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Estimating Exponential Affine Models With Correlated Measurement Errors: Applications to Fixed Income and Commodities

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Outline

- Introduction
- Brief indication of cross-sectional and serial correlation of measurement errors
- State-space formulation of the exponential affine model
- Measurement errors and parameter estimates
- The augmented state space form
- Empirical results

Motivation

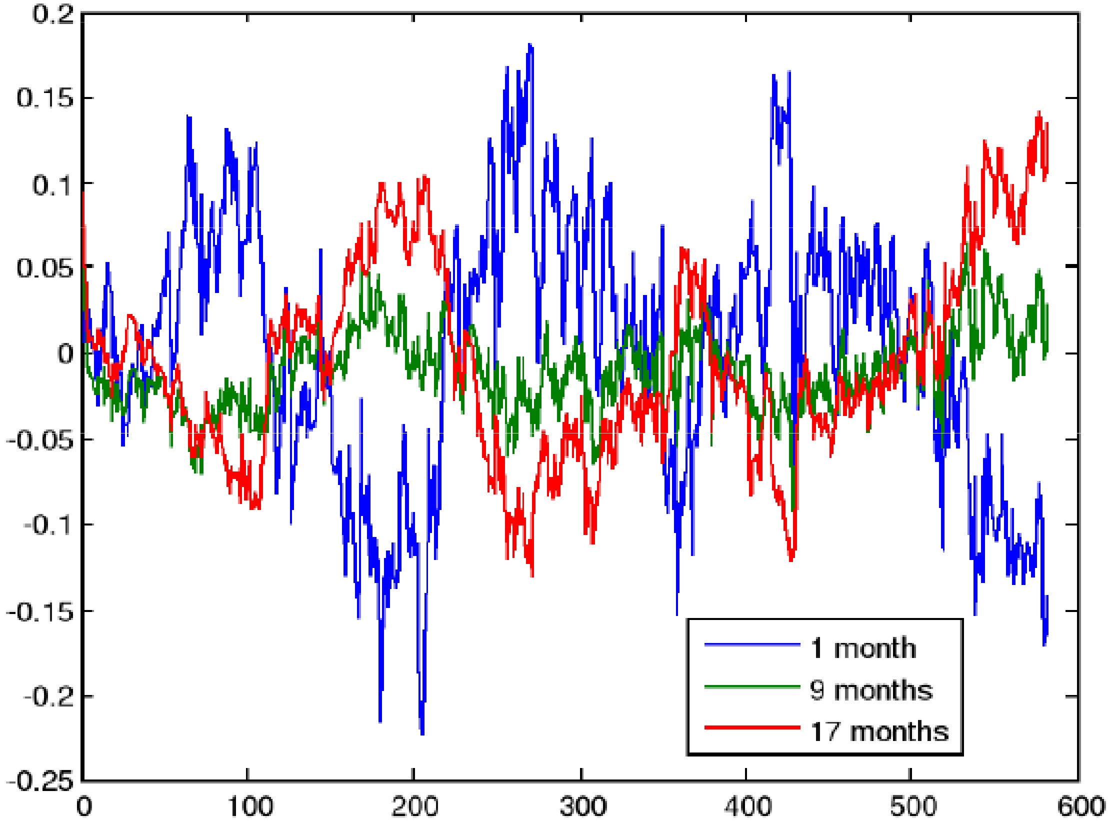
- **Exponential affine models** (EAMs) are important in the term-structure modelling of fixed income securities and commodity futures
- However they are difficult to estimate because many of their factors are latent
- Proxies, the efficient method of moments and versions of the Kalman filter are all employed
- The Kalman filter is the most popular

  Duffee and Stanton (2004)
- **Measurement errors** are assumed for computational convenience to be independent identically distributed but in applications this is seldom true

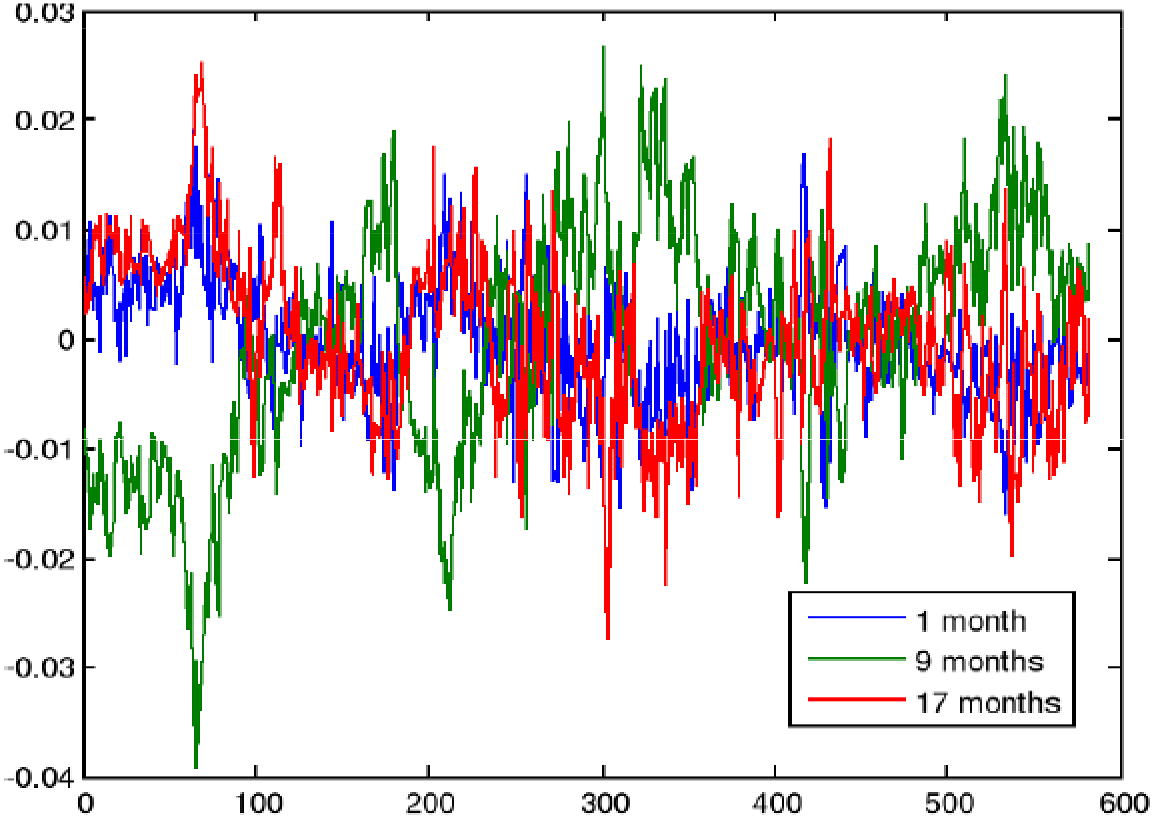
Related literature

- Schwartz (1997) finds serial correlation in the estimated measurement errors of commodity future models
- De Jong (2000) finds that both strong cross-sectional and serial correlation exists in the estimated measurement errors of fixed income models

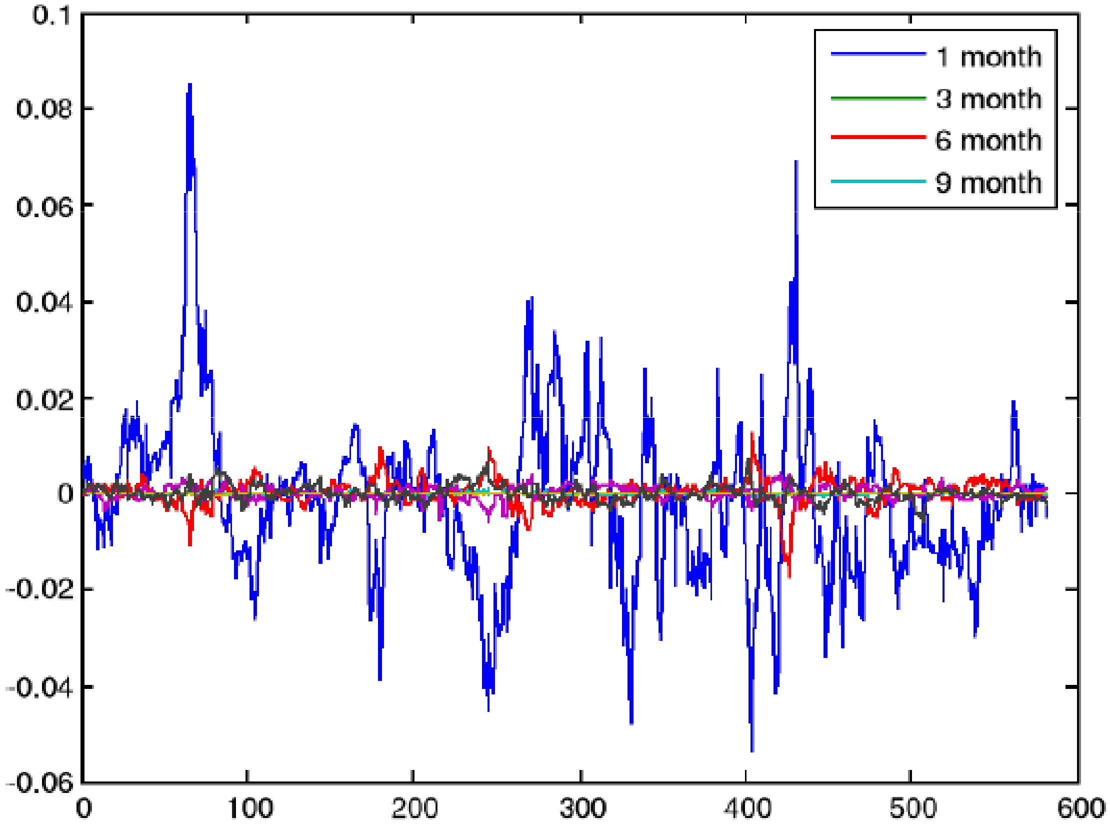
Measurement errors of a one-factor commodity model applied to oil futures



Measurement errors of a two-factor commodity model applied to oil futures



Measurement errors of a three-factor commodity model applied to oil futures



The exponential affine model

$$F_t(\tau) = \mathbb{E}_t^Q[\exp(\boldsymbol{v}_T)]$$

$$P_t(\tau) = \mathbb{E}_t^Q[\exp(-\int_t^T \boldsymbol{r}_s ds)],$$

$$\boldsymbol{x}_t = \phi_0 + \phi_Y{}'\boldsymbol{Y}_t,$$

$$d\boldsymbol{Y}_t = K(\Theta - Y_t)dt + \Sigma\sqrt{\Upsilon}d\boldsymbol{W}_t^Q,$$

$$Z_t(\tau) = \exp[A(\tau) + B'(\tau)Y_t],$$

State-space formulation of the exponential affine model

Transition Equation

$$\boldsymbol{Y}_t = d + \Phi Y_{t-1} + \boldsymbol{\eta}_t,$$

$$\begin{aligned} E[\boldsymbol{Y}_t | Y_{t-1}] &= d + \Phi Y_{t-1} \\ var(\boldsymbol{\eta}_t) &= var(\boldsymbol{Y}_t | Y_{t-1}) = \Omega(Y_{t-1}) := \Omega_t \end{aligned}$$

Measurement Equation

$$\mathcal{Z}_t = \mathcal{A} + \mathcal{B} Y_t + \varepsilon_t$$

Are three factors sufficient?

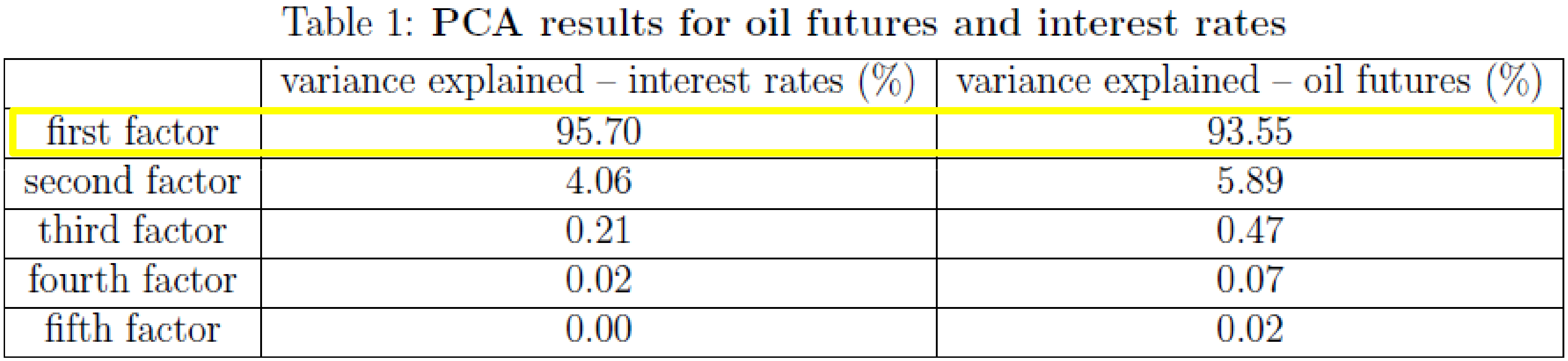


Table 1: **PCA results for oil futures and interest rates**

	variance explained – interest rates (%)	variance explained – oil futures (%)
first factor	95.70	93.55
second factor	4.06	5.89
third factor	0.21	0.47
fourth factor	0.02	0.07
fifth factor	0.00	0.02

Test of IID commodity measurement errors

Maturity	Covariance matrix									DW	Serial Correlation
Basic SSF											
3 months	1.00	-0.62	-0.66	-0.40	0.26	0.30	0.12	-0.31	-0.37	**0.33**	**0.84** (0.017)
6 months	-0.62	1.00	0.17	-0.16	-0.20	0.06	0.12	0.06	0.05	**1.42**	**0.29** (0.029)
12 months	-0.66	0.10	1.00	0.58	-0.39	-0.31	-0.14	0.39	0.37	**0.54**	**0.73** (0.021)
24 months	-0.40	-0.16	0.58	1.00	-0.13	-0.56	-0.37	0.50	0.53	**0.37**	**0.82** (0.018)
36 months	0.26	-0.28	-0.39	-0.13	1.00	-0.53	-0.51	0.04	0.27	**1.27**	**0.37** (0.028)
48 months	0.31	0.06	-0.31	-0.56	-0.53	1.00	0.45	-0.59	-0.62	**0.83**	**0.58** (0.025)
60 months	0.12	0.12	-0.14	-0.37	-0.51	0.45	1.00	-0.58	-0.58	**0.49**	**0.75** (0.020)
72 months	-0.31	0.06	0.39	0.50	0.04	-0.59	-0.58	1.00	0.33	**0.67**	**0.67** (0.023)
120 months	-0.37	0.05	0.37	0.53	0.27	-0.62	-0.58	0.33	1.00	**0.29**	**0.86** (0.016)

Maturity	Covariance matrix			DW	Serial Correlation
Basic SSF					
1 month	1.00	-0.39	-0.81	**0.19**	**0.90** (0.018)
9 month	-0.39	1.00	0.76	**0.40**	**0.80** (0.024)
17 month	-0.81	0.76	1.00	**0.06**	**0.97** (0.011)

Impact of cross-sectional and serial correlations on parameter estimates

We simulate three groups of data for the **two factor** Gibson-Schwartz (1990) model using the basic state-space form

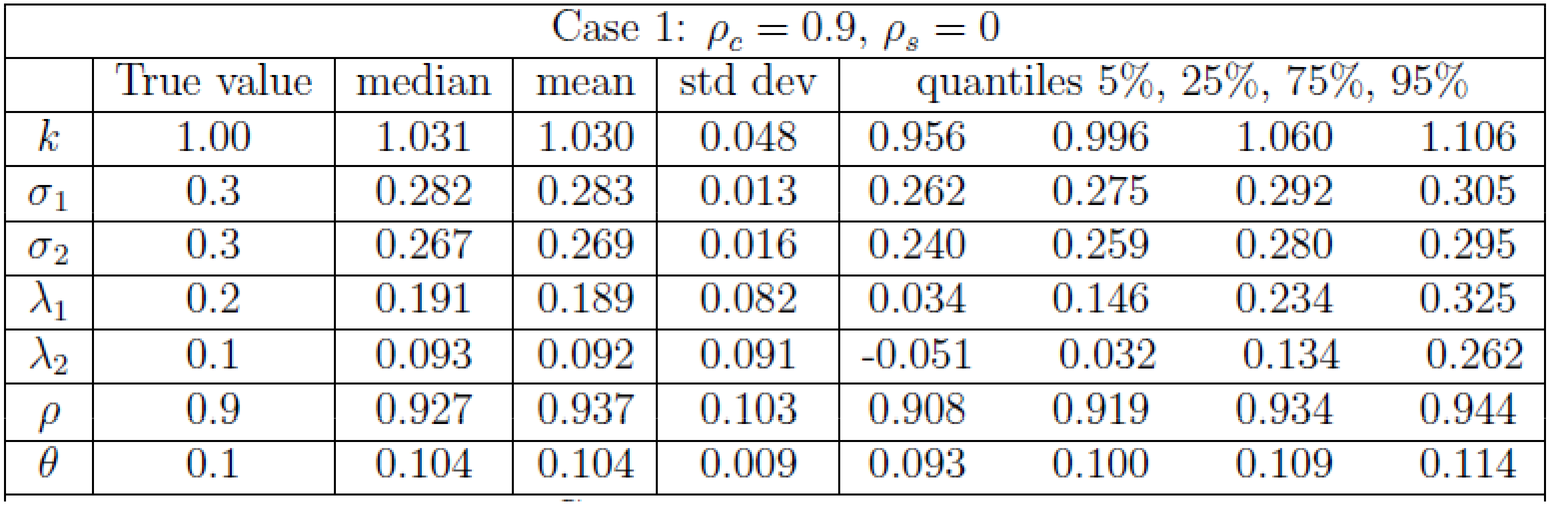
$$
\begin{aligned}
d\boldsymbol{v}_t &= (r - \delta_t - \frac{1}{2}\sigma_1^2 + \lambda_1)dt + \sigma_1 d\boldsymbol{W}_1^P \\
d\boldsymbol{\delta}_t &= k(\theta - \delta_t)dt + \sigma_2 d\boldsymbol{W}_2^P
\end{aligned}
$$

$$E[d\boldsymbol{W}_1^P d\boldsymbol{W}_2^P] = \rho dt,$$

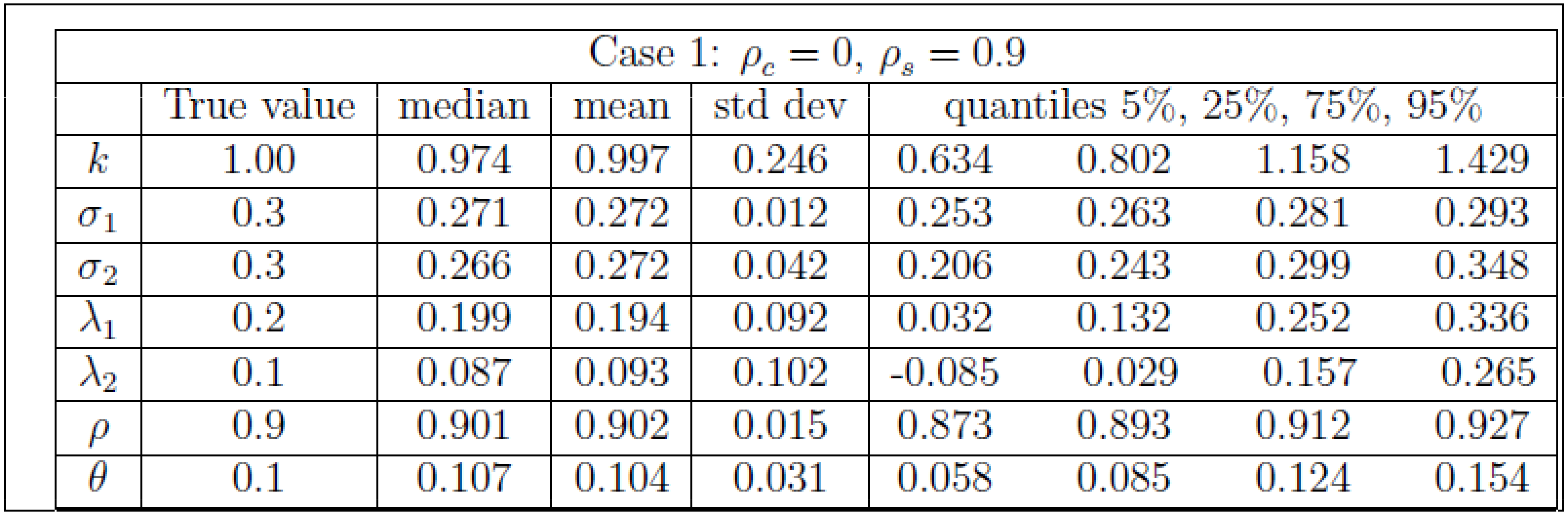
$$
\begin{aligned}
d\boldsymbol{v}_t &= (r - \delta_t - \frac{1}{2}\sigma_1^2)dt + \sigma_1 d\boldsymbol{W}_1^Q \\
d\boldsymbol{\delta}_t &= [k(\theta - \delta_t) - \lambda_2]dt + \sigma_2 d\boldsymbol{W}_2^Q
\end{aligned}
$$

$$E[d\boldsymbol{W}_1^Q d\boldsymbol{W}_2^Q] = \rho dt.$$

Simulation results for Groups 1 and 2

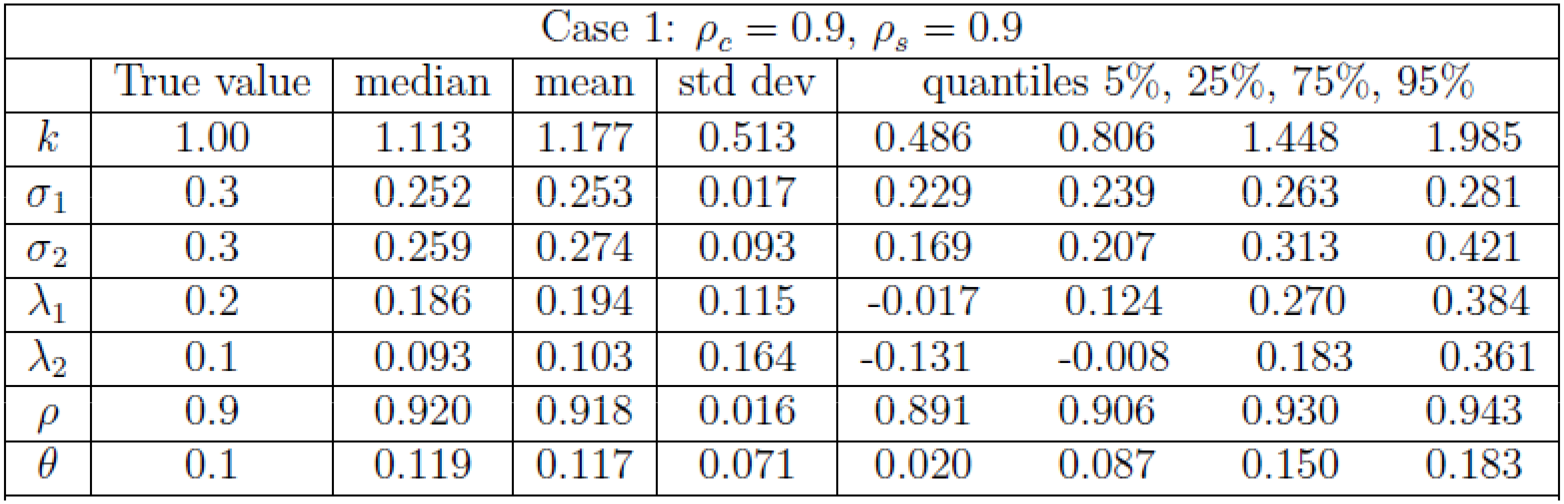


Case 1: $\rho_c = 0.9$, $\rho_s = 0$								
	True value	median	mean	std dev	quantiles 5%, 25%, 75%, 95%			
k	1.00	1.031	1.030	0.048	0.956	0.996	1.060	1.106
σ_1	0.3	0.282	0.283	0.013	0.262	0.275	0.292	0.305
σ_2	0.3	0.267	0.269	0.016	0.240	0.259	0.280	0.295
λ_1	0.2	0.191	0.189	0.082	0.034	0.146	0.234	0.325
λ_2	0.1	0.093	0.092	0.091	-0.051	0.032	0.134	0.262
ρ	0.9	0.927	0.937	0.103	0.908	0.919	0.934	0.944
θ	0.1	0.104	0.104	0.009	0.093	0.100	0.109	0.114



Case 1: $\rho_c = 0$, $\rho_s = 0.9$								
	True value	median	mean	std dev	quantiles 5%, 25%, 75%, 95%			
k	1.00	0.974	0.997	0.246	0.634	0.802	1.158	1.429
σ_1	0.3	0.271	0.272	0.012	0.253	0.263	0.281	0.293
σ_2	0.3	0.266	0.272	0.042	0.206	0.243	0.299	0.348
λ_1	0.2	0.199	0.194	0.092	0.032	0.132	0.252	0.336
λ_2	0.1	0.087	0.093	0.102	-0.085	0.029	0.157	0.265
ρ	0.9	0.901	0.902	0.015	0.873	0.893	0.912	0.927
θ	0.1	0.107	0.104	0.031	0.058	0.085	0.124	0.154

Simulation results for Group 3



Case 1: $\rho_c = 0.9$, $\rho_s = 0.9$								
	True value	median	mean	std dev	quantiles 5%, 25%, 75%, 95%			
k	1.00	1.113	1.177	0.513	0.486	0.806	1.448	1.985
σ_1	0.3	0.252	0.253	0.017	0.229	0.239	0.263	0.281
σ_2	0.3	0.259	0.274	0.093	0.169	0.207	0.313	0.421
λ_1	0.2	0.186	0.194	0.115	-0.017	0.124	0.270	0.384
λ_2	0.1	0.093	0.103	0.164	-0.131	-0.008	0.183	0.361
ρ	0.9	0.920	0.918	0.016	0.891	0.906	0.930	0.943
θ	0.1	0.119	0.117	0.071	0.020	0.087	0.150	0.183

The Augmented State Space Form

We define the new state vector X in terms of Y and ε, i.e. $X_t = \binom{Y_t}{\varepsilon_t}$

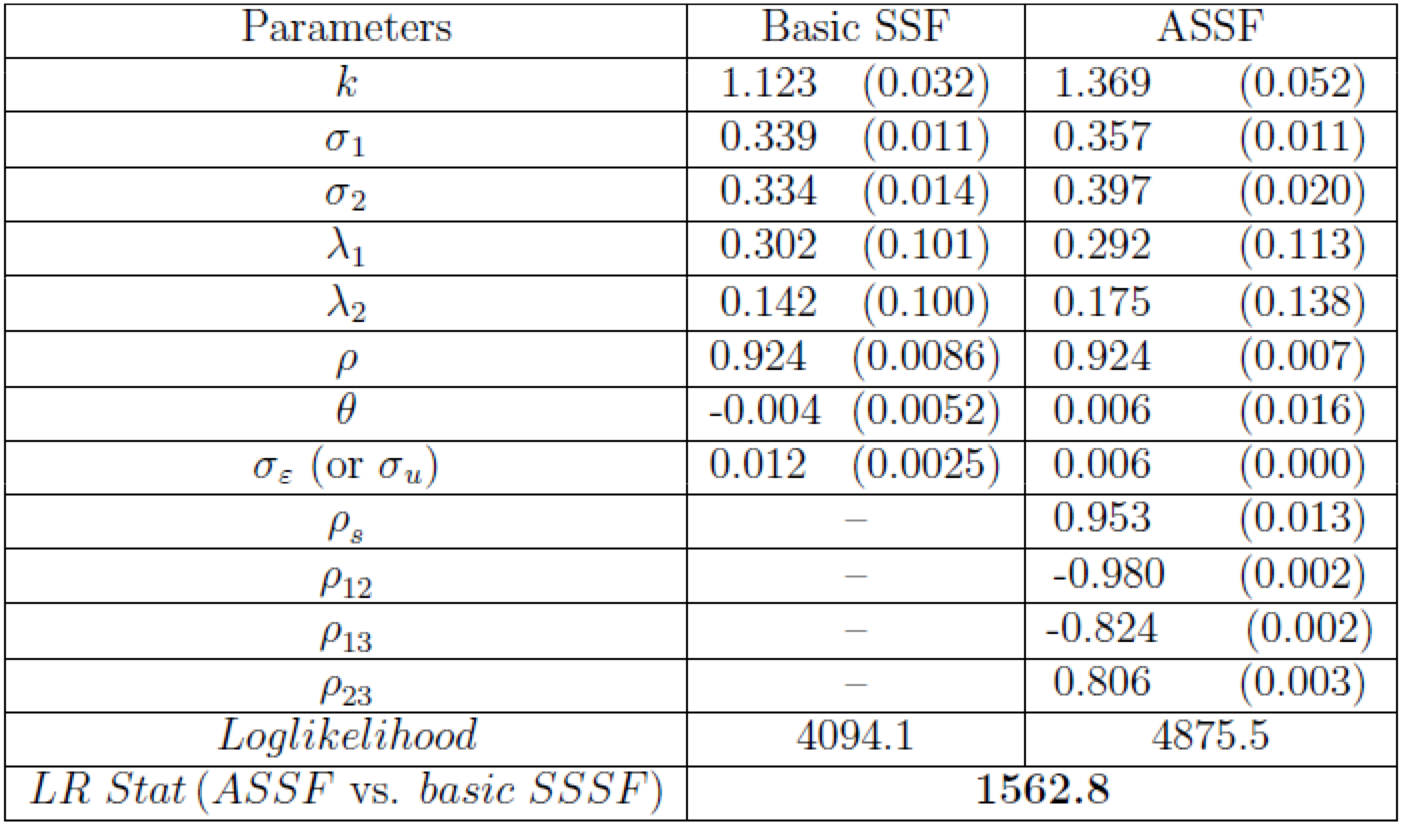
and assume that ε_t follows a first order vector auto-regressive process

$$\varepsilon_t = \Pi \varepsilon_{t-1} + u_t$$

Then the **augmented** state space form changes to

$$\begin{aligned} X_t &= f + G X_{t-1} + w_t \\ Z_t &= A + C X_t , \end{aligned}$$

Results for the 2-Factor Gibson-Schwartz Model for Commodity Data



Parameters	Basic SSF		ASSF	
k	1.123	(0.032)	1.369	(0.052)
σ_1	0.339	(0.011)	0.357	(0.011)
σ_2	0.334	(0.014)	0.397	(0.020)
λ_1	0.302	(0.101)	0.292	(0.113)
λ_2	0.142	(0.100)	0.175	(0.138)
ρ	0.924	(0.0086)	0.924	(0.007)
θ	-0.004	(0.0052)	0.006	(0.016)
σ_ε (or σ_u)	0.012	(0.0025)	0.006	(0.000)
ρ_s	–		0.953	(0.013)
ρ_{12}	–		-0.980	(0.002)
ρ_{13}	–		-0.824	(0.002)
ρ_{23}	–		0.806	(0.003)
Loglikelihood	4094.1		4875.5	
LR Stat (*ASSF* vs. *basic SSSF*)	**1562.8**			

Table 6: **Statistics of residuals $\widehat{\varepsilon}_t$ in the basic SSF and $\widehat{u}_t$ in the augmented SSF for the two factor commodity model**

	Covariance matrix			DW	Serial correlation	
Basic SSF						
1 month	1.00	-0.79	0.25	**0.92**	**0.54**	(0.034)
9 months	-0.79	1.00	-0.64	**0.22**	**0.89**	(0.018)
17 months	0.25	-0.64	1.00	**0.48**	**0.76**	(0.026)
ASSF						
1 month	1.00	-0.99	-0.91	2.04	-0.02	(0.040)
9 months	-0.99	1.00	0.90	2.03	-0.02	(0.041)
17 months	-0.91	0.90	1.00	2.06	-0.03	(0.040)

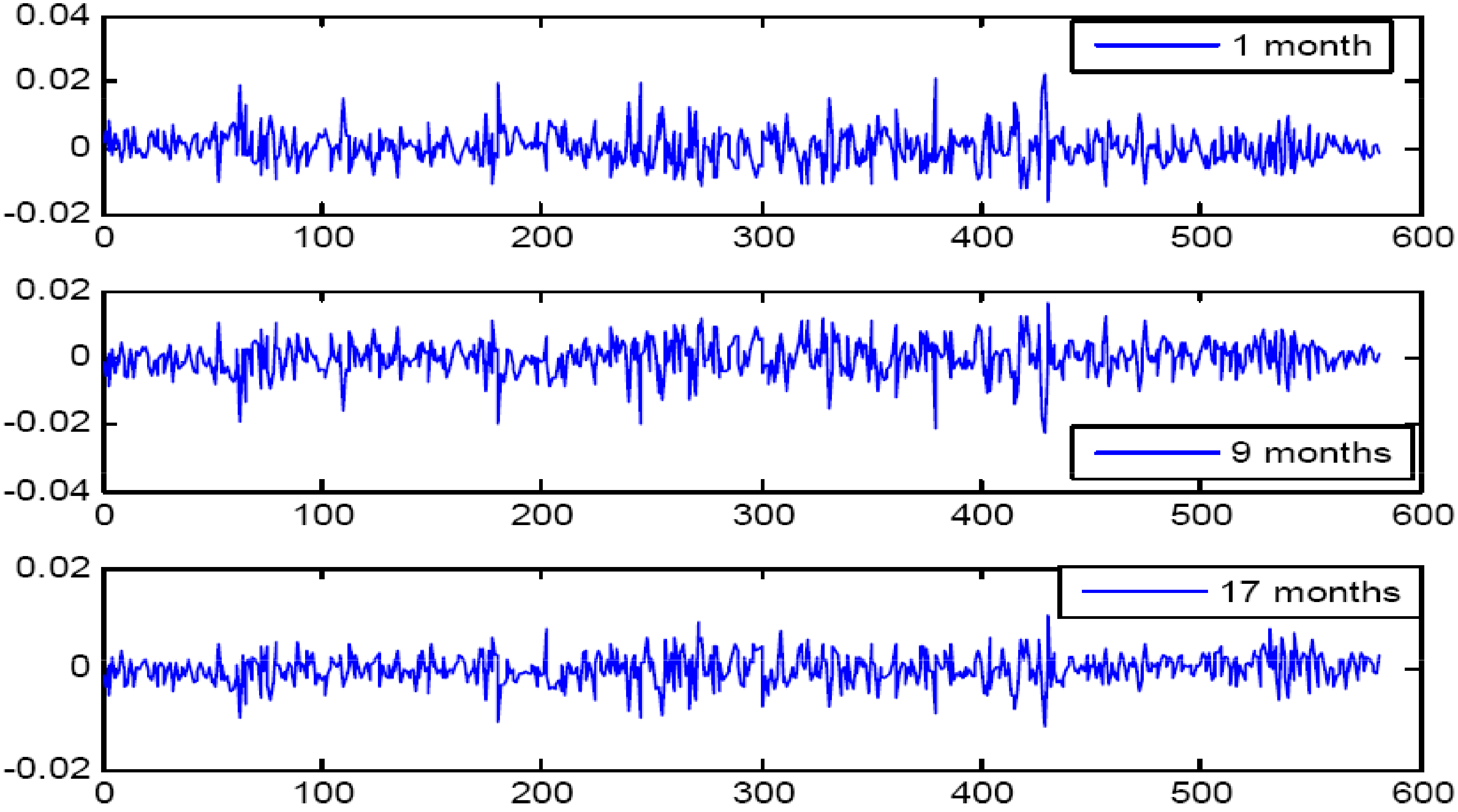


Figure 5: The measurement errors for the augmented state space form of the GS commodity model

3 Factor Commodity Model Dempster, Medova & Tang (2008)

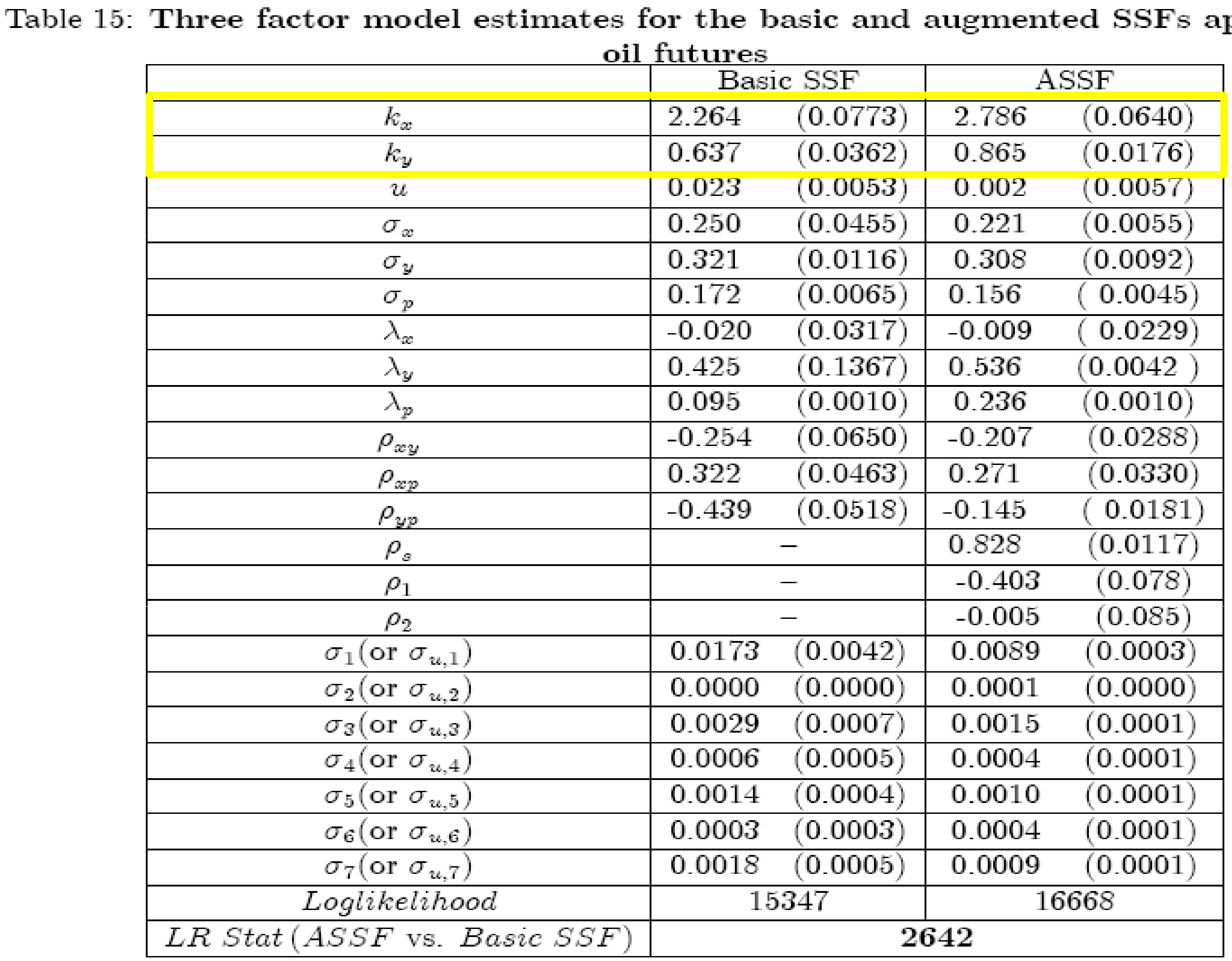


Table 15: **Three factor model estimates for the basic and augmented SSFs applied to oil futures**

	Basic SSF		ASSF	
k_x	2.264	(0.0773)	2.786	(0.0640)
k_y	0.637	(0.0362)	0.865	(0.0176)
u	0.023	(0.0053)	0.002	(0.0057)
σ_x	0.250	(0.0455)	0.221	(0.0055)
σ_y	0.321	(0.0116)	0.308	(0.0092)
σ_p	0.172	(0.0065)	0.156	(0.0045)
λ_x	-0.020	(0.0317)	-0.009	(0.0229)
λ_y	0.425	(0.1367)	0.536	(0.0042)
λ_p	0.095	(0.0010)	0.236	(0.0010)
ρ_{xy}	-0.254	(0.0650)	-0.207	(0.0288)
ρ_{xp}	0.322	(0.0463)	0.271	(0.0330)
ρ_{yp}	-0.439	(0.0518)	-0.145	(0.0181)
ρ_s	–		0.828	(0.0117)
ρ_1	–		-0.403	(0.078)
ρ_2	–		-0.005	(0.085)
σ_1(or $\sigma_{u,1}$)	0.0173	(0.0042)	0.0089	(0.0003)
σ_2(or $\sigma_{u,2}$)	0.0000	(0.0000)	0.0001	(0.0000)
σ_3(or $\sigma_{u,3}$)	0.0029	(0.0007)	0.0015	(0.0001)
σ_4(or $\sigma_{u,4}$)	0.0006	(0.0005)	0.0004	(0.0001)
σ_5(or $\sigma_{u,5}$)	0.0014	(0.0004)	0.0010	(0.0001)
σ_6(or $\sigma_{u,6}$)	0.0003	(0.0003)	0.0004	(0.0001)
σ_7(or $\sigma_{u,7}$)	0.0018	(0.0005)	0.0009	(0.0001)
Loglikelihood	15347		16668	
LR Stat (*ASSF* vs. *Basic SSF*)	**2642**			

3 Factor Generalized Vasicek Model Dempster, Medova & Villaverde 2010

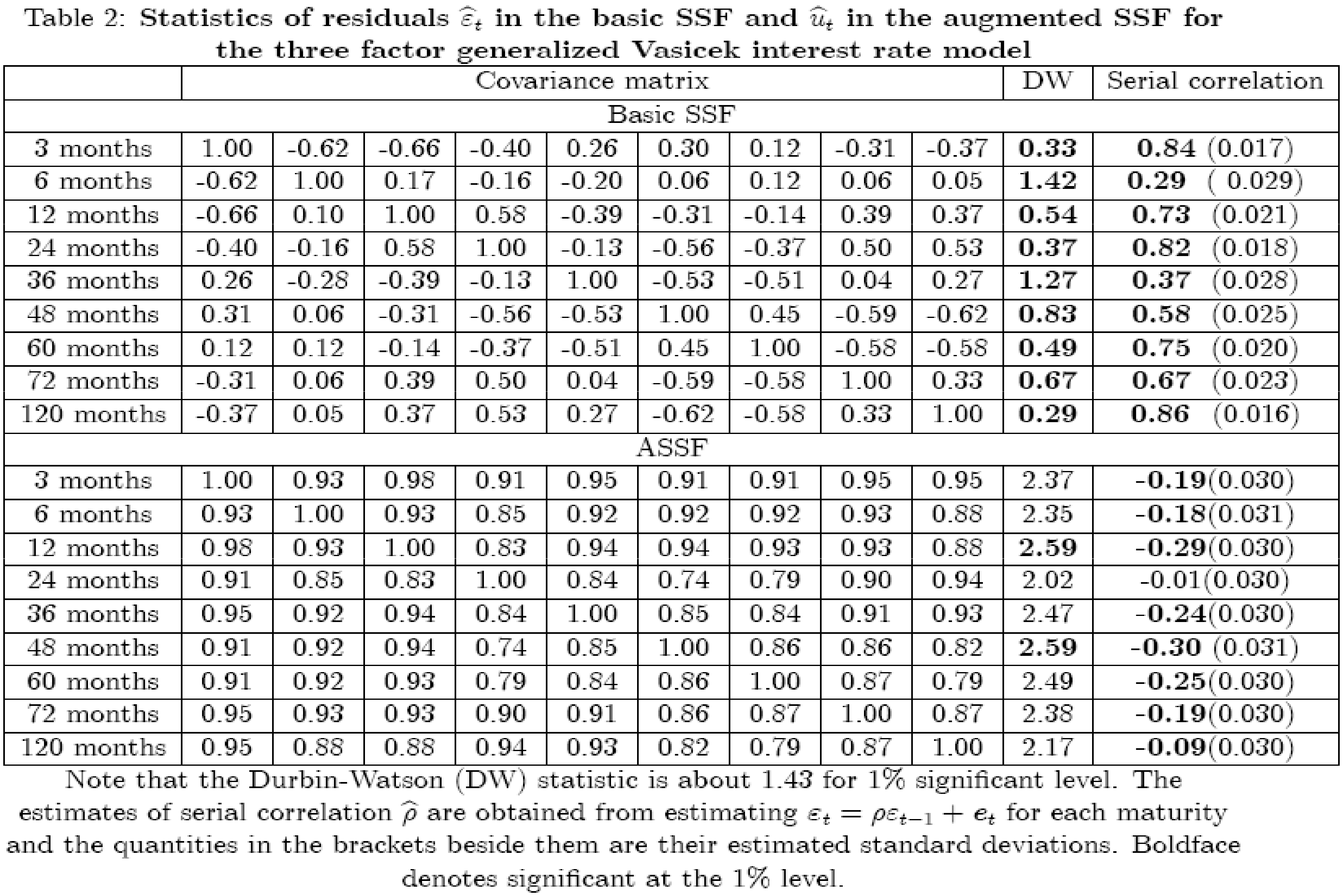


Table 2: Statistics of residuals $\widehat{\varepsilon}_t$ in the basic SSF and $\widehat{u}_t$ in the augmented SSF for the three factor generalized Vasicek interest rate model

	Covariance matrix									DW	Serial correlation
	Basic SSF										
3 months	1.00	-0.62	-0.66	-0.40	0.26	0.30	0.12	-0.31	-0.37	**0.33**	**0.84** (0.017)
6 months	-0.62	1.00	0.17	-0.16	-0.20	0.06	0.12	0.06	0.05	**1.42**	**0.29** (0.029)
12 months	-0.66	0.10	1.00	0.58	-0.39	-0.31	-0.14	0.39	0.37	**0.54**	**0.73** (0.021)
24 months	-0.40	-0.16	0.58	1.00	-0.13	-0.56	-0.37	0.50	0.53	**0.37**	**0.82** (0.018)
36 months	0.26	-0.28	-0.39	-0.13	1.00	-0.53	-0.51	0.04	0.27	**1.27**	**0.37** (0.028)
48 months	0.31	0.06	-0.31	-0.56	-0.53	1.00	0.45	-0.59	-0.62	**0.83**	**0.58** (0.025)
60 months	0.12	0.12	-0.14	-0.37	-0.51	0.45	1.00	-0.58	-0.58	**0.49**	**0.75** (0.020)
72 months	-0.31	0.06	0.39	0.50	0.04	-0.59	-0.58	1.00	0.33	**0.67**	**0.67** (0.023)
120 months	-0.37	0.05	0.37	0.53	0.27	-0.62	-0.58	0.33	1.00	**0.29**	**0.86** (0.016)
	ASSF										
3 months	1.00	0.93	0.98	0.91	0.95	0.91	0.91	0.95	0.95	2.37	**-0.19**(0.030)
6 months	0.93	1.00	0.93	0.85	0.92	0.92	0.92	0.93	0.88	2.35	**-0.18**(0.031)
12 months	0.98	0.93	1.00	0.83	0.94	0.94	0.93	0.93	0.88	**2.59**	**-0.29**(0.030)
24 months	0.91	0.85	0.83	1.00	0.84	0.74	0.79	0.90	0.94	2.02	-0.01(0.030)
36 months	0.95	0.92	0.94	0.84	1.00	0.85	0.84	0.91	0.93	2.47	**-0.24**(0.030)
48 months	0.91	0.92	0.94	0.74	0.85	1.00	0.86	0.86	0.82	**2.59**	**-0.30** (0.031)
60 months	0.91	0.92	0.93	0.79	0.84	0.86	1.00	0.87	0.79	2.49	**-0.25**(0.030)
72 months	0.95	0.93	0.93	0.90	0.91	0.86	0.87	1.00	0.87	2.38	**-0.19**(0.030)
120 months	0.95	0.88	0.88	0.94	0.93	0.82	0.79	0.87	1.00	2.17	**-0.09**(0.030)

Note that the Durbin-Watson (DW) statistic is about 1.43 for 1% significant level. The estimates of serial correlation $\widehat{\rho}$ are obtained from estimating $\varepsilon_t = \rho\varepsilon_{t-1} + e_t$ for each maturity and the quantities in the brackets beside them are their estimated standard deviations. Boldface denotes significant at the 1% level.

Table 16: **Three factor generalized Vasicek interest rate model estimates for the basic and augmented SSFs**

	Basic SSF	ASSF
k_s	0.8250 (0.0578)	1.3363 (0.1195)
k_l	0.0142 (0.0015)	0.0226 (0.0020)
k_r	1.1815 (0.1022)	0.7699 (0.0519)
σ_1	0.0210 (0.0018)	0.0308 (0.0033)
σ_2	0.0101 (0.0003)	0.0110 (0.0003)
σ_3	0.0104 (0.0003)	0.0077 (0.0002)
u_s	-0.1899 (0.0332)	0.0314 (0.0078)
u_l	0.0049 (0.0000)	0.0016 (0.0002)
γ_1	-0.4003 (0.2048)	-0.3542 (0.2054)
γ_2	-0.0580 (0.0788)	0.0386 (0.0961)
γ_3	-0.7120 (0.2219)	-0.8531 (0.2196)
ρ_{12}	0.2204 (0.0533)	0.2613 (0.0395)
ρ_{13}	-0.4080 (0.0323)	-0.1395 (0.0384)
ρ_{23}	0.0190 (0.0351)	0.2673 (0.0328)
ρ_s	–	0.7667 (0.0328)
ρ_1	–	0.9253 (0.0058)
ρ_2	–	0.5517 (0.0299)
ρ_3	–	0.8671 (0.0150)
ρ_4	–	0.5666 (0.0302)
σ_1(or $\sigma_{u,1}$)	0.00097 (0.00021)	0.00050 (0.00018)
σ_2(or $\sigma_{u,2}$)	0.00001 (0.00000)	0.00074 (0.00020)
σ_3(or $\sigma_{u,3}$)	0.00072 (0.00016)	0.00091 (0.00022)
σ_4(or $\sigma_{u,4}$)	0.00034 (0.00008)	0.00048 (0.00014)
σ_5(or $\sigma_{u,5}$)	0.00008 (0.00004)	0.00040 (0.00011)
σ_6(or $\sigma_{u,6}$)	0.00015 (0.00005)	0.00032 (0.00010)
σ_7(or $\sigma_{u,7}$)	0.00021 (0.00007)	0.00033 (0.00011)
σ_8(or $\sigma_{u,8}$)	0.00020 (0.00009)	0.00044 (0.00012)
σ_9(or $\sigma_{u,9}$)	0.00046 (0.00013)	0.00053 (0.00014)
Loglikelihood	57788	61454
LR Stat (ASSF vs. Basic SSF)	**7332**	

3 Factor Generalized CIR Model 1 Dai & Singleton (2000)

Table 3: Statistics of residuals $\widehat{\varepsilon}_t$ in the basic SSF and $\widehat{u}_t$ in the augmented SSF for the three factor $A_{1,DS}(3)$ interest rate model

	Covariance matrix									DW	Serial correlation
Basic SSF											
3 months	1.00	0.95	0.50	-0.50	-0.46	0.52	0.50	-0.03	-0.58	**0.14**	**0.93** (0.011)
6 months	0.95	1.00	0.69	-0.36	-0.59	0.47	0.49	0.10	-0.60	**0.26**	**0.87** (0.015)
12 months	0.50	0.69	1.00	0.19	-0.75	0.15	0.24	0.42	-0.37	**0.98**	**0.51** (0.026)
24 months	-0.50	-0.36	0.19	1.00	-0.29	-0.51	-0.41	0.58	0.31	**0.50**	**0.75** (0.020)
36 months	-0.46	-0.59	-0.75	-0.29	1.00	-0.52	-0.54	-0.16	0.51	**0.80**	**0.60** (0.024)
48 months	0.52	0.47	0.15	-0.51	-0.52	1.00	0.71	-0.59	-0.49	**0.72**	**0.64** (0.023)
60 months	0.50	0.49	0.24	-0.41	-0.54	0.71	1.00	-0.52	-0.57	**0.34**	**0.82** (0.017)
72 months	-0.03	0.10	0.42	0.58	-0.16	-0.59	-0.52	1.00	-0.17	**0.79**	**0.60** (0.024)
120 months	-0.58	-0.60	-0.37	0.31	0.51	-0.49	-0.57	-0.17	1.00	**0.19**	**0.90** (0.013)
ASSF											
3 months	1.00	0.90	0.73	-0.10	-0.12	0.12	0.21	0.41	0.38	2.23	-**0.12** (0.030)
6 months	0.90	1.00	0.88	-0.02	-0.06	0.16	0.25	0.46	0.44	2.46	-**0.23** (0.030)
12 months	0.73	0.88	1.00	0.09	0.06	0.19	0.29	0.46	0.45	**2.77**	-**0.39** (0.028)
24 months	-0.10	-0.02	0.09	1.00	0.84	0.85	0.73	0.76	0.82	2.25	-**0.13** (0.029)
36 months	-0.12	-0.06	0.06	0.84	1.00	0.78	0.58	0.65	0.73	2.44	-**0.22** (0.029)
48 months	0.12	0.16	0.19	0.85	0.78	1.00	0.43	0.65	0.78	2.37	-**0.18** (0.030)
60 months	0.21	0.25	0.29	0.73	0.58	0.43	1.00	0.81	0.85	2.42	-**0.21** (0.029)
72 months	0.41	0.46	0.46	0.76	0.65	0.65	0.81	1.00	0.85	2.37	-**0.19** (0.030)
120 months	0.38	0.44	0.45	0.82	0.73	0.78	0.85	0.85	1.00	2.24	-**0.12** (0.030)

Table 17: **Three factor $A_{1,DS}(3)$ interest rate model estimates for the basic and augmented SSFs**

	Basic SSF	ASSF
a	0.0802 (0.0006)	0.2595 (0.0348)
k	0.3540 (0.0042)	0.2132 (0.0312)
u	0.0831 (0.0020)	0.6003 (0.18353)
η	0.0410 (0.0011)	0.0035 (0.0007)
$\overline{v}$	0.0523 (0.0021)	0.0011 (0.0001)
$\overline{\theta}$	0.2941 (0.0039)	0.0780 (0.0016)
ζ	0.0183 (0.0042)	0.0280 (0.0037)
b	-0.0132 (0.0020)	0.0026 (0.0014)
σ_{rv}	0.1606 (0.0035)	-0.1254 (0.2181)
c	0.0073 (0.0032)	-0.1519 (0.0544)
λ_1	6.8730 (3.5396)	-1.0708 (3.7815)
λ_2	-1.4138 (0.7281)	-0.1604 (0.0655)
λ_3	0.2131 (0.0038)	0.2706 (0.1304)
ρ_s	–	0.9592 (0.0046)
ρ_1	–	0.8390 (0.0125)
ρ_2	–	0.1471 (0.0580)
ρ_3	–	0.6738 (0.0340)
ρ_4	–	0.3222 (0.0415)
σ_1(or $\sigma_{u,1}$)	0.00356 (0.00006)	0.00102 (0.00028)
σ_2(or $\sigma_{u,2}$)	0.00236 (0.00004)	0.00087 (0.00025)
σ_3(or $\sigma_{u,3}$)	0.00101 (0.00012)	0.00090 (0.00024)
σ_4(or $\sigma_{u,4}$)	0.00000 (0.00000)	0.00037 (0.00011)
σ_5(or $\sigma_{u,5}$)	0.00031 (0.00001)	0.00030 (0.00009)
σ_6(or $\sigma_{u,6}$)	0.00019 (0.00001)	0.00022 (0.00009)
σ_7(or $\sigma_{u,7}$)	0.00028 (0.00002)	0.00024 (0.00007)
σ_8(or $\sigma_{u,8}$)	0.00000 (0.0000)	0.00035 (0.00008)
σ_9(or $\sigma_{u,9}$)	0.00056 (0.00003)	0.00046 (0.00010)
Loglikelihood	52624	60448
LR Stat (*ASSF* vs. *Basic SSF*)	**14568**	

3 Factor Generalized CIR Model 2 Dai & Singleton (2000)

Table 4: Statistics of residuals $\widehat{\varepsilon}_t$ in the basic SSF and $\widehat{u}_t$ in the augmented SSF for the three factor $A_{2,DS}(3)$ interest rate model

	Covariance matrix									DW	Serial correlation
Basic SSF											
3 months	1.00	0.97	0.69	-0.18	-0.52	0.27	0.37	0.15	-0.43	**0.11**	**0.94** (0.010)
6 months	0.97	1.00	0.82	-0.04	-0.60	0.19	0.36	0.24	-0.45	**0.19**	**0.91** (0.013)
12 months	0.69	0.82	1.00	0.34	-0.66	-0.03	0.22	0.41	-0.39	**0.64**	**0.68** (0.022)
24 months	-0.18	-0.04	0.34	1.00	-0.31	-0.52	-0.27	0.52	0.02	**0.53**	**0.73** (0.021)
36 months	-0.52	-0.60	-0.66	-0.31	1.00	-0.48	-0.54	-0.17	0.54	**0.70**	**0.65** (0.023)
48 months	0.27	0.19	-0.03	-0.52	-0.48	1.00	0.46	-0.50	-0.23	**1.00**	**0.50** (0.026)
60 months	0.37	0.36	0.22	-0.27	-0.54	0.46	1.00	-0.42	-0.27	**0.51**	**0.73** (0.020)
72 months	0.15	0.24	0.41	0.52	-0.17	-0.50	-0.42	1.00	-0.59	**0.60**	**0.70** (0.022)
120 months	-0.43	-0.45	-0.39	0.02	0.54	-0.23	-0.27	-0.59	1.00	**0.33**	**0.84** (0.017)
ASSF											
3 months	1.00	0.95	0.81	0.37	0.53	0.39	0.29	0.31	0.21	2.16	**-0.08** (0.030)
6 months	0.95	1.00	0.91	0.39	0.54	0.39	0.30	0.37	0.26	2.38	**-0.19** (0.030)
12 months	0.81	0.91	1.00	0.34	0.48	0.32	0.30	0.41	0.33	**2.78**	**-0.29** (0.028)
24 months	0.37	0.39	0.34	1.00	0.91	0.63	0.54	0.49	0.30	**2.64**	**-0.32** (0.029)
36 months	0.53	0.54	0.48	0.91	1.00	0.88	0.41	0.22	0.05	**2.74**	**-0.37** (0.028)
48 months	0.39	0.39	0.32	0.63	0.88	1.00	0.12	-0.23	-0.34	2.32	**-0.31** (0.028)
60 months	0.29	0.30	0.30	0.54	0.41	0.12	1.00	0.41	0.26	**2.64**	**-0.32** (0.029)
72 months	0.31	0.37	0.41	0.49	0.22	-0.23	0.41	1.00	0.70	2.43	**-0.26** (0.029)
120 months	0.21	0.26	0.33	0.30	0.05	-0.34	0.26	0.70	1.00	2.18	**-0.09** (0.030)

Table 18: **Three factor $A_{2,DS}(3)$ interest rate model estimates for the basic and augmented SSFs**

	Basic SSF	ASSF
a	0.0224 (0.0005)	0.0353 (0.0006)
k	0.2776 (0.0092)	0.4094 (0.0070)
u	0.0429 (0.0040)	0.1458 (0.0601)
η	0.0225 (0.0001)	0.0913 (0.0052)
$\overline{v}$	0.0103 (0.0014)	0.0329 (0.0005)
$\overline{\theta}$	0.2297 (0.0099)	0.3158 (0.0003)
ζ	0.0474 (0.0009)	0.0322 (0.0013)
$k_{\theta v}$	0.1324 (0.0141)	0.1114 (0.0442)
k_{rv}	0.1298 (0.0211)	0.1379 (0.0236)
σ_{rv}	-0.0942 (0.0297)	0.0843 (0.0145)
λ_1	-0.1348 (3.3137)	-0.0011 (0.0010)
λ_2	0.2911 (0.7637)	-0.0067 (0.0014)
λ_3	0.0766 (0.2874)	-0.0018 (0.0013)
ρ_s	–	0.9292 (0.0161)
ρ_1	–	0.7197 (0.0040)
ρ_2	–	0.4014 (0.0902)
ρ_3	–	0.3107 (0.0034)
ρ_4	–	0.3016 (0.0046)
σ_1(or $\sigma_{u,1}$)	0.00387 (0.00082)	0.00259 (0.00016)
σ_2(or $\sigma_{u,2}$)	0.00265 (0.00057)	0.00188 (0.00012)
σ_3(or $\sigma_{u,3}$)	0.00129 (0.00029)	0.00120 (0.00023)
σ_4(or $\sigma_{u,4}$)	0.00027 (0.00006)	0.00029 (0.00002)
σ_5(or $\sigma_{u,5}$)	0.00009 (0.00004)	0.00000 (0.00000)
σ_6(or $\sigma_{u,6}$)	0.00014 (0.00003)	0.00017 (0.00002)
σ_7(or $\sigma_{u,7}$)	0.00021 (0.00005)	0.00019 (0.00001)
σ_8(or $\sigma_{u,8}$)	0.00009 (0.00004)	0.00026 (0.00001)
σ_9(or $\sigma_{u,9}$)	0.00039 (0.00008)	0.00040 (0.00004)
Loglikelihood	53155	57124
LR Stat (*ASSF* vs. *Basic SSF*)	**7938**	

Conclusion

- Investigating measurement errors in three factor yield curve models and one, two and three factor commodity futures models we find that residual measurement errors are not independently and identically distributed but rather show strong contemporaneous cross-sectional and serial correlations
- This is inconsistent with the usual iid assumptions in many studies employing EAMs in the literature
- When serial correlation exists the basic SSF estimation procedure performs very poorly – especially regarding the estimation of the parameters of mean-reverting processes
- We propose an augmented SSF to replace the original and employ the Kalman filter to estimate the ASSF parameters
- The empirical results with interest rate and commodity futures data demonstrate that the new ASSF performs much better than the original SSF in estimating EAM parameters
- Further experimentation with measurement error specification is both possible and needed