bond market data

- If statistic value > 1 implies more mass in tails during the crisis period

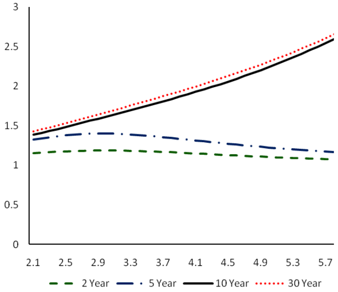


total tails

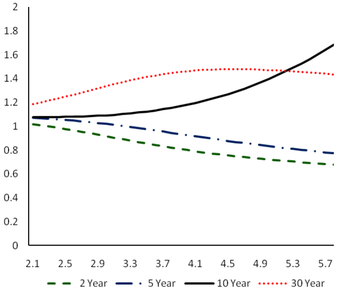
- more mass into tails in the crisis, most pronounced for the longer maturities

Application to the bond market data

- If statistic value > 1 implies more mass in tails during the crisis period



positive tails



negative tails

- Flight to cash is clearly apparent, there is a reduction in mass in the negative tail for short dated maturities

Concluding Remarks

- This paper is about
  - reconciling changes in behavior in high frequency data with characterizations of crisis
- Application of Ait-Sahalia and Jacod method
  - show the importance of recongising power dimension of the problem
  - develop a new measure of change across sample periods
- Empirical application to Treasury bonds
  - first application, shows infinite activity jumps process.
  - Its the tail behavior which changes in crises
  - Other aspects of DGP remain intact
  - *means that crises can be characterized as an underlying process with additional peculiarities*
  - this is also supported by applications not reported in this paper (equity futures market data)

Heston Model

$$dX_t = v^{1/2} dW_t + \theta dY_t$$
$$dv_t = \kappa(\eta - v_t)dt + \gamma v_t^{1/2} dB_g + dJ_t$$

with $E(dW_t dB_t] = \rho dt$ the correlation between the Brownian motion processes, W_t and B_t and J_t discrete jumps.

$\eta^{1/2}$	0.25
γ	0.5
κ	5
ρ	-0.5

scaled to our data samples (/201).